Public release copy. Source: `34a_E1R_protected_sector_note.md`, Gravity Frontier Research, David Vávra, Independent Researcher, Prague. SHA-256 of the Markdown source: `f71bc93afa68fb6fc3b09c48f1e5a8c31e8862aa3e5160ff49f947247ce2c449`. First published at vavradev.com/research on 2026-11-15. The Markdown file published beside this PDF is byte-identical to the source.

E1-R: the common-mode protected sector of the gravitational entangling gate. At the 2026 benchmark the protection does not pay (a fixed protected-sector witness against the best two-path witness, per setting; riders R5 and R11 of the gate travel with this line); a lemma with its proof, its closed forms, the certified cells, and the law each rung reads

The finding that leads (NOTE grade; not a discovery claim; ruled LICENSED as worded, and conservative, by the steelman leg [S], its item HS-0). Outcome word of record, as the gate [G] ruled it: **PROTECTION-DOES-NOT-PAY** at the 2026 white-paper benchmark (labelled "not a design of record"), under the white paper's stated noise allowance and for every registered split of that allowance into shared and independent noise: the three-path protected-sector witness needs 23.2 to 57.7 times more shots per setting than the two-path witness (truncated ratios of the counts of item 0.1). Rider R11 of [G], which travels with every printing of the word: it compares the fixed protected-sector witness against the best two-path witness, per setting, under RULE-SC; it is not a comparison of the best three-path certificate with the best two-path certificate at the stated allowance (NOT COMPUTED; the noise-free three-path negativity is 8.83 times the protected one). The companion prediction P-R4 of the order of record [O], a different sentence, is TRUE on its registered primary reading (sigma* = 3/5 rad) and is printed only with rider R5 of [G]: the registered three-path row spans 20 um, twice the white paper's superposition size, and at equal span (40, 5, 1) there is no certified positive at local coherence 49/50; the window is 3/5 to 205.6 rad at mismatch 1/1000 (certified here on the continuum up to 30 rad by monotonicity, and to 205.6 rad because the two-path state stays PPT); the count is 80 times the noise-free two-path count; at the stated allowance the word is the other one (precisions of the two adversarial legs, wherever this text is printed: its "noise-free two-path count" is the count at zero common-mode noise and local coherence 49/50 (defect D-3 of [R]; section 6.2), and its 205.6 rad is a number of the registered mismatch 1/1000, the pilot's value and not a design's (rider RW-3 of [S]; section 6.4), and its "80 times" is the count while the product of rms phase and mismatch is negligible: along the window the count grows with that product, to 225,523 at 30 rad, and without bound toward the end of the window (closing audit, item B-1 of Appendix D; section 6.2)). "Protected" carries the rider block of section 1 wherever it stands in this note. Details and numbers: item 0.1 and sections 6.1 and 6.2.

Final v1.1, 2026-11-14 (Final v1.0 of 2026-11-12 with the replacement texts of its closing audit: item 0.9 and Appendix D). In-house, experiment-facing note of program RELATIVE (order of record [O]: `33_dream_question_6.md`, sections 5 and 6, mandates M-R4 and M-R5; the program's report is doc 34). Assembled after the pilot [P], its verifier gate [G] (ruling PASS with eleven repairs, each proven verdict-neutral), the scout lanes [L1], [L2], [L34], the mediator column [M] and the control universe [C] with its verifier [CV], and finalized after the two adversarial legs that [O] owes on the note: a certificate-rigor leg [R] (ruling PASS WITH LISTED DEFECTS: 298 own checks, none failed; four wording defects, two of them with a numeric clause; no number of the draft without a source; Theorem R not broken) and an experimentalist steelman [S] (ruling PASS WITH REWORDINGS: of thirteen headline sentences 7

LICENSED, 5 REWORD-TO, 1 OVER-REACH; no cell wrong under the note's own premises; the leading finding stands and is conservative). **Reported first (item 0.8; the ledger is Appendix C):** two statements of the draft were false as worded and are replaced (the bound that a protected-sector null certifies was attached to the whole reach although it holds at matched spacing only; two "between" phrases built from truncated digits); one phrase misnamed its divisor ("80 times the noise-free two-path count"); and the advice at the end of the one-line summary said more than the record carries and is withdrawn. **No repair touches a verdict, an outcome word, the truth value of a prediction, the status of a kill criterion or a printed digit of a certified cell:** every one is a wording, a rider or a data row (Appendix C). **Grade: NOTE. Nothing in this note is a discovery claim, and no sentence of it may be read as one.** Theorem R below is a corollary of a three-line lemma and a polynomial identity, labelled so before any run (KR-0 of [O]: "a note, not a front"). **Status:** in-house, and final for the binding order of [O] section 6 (pilot, gate, lanes, assembly, two adversarial legs); the sentence of section 13 stands as amended by the two legs, whole with its riders; the closing audit of [O] at its untouched reserve was run on the amended whole on 2026-11-14 (record [K]; ruling PASS WITH LISTED DEFECTS; item 0.9), with a literature re-sweep at that date; the replacement texts that it supplied are entered here and have had no further reading, and a quotation event at a later date owes a fresh literature re-sweep at its date. Never posted by this program; licensing is never posting; release is the owner's call. Sibling of the E1 visibility note [E1] and of the E1-M note [E1-M]; an additive layer over the Witness Theorem [W], which it inherits unamended. Self-contained: no reading of the program documents is assumed. Every number of columns C and L below was regenerated for this note from the pilot's engine by an isolated, sha-verified copy (all 17 regenerated logs equal the logs of record line for line, at the assembly and again, from a fresh copy, at the finalization); every number that the finalization added was certified by an own interval or exact route before it entered (record [F]; Appendix C); every other number is copied digit for digit from a named file of record of the leg that owns it (Appendix A).

Companion documents: [W] `06_witness_theorem.md` (Theorems W and W', premises W1 to W6, Definitions 1 and 2); [E1] `05a_E1_visibility_note.md`; [E1-M] `21a_E1M_magic_note.md`; [T] `30a_exclusion_table.md`; [O] `33_dream_question_6.md`; [P] `relative_pilot/` (`BUILD_REPORT.md`, `PILOT_REPORT.md`, `PROOF_R_general_n.md`, `literature/LITERATURE.md`); [G] `relative_pilot/VERIFIER_RULING.md` with `relative_pilot/verifier_MR2/`; [L1] `relative_lanes/L1_prior_art.md`; [L2] `relative_lanes/L2_short_range.md`; [L34] `relative_lanes/L3L4_occupancy.md`; [M] `relative_mediator/MEDIATOR_COLUMN.md`; [C] `relative_control/CONTROL_REPORT.md`; [CV] `relative_control/verify/VERIFIER_REPORT.md`; [B] `relative_note/` (the build record of the draft); [R] `relative_verify/RIGOR_REPORT.md` with `relative_verify/rigor/` (the certificate-rigor leg of M-R5); [S] `relative_verify/STEELMAN_REPORT.md` with `relative_verify/steelman/` (the experimentalist steelman of M-R5); [F] `relative_note/finalize/` (the record of the finalization: registration, certifications, patcher, repairs ledger; the draft is kept there as the pre-image); [K] `owner_blocks/B34/` with its report `owner_blocks/B34/B34_REPORT.md` (the closing audit of [O] and the patcher of Final v1.1; the text of Final v1.0 is kept as `analysis/34a_E1R_protected_sector_note.v1.0.md`).

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0. Reported first

0.1 The finding that leads: at the 2026 benchmark the protection does not pay. Outcome word of record, as the gate ruled it: **PROTECTION-DOES-NOT-PAY** at the white paper's benchmark (labelled "not a design of record") under the white paper's stated noise allowance (a phase variance of 3/100 per interferometer), for every registered split of that allowance into noise shared by the two interferometers

and independent noise (correlation f from 0 to 1): the three-path protected-sector witness ("protected" as in the rider block of section 1) needs 23.2 to 57.7 times more shots per setting than the two-path witness (truncated ratios of the counts: 53,591 against 2,306 at f = 1; 136,907 against 2,370 at f = 0; 303 times with force-type independent noise). Typing (repair R4 (i) of [G]): decided at the sealing of RULE-B by hand item H-B2; counts COMPUTED. Rider R11 of [G], which travels with every printing of the word: it compares the fixed protected-sector witness against the best two-path witness, per setting, under RULE-SC; it is not a comparison of the best three-path certificate with the best two-path certificate at the stated allowance (NOT COMPUTED; the noise-free three-path negativity is 8.83 times the protected one). The judge of [O] had priced the other word at about 60 percent; a typed result of equal standing, and by the order of this assembly the finding that leads. The companion prediction P-R4 of [O], a different sentence, is TRUE beside it (sigma* = 3/5 rad) and is printed only with rider R5 of [G]: the registered three-path row spans 20 um, twice the white paper's superposition size, and at equal span (40, 5, 1) there is no certified positive at local coherence 49/50; the window is 3/5 to 205.6 rad at mismatch 1/1000 (certified here on the continuum up to 30 rad by monotonicity, and to 205.6 rad because the two-path state stays PPT); the count is 80 times the noise-free two-path count; at the stated allowance the word is the other one (precisions of the two adversarial legs, wherever this text is printed: its "noise-free two-path count" is the count at zero common-mode noise and local coherence 49/50 (defect D-3 of [R]; section 6.2), and its 205.6 rad is a number of the registered mismatch 1/1000, the pilot's value and not a design's (rider RW-3 of [S]; section 6.4), and its "80 times" is the count while the product of rms phase and mismatch is negligible: along the window the count grows with that product, to 225,523 at 30 rad, and without bound toward the end of the window (closing audit, item B-1 of Appendix D; section 6.2)) (section 6.2).

0.2 Refuted forecasts of the record, transcribed from the legs that own them (none refutes a prediction of record P-R1 to P-R7). Pilot [P]: F-X1 in two of four sub-clauses (a hand estimate of the two-path threshold at perfect local coherence: 3.028190 rad at mismatch 1/1000 and 2.270484 rad at mismatch 1/100, outside the forecast intervals); F-L in one conjunct (the judge's "0.0689263 at 20.18 um" is the value at a grid point, not a minimum: section 8.3). Gate [G]: none refuted. Lane [L1]: F-9 (a posting of the gravitational-entanglement literature does state an entanglement consequence of correlated noise for a path-encoded pair: item 0.3) and F-6 (no second printed criterion at local dimension three was found). Lane [L2]: F-3 in its first clause (the published sign-resolved limits NARROW the window: item 0.4), F-7 in its gloss, F-10, and two data forecasts of its own reading. Lane [L34]: F8 on the entry reading and FB3 in its first clause (item 0.3). Mediator column [M]: one regression of print (a rounding in arXiv:2607.20867; it enters no cell). Control universe [C]: one clause of its own registration (BR-1 on its cells (iii)); its verifier found the registered sentence U-1 false as worded and used the report's wording with its proviso (section 10.2). For [M] and [C] this item lists forecast records only; no sentence of column M is stated here. Rigor leg [R]: its forecast F-5 in one count clause (twenty short figures of the draft, not at most five, needed an arithmetic derivation; none was without a source). Steelman leg [S]: two hand values of its item H-ST4 (the tilt of section 4.3), one clause of its item H-ST10 (ill-posed, and refuted against the note's favour), its forecast F-ST6 in the first clause (it had expected a printed contrast loss from uniform acceleration noise; the cited text prints the opposite) and F-ST5 with a qualification. Finalize leg [F]: its forecast records are in Appendix C.

0.3 What is printed elsewhere (cells that become cited rows). Lanes [L1] and [L34], after the gate: Wang, Pedernales and Plenio, arXiv:2611.05044 v1 (2026-11-03), eq. (A31) of its Supplemental Material, print a closed-form concurrence for the two-path pair under local plus correlated dephasing. Under an exact dictionary the two-path entries of the pilot's cell C5 at matched spacing, and the pilot's hand item H-C1, are evaluations of that printed expression. This note certifies the identity by an own derivation and an own interval route (section 5.5; check A-2 of [B]), types those entries **CITED**, and types the outcome

word **SCOOPED-CELL for those entries only** (the entry reading of [L34], the reading least favourable to the program; on the group reading, where "cell" is the whole labelled cell C5 with its mismatch columns and three-path rows, it is not typed; both readings are the owner's to rule). **KR-1 of [O] is NOT FIRED by its letter**, as [L1] and [L34] ruled: no posting was found that states the path-encoded fact, Theorem R, its closed form at three paths or the two-against-three contrast (a net-grade and claim-grade absence over 102 fetched texts in the pilot's leg and [L1], and 18 further texts in [L34]: not a proof). Both halves of that contrast at two and three levels follow by substitution from one 2005 paper (Li and Xu, arXiv:quant-ph/0504228), and the note says so wherever it states them (sections 4.5 and 11).

0.4 The window of the law column is narrower for the registered sign. Lane [L2] read the sign-resolved limits of the 2020 torsion-balance measurement in the first author's dissertation (TABLE (thesis); the journal's Supplemental Material is NOT READ): for one Yukawa term of strength -4/3, the sign of the registered spin-2 term, the open ranges end at about 24.7584 um (between the printed rows at 22 and 25 um), not at 35.742 um, the pilot's reading of the modulus curve (CURVE-READ), and not at the 38.6 um that [O] used. No cell of the pilot is re-scored and no outcome word is re-ruled from this; every window sentence below names both windows (section 8.1).

0.5 A witness null is a statement modulo 2 pi (repair R1 of [G]). For one Yukawa term of large strength both witnesses can be exactly null while neither reading vanishes; the null rule and the former words "no blind point" carry the alias rider verbatim (section 8.5).

0.6 Corrections to numbers and sentences of the order of record, for the owner's fold (repair R9 of [G]; nothing applied to any committed document). "Never falls below 0.069" is 0.0688 (certified minimum 0.068865094 at 20.17795 um). The strength-4/3 edge is 35.742 um for the modulus curve (CURVE-READ), not 38.6 um; for the negative sign about 24.7584 um (TABLE (thesis); [L2]). The figures of row R21 of [T] at 52 um lie outside the cited single-term windows. Under P-R4, the sentence of [O] that equates the FALSE branch of that prediction with the negative outcome word does not match the outcome clause of [O] itself (the two sentences came out differently; riders R5 and R11: sections 6.1 and 6.2). "Theorem W applies verbatim" does not hold (repair R3: section 7). One digit (repair R7): the pilot's printed 18.1640474040 for the zero of the three-path ratio is a rounding of 18.16404740397856, not a truncation. From [L34], for the owner's block on [T]: the path-dephasing threshold r* = 0.855798920 of [O] section 8 is the zero of the printed eq. (A24) of arXiv:2611.05044 v1 and must be cited there, not claimed.

0.7 Defects and disclosures of the assembly leg (transcribed from the draft; "this leg" in this item is that leg; item (11) is updated). (1) One registered forecast of this leg was refuted and is reported first in Appendix B: check A-6 (lane [L2]'s float pointers do not all regress to their printed digits at the lane's four-decimal locators; the registered criterion was kept, and the rows marked (a) are this note's own enclosures). (2) A slip of the registration: it named lines 74 and 76 of [W] for two verbatim lines that are lines 75 and 77; corrected by a dated addendum before any script pulled them. (3) Four check-logic slips of the assembly-check script were found by two development runs in the scratchpad and repaired before its one run of record (a symbolic substitution that substituted nothing; an interval function that the library lacks, replaced by a bracketing argument for the same statement; a print format; a data comparison of truncations with roundings); the development logs and the superseded script are kept under `relative_note/dev/`. (4) Parts of the template of this note were drafted while the regeneration ran, before the run of record of the assembly checks; every number entered afterwards through placeholders, and the typing CITED of the two-path rows, which is conditional on check A-2, was drafted after A-2 had passed in a development run and before its run of record. (5) The gate's stage v7 (its audit of file times) cannot be repeated from an isolated copy and was NOT RUN here; its log of record was copied as data for stage v8. (6) Reading grade: the quotations of papers in this note were re-found by this leg on their

stated pages of the texts that the quoting lanes had left in the session scratchpad; no paper was read anew in full by this leg; `relative_control/DRAFT_RIDERS.md` was NOT READ (its riders are not entered here); of the co-runner's record one line of its verifier's report was read, for the KR-7 row of section 12, and nothing else. (7) The leg ran in two sittings (Appendix B): a usage limit cut the first off before the document checker had ever run. (8) The first development run of the document checker, in the second sitting, returned two DEFECT lines on the first sitting's draft: one paragraph printed P-R4 without naming rider R5, and five sections used the word "protected" without a pointer to its rider. Both were defects of the template and were repaired in the template, never in generated text. (9) Found at the resume by the author's own reading, not by a check, and registered in a second dated addendum before any repair: the first paragraph of the first sitting's draft did not state the leading finding and its title line named no rider (assembly rule AR-6 of [B]); the first version of the document checker passed "repair R10 is named" through the string H-R10 while the draft did not list that repair, and it accepted a looser rider test than its registered letter; six lines printed the outcome word, or the verdict of the companion prediction, with a pointer or a paraphrase where repairs R5 and R11 of [G] ask for the rider in the same sentence; hand-typed values of column C and rounded "about" figures stood in the prose against rule AR-1 of [B]; one clause about the co-runner had no named source; one sentence ("the same products to six digits") was false by its letter for one pair of thirteen. Each is repaired in the template or in a second version of a script; the first versions are kept under `relative_note/dev/`; three new document checks and a control of the checker by nineteen mutations were registered for them. (10) A precision made while applying those repairs (section 6.2; listed in a third dated addendum before the build of record): one sentence of [P] that [G] kept is exact only from the two-path threshold on; no verdict uses it. (11) When this list was written the draft had had no adversarial reading; the two adversarial legs have since read it (item 0.8).

0.8 What the two adversarial legs found in the draft, and what the finalization did (ledger: Appendix C). *Statements that were false as worded: removed and replaced.* (i) Section 7.2 attached the bound that a protected-sector null certifies ("protected" as in the rider block of section 1), the modulus of J below pi - 3 arccos(1/(2 lam^2)), to every product sigma times eps inside the reach; it holds at matched spacing only, and inside the reach the bound runs up to the Newtonian J itself (defect D-1 of [R]). (ii) Section 8.3 printed two "between" phrases built from truncated digits, each false as an interval for one of its two numbers (defect D-2 of [R]). *A misnomer.* (iii) "80 times the noise-free two-path count" divides by the two-path count at zero common-mode noise and local coherence 49/50; a pair with no noise at all needs fewer shots, and the flat three-path count is 127.5 times that (defect D-3 of [R]; the phrase is verbatim from the gate and keeps its place, with the precision beside it). *An over-reach.* (iv) The one-line summary advised keeping the three-path reading "as a fallback for shared noise and as a second, independent reading of the short-distance law": advice that the record does not carry (item HS-8 of [S]). It is withdrawn; the summary now says what the record carries. *Rewordings: true as drafted, but misleading for a lab reader or short of a rider (items HS-3, HS-5, HS-9, HS-10, HS-11 of [S]).* (v) "Shared" is not "common-mode": a shield, or any source midway between the registers, acts on them with opposite sign and is outside the protected class, and the shield that the benchmark presupposes was never named (the clarifications of section 1; rider RW-3 of section 6.4; section 11). (vi) The mismatch is one of force histories, not of masses, and 1/1000 is the pilot's value, not a design's (sections 1, 2 and 6.4). (vii) "A fallback for" read as available; what the cell shows is "would pay only for" (sections 2 and 6.1). (viii) The ladder of two readings exists inside the two-path rung already, at fewer shots (section 8.3). *A cosmetic defect.* (ix) The character map of section 7 was stated incompletely (defect D-4 of [R]). *Precisions and data rows* that neither change nor support a verdict are listed in Appendix C; one of them was found by the finalize leg's own reading (a null of the fixed witness bounds the modulus of J only when the witness is read with the phases of the state: section 7.2). *What did not move:* every verdict, every outcome word, every truth value of a prediction, every status of a kill criterion, every type and printed digit of a certified

cell, the 21 verbatim blocks, and Theorem R, which the rigor leg tried to break by its registered attempts and did not. *What the legs did not reproduce:* the rigor leg opened no paper, thesis or published curve and did not repeat the enumerations of the pilot and the gate; section 14 item 8 and Appendix B say what was regenerated at the finalization and what stands on its lane alone.

0.9 What the closing audit found, and what Final v1.1 changed (ledger: Appendix D; record [K]). The closing audit of [O] (block B34 of the owner's batch of 2026-11-12, run on 2026-11-14) read every sentence that the finalization had amended or added, the finalize leg's own item on the phases of the witness first; re-ran the certificates by own routes, with two certified primes of the closing reserve as rank pins; and re-swept the literature at its date. Ruling by its sealed rule: PASS WITH LISTED DEFECTS. Every printed number that it tested was reproduced; no verdict, outcome word, truth value of a prediction, status of a kill criterion, type or printed digit of a certified cell changes. *Sentences that said more than their certificates carry, replaced.* (B-1) Sentence S-R attached "80 times" to the whole window of P-R4; the count of the fixed witness is flat only while the product of rms phase and mismatch is negligible, and it grows without bound toward the end of the window (sections 6.2 and 13, and beside every printing of rider R5). (B-2) Six "between" phrases built from truncated digits were false as intervals for their larger end value, the class of defect D-2 of [R]; one of them had been added by the finalization in the paragraph that follows the repair of that defect (the lead, item 0.1, sections 2, 8.2, 8.3 and 13). (B-3) Section 12 said that the rigor leg had scored every prediction and every outcome word by its own routes; its report names three exceptions. (B-4) Section 7.1 printed a two-path figure "at the allowance" without its split. (B-5) Section 6.2 printed one count without its condition. (B-6) The one-line summary used the word "protected" without its rider ("protected" as in the rider block of section 1) and a hand-grade clause without its label. *Precisions.* The sign of the offset in the finalize leg's item on the phases of the witness (section 7.2); one sentence on truncated bounds, thresholds and ranges in the reading conventions of section 1. *A rider of a later date.* Row KR-7 of section 12 dates the co-runner's typing. *The re-sweep of 2026-11-14.* The six citer lists of mandate M-R9 are unchanged by value since 2026-11-12; arXiv:2611.05044 stands at its first version; no posting was found that states the path-encoded fact or a closed form for the negativity of the sector that complete common-mode dephasing spares (net grade: not a proof). *Not done by the closing audit:* of the sources it re-found the nine quotations that the finalization added on their stated pages and read two cited passages at reading grade, and it opened no other paper, thesis or curve; it did not re-derive the sizes of column M other than the census, the enumerations of the pilot and the gate, the alias typings beyond the alias of least strength, the zero of the parallel reading or the two committed anchors that have no stated formula here; the hand estimates stay HAND. The texts of this item and of Appendix D have had no further reading.

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1. Standing wording (it travels into every sentence below)

Never, in any sentence of this note or quoted from it: "gravity is quantum"; "gravity is classical"; "immune to noise"; "decoherence-free" bare for the pair; "the experiment tests quadratic gravity" bare; "loophole-free" bare; "undecidable"; "gravity is a magic factory"; "QM derived"; "requires QM"; "cannot write history and keep perfect time" bare; "no nonnegative model exists"; "not simulable"; "efficient" for any classical model. No sentence about the protected sector is a sentence about the mediator.

"Protected" always means, in every section of this note (the rider block): against COMPLETE common-mode dephasing; in the path-diagonal model (equally spaced branches with one spacing history in both interferometers, closed at readout; noise entering as a force uniform over both masses, hence a phase linear in the branch index; gradient noise and imperfect closure are outside the model); at

matched mass times spacing; and nothing against local dephasing (the protected state needs MORE local coherence than the unprotected benchmark, not less).

Three clarifications for the bench (text RW-9 of the steelman leg [S], verbatim; hand grade for the physics; the algebra behind them is stated after them and was checked exactly, record [F]). (i) Common-mode means generated by the SUM of the two path positions, that is a force uniform over both masses. Noise that the two registers merely share is not thereby common-mode: a shield or any source midway between them acts on the two registers with opposite sign (printed for the two-path pair by Bulling, Steiner, Pedernales and Plenio, arXiv:2604.22593) and is outside the protected class; a shared field is inside it only for registers that are translated copies of each other. (ii) In a Stern-Gerlach interferometer 'matched mass times spacing' is a matching of the two spin-force histories (gradient, spin projection, timing); the masses drop out. (iii) A three-path register whose middle path is the unforced spin state gives up the symmetry between the two forced paths on which the two-path loop relies (kinetic phase, zero-field splitting, closure in a curved field); none of that is in the model. The algebra behind (i): a generator of the form f(k) + f(l) has equal eigenvalues on |k l> and |l k>, so no noise of that form, of any strength and with any number of independent generators, touches the block d = 0 of the partial transpose, which is built from the coherences between |k l> and |l k> alone (for identical, translated registers the premise "equally spaced, one spacing history" can therefore be weakened for that block); a generator v(2 - k) - v(l), a source midway between the registers, leaves the outer coherence of that block alone and dephases the two inner ones through the second difference v(0) - 2 v(1) + v(2); when that dephasing too is complete, on top of the complete common-mode dephasing, the state is PPT (the margin under a partial one is in section 5.3). The printed sentence behind (i): [2604.22593v1 p9] "The previous case of shield variations can actually be expressed by using the same fluctuations in a" correlated way, with opposite signs on the two masses. Behind (ii): [2007.15029v2 p6] "phase fluctuations in Eq. (30) are completely independent of the particle mass". Behind (iii): the hand check of section 3.

The ceiling riders of the order of record, verbatim:

> **Ceiling riders, fused from day one.** NOTE grade: no sentence of this program claims a discovery; Theorem R is a corollary of a three-line lemma and an integer identity (computed to n = 8 by the judge; general n is a derivation target) and is labelled so before the run. "Protected" always travels with: against complete common-mode dephasing; in the path-diagonal model (equally spaced branches with one spacing history in both interferometers, closed at readout; noise entering as a force uniform over both masses, hence a phase linear in the branch index; gradient noise and imperfect closure are outside the model); matched mass times spacing; nothing against local dephasing (the protected state needs more local coherence than the unprotected benchmark, not less). The principle (the interaction depends on the relative position alone, so noise on the common mode can be rejected) is printed for the Gaussian setting (Datta and Miao, arXiv:2104.04414): cited first, never claimed; the house's statement is the path-encoded one. A positive certifies "non-classical" in Definition 2's sense and Theorem W's disjunction, never "quantum" and never "gravity is quantum"; a null never feeds "gravity is classical"; a protected-sector reading is a statement about a channel and its detection, never about the mediator. The law column is static and tree-level; torsion-balance bounds are cited, never re-derived (doc 29 section 4.2 (v)); the benchmark is the white paper's representative point; "the experiment tests quadratic gravity" is never used bare; Matsui's trilemma (2610.25994) is cited, never re-derived. "Loophole-free" never bare; "undecidable" never; licensing is never posting; release is the owner's call.

Its wording-ban criterion and its standing bans, verbatim:

> - **KR-5 (wording bans).** Any drift toward "gravity is quantum", "gravity is classical", "immune to noise", "decoherence-free" bare for the pair, "the experiment tests quadratic gravity" bare, "loophole-free" bare, or a protected-sector sentence about the mediator: stop the sentence at the licensed wording.

> *Every claim in this document inherits the corpus's standing wording bans: "undecidable", "gravity is a magic factory", "QM derived", "requires QM", the bare "cannot write history and keep perfect time", "no nonnegative model exists", "not simulable", "loophole-free" bare, "efficient" for any classical model. A null anywhere never feeds "gravity is classical"; a positive never feeds "gravity is quantum" (Theorem W's disjunction is the licensed form); a hosted row never feeds "classical" and an outside row never feeds a cost or advantage sentence; no 2+1 sentence is a sentence about 3+1 gravity; the margin theorem is never a gravity sentence. Licensing is never posting; release is the owner's call.*

Reading conventions of this note. Types (one per cell): **COMPUTED** (an exact or interval-certified number of the pilot's own engine, regenerated here); **COROLLARY** (it follows from Theorem R, from a lemma or from a polynomial identity; never a finding); **CITED** (a printed number or formula, or a transcription of a printed curve or table; never re-derived); **SATURATED** (certified below the resolution registered for it). Status words that are not types: NOT DECIDED, NOT RUN, NOT BUILT, NOT CHECKED, NOT DERIVED, NOT COMPUTED. Labels of the cited short-range data, as lane [L2] defines them: PRINTED, CURVE-READ (an own transcription of the vector graphics of a published figure), TABLE (thesis), LOCATOR (a float interpolation between printed points; its certificate is the two bracketing printed rows). HAND marks a paper derivation that no script of record tested. Decimals are printed as prefixes of the digits the logs print (the engine's logs round half up at their last printed digit; a shortened copy is a truncation of that; nothing is re-rounded here). A truncated decimal that stands as a one-sided bound or as a threshold (after "below", "above", "at most", "only while") is the number whose digits begin as printed: it is exact to less than one unit of its last digit and is not rounded toward the safe side; a range of such decimals printed "a to b" names the truncated digits of its two end values and is not an enclosing interval (closing audit, items B-2 and R-2 of Appendix D). "The gate" is the pilot verifier's ruling [G]; "repair Rn" is repair n of its section 5. "Defect D-n" and "remark R-x of [R]", and "item HS-n", "text RW-n" and "defect Dn of [S]", are the items of the two adversarial legs. A row typed HAND with the steelman leg as its source is a hand estimate of [S] on stated premises, re-derived for this note by own arithmetic from those premises and from no measurement: it is data and re-scores no cell.

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2. The question and the answers, in one page

The question (the dream form of [O], section 5). Gravity acts on two masses only through their relative position, and a uniform disturbance falls on both alike. Which part of gravity's entangling action survives a noise that the two interferometers share, which law of gravity does that part read near thirty micrometres, and does it certify what the first experiment was built to certify?

The answers, in the order of their weight. "Protected" carries the rider block of section 1 in every item.

1. **At the benchmark the protection does not pay** (item 0.1; section 6.1). Outcome word PROTECTION-DOES-NOT-PAY, with rider R11 of [G]: the comparison is the fixed protected-sector witness against the best two-path witness, per setting, under RULE-SC; it is not a comparison of the best three-path certificate with the best two-path certificate at the stated allowance (NOT COMPUTED; the noise-free three-path negativity is 8.83 times the protected one). Under the registered split the three-path design would pay only for an experiment whose uniform shared noise misses the white paper's allowance by more than an order of magnitude in variance while its

independent noise meets it, with a 20 um register and force histories matched at the registered 1/1000; it is not a way to relax the white paper's acceleration-noise requirement by the eight orders that separate it from tabletop isolation. (text RW-10 of [S]).

2. **The lemma (Theorem R; section 4).** In the path-diagonal model, after complete common-mode dephasing the state of two path registers is separable iff it is PPT iff the phase table of the gate is LU-trivial up to sector constants iff every generator of an explicit invariant lattice vanishes modulo 2 pi. With at least three paths in each register "LU-trivial up to sector constants" is "quadratic up to sector constants". Consequences: a two-path register against any register keeps nothing in the complete limit; three paths in the linear geometry keep an entangled state governed by ONE third-difference invariant J, unless J vanishes modulo 2 pi; the parallel geometry needs four paths. No bound entanglement in this gate family. A corollary of a three-line lemma and a polynomial identity, proved for every number of paths (the proof was checked line by line by the gate), computed by two legs to 12 paths (to 14 by the gate's lattice code). At two and three paths the statement follows from the Peres-Horodecki blocks printed by Li and Xu (2005) for two qubits and two spin-1 systems under collective dephasing, evaluated on the gate state; the note adds the converse, the closed form with the trace, every size, and the invariant lattice (the licensed form of [L1]).
3. **The price of the protected sector (sections 5 and 6).** Closed forms: protected negativity N_3(J) = (2 cos((pi - |J|)/3) - 1)/9; the protected three-path state with uniform amplitudes is entangled iff the local coherence of each register exceeds (2 cos((pi - |J|)/3))^(-1/2), for every J (a lemma of the gate). At the benchmark that threshold is 0.9597 against 0.9002 for the unprotected two-path pair: the protected state needs MORE local coherence. The noise-free three-path negativity is 8.83 times the protected one. A mismatch eps of the two spin-force histories (the masses drop out: clarification (ii) of section 1) limits the tolerable common-mode rms phase to about 0.28/eps rad for the fixed witness: two to two and a half orders of tolerable phase at the registered eps = 1/1000 (the pilot's value, not a design's), one to one and a half orders at eps = 1/100, the size that the steelman leg's hand estimates give for a shield midway or for a field whose gradient varies over a few millimetres (section 6.4); never eight.
4. **The law (section 8).** The two rungs read different functionals of a static, tree-level potential: two paths read the second difference, three paths the third-difference invariant J. On the scalar-decoupled Stelle member their ratios to the Newtonian values vanish at ranges 22.4514 um and 18.1640 um; the larger modulus of the two never falls below 0.0688; for one Yukawa term of any strength the two ratio functions have no common zero at any finite range (ten polynomial identities), and neither have any two different two-path sub-registers of one apparatus (fifteen identities of the same kind: the ladder exists inside the two-path rung already; section 8.3). LADDER-INDEPENDENT, a corollary; with the alias rider: witness nulls are statements modulo 2 pi. At the benchmark the two-path reading is of order one inside the window that the cited torsion-balance limits leave open (from 0.94 of Newtonian at the lower end of the drawn curve down to -0.20 at the modulus-curve edge, in truncated digits; down to about -0.055 at the sign-resolved edge); at the committed geometry of record the law column is SATURATED (it reads nothing).
5. **What a result on the protected sector certifies (section 7).** A certified positive certifies "non-classical" in the sense of Definition 2 of [W] and the disjunction of Theorem W, once premises W1 to W6 and clauses 2 and 3 of Definition 1 are re-established for a three-path register at the benchmark; it certifies nothing about the mediator beyond that disjunction. A null never transfers Theorem W' verbatim: inside the model it certifies only a bound on |J|, which weakens with the mismatch and needs the witness read with the phases of the state (section 7.2); outside the model it certifies nothing. A null of either kind never feeds a classicality sentence about gravity.
6. **Column M (section 10), assembled, never headline:** what stays invisible to matter, with its sizes, and a control universe in which the experiment is exactly null.

Why this is a note and not a front. The theorem was decided at the freeze of [O] by a three-line lemma; the principle behind it (the interaction depends on the relative position alone, so noise on the common mode can be rejected) is printed for the Gaussian setting by Datta and Miao, arXiv:2104.04414, cited first and never claimed: [2104.04414v2 p1] "This is because the gravitational interaction depends solely on the relative position of the two masses." and [2104.04414v2 p2] "a significant part of the seismic noise acts on the common mode of the masses without affecting the differential mode". The house's statement is the path-encoded one. Every cell closed as a closed form, a certified number under a registered convention, or a cited number (the gate keeps this status line and labels it wider than KR-0's letter, "a closed form or a cited number": repair R6).

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3. Setup: registers, gate, noise, design points

Registers and gate. Register A has paths k = 0..n_A-1, register B has paths l = 0..n_B-1 (the family of [O] has n_A = n_B = n). A product input with nonzero amplitudes a_k b_l passes a diagonal gate exp(i phi(k,l)): Psi(k,l) = a_k b_l exp(i phi(k,l)). In the linear geometry (both registers along one line, equal spacing s, distance D between equal-index paths) phi(k,l) = g(l - k) with g(m) = (G m_A m_B tau / hbar) U(D + m s); in the parallel geometry phi(k,l) = h(|l - k|) with h(r) = (G m_A m_B tau / hbar) U(sqrt(D^2 + r^2 s^2)). U is a static potential of a registered family: 1/r; one Yukawa term, (1/r)(1 + alpha exp(-r/lambda)); the Stelle form (1/r)(1 - (4/3) exp(-r/l2) + (1/3) exp(-r/l0)). For Newton and m_A = m_B = 10^-14 kg the pin is G m^2/hbar = 633/10 um rad/s (the E1 pin of the record [E1], quoted, never re-derived), so g(m) = (633/10) tau/(D + m s) with lengths in um and tau in s.

The path-diagonal model (the rider of every "protected": the rider block of section 1). Equally spaced branches with one spacing history in both interferometers, closed at readout; the gate diagonal in the path labels; noise entering as a force uniform over both masses, hence a phase linear in the branch index, theta (mu_A k + mu_B l) with mu = mass times spacing. Gradient noise, imperfect closure and unequal spacing histories are OUTSIDE the model and nothing below speaks about them.

The noise rule of every cell (RULE-N of [P], sealed before its engine existed). Uniform amplitudes. The coherence between (k,l) and (k',l') is multiplied by exp(-sigma^2 (Dk + (1 + eps) Dl)^2 / 2) lam^[k != k'] lam^[l != l'], with Dk = k - k', Dl = l - l'; sigma is the rms common-mode phase per unit of path index, eps the mismatch of mass times spacing between the registers, lam the local coherence of one register (independent local dephasing). **Complete common-mode dephasing** (the twirl) is defined at eps = 0: only the coherences with Dk + Dl = 0 are kept, i.e. the state is projected onto the sectors k + l = S: rho_tw = sum_S P_S |Psi><Psi| P_S. The twirl is a mixture of correlated local unitaries, never a postselection (a single postselected sector of a product state is entangled: check S2-h5 of [P]).

Vocabulary guard. In this note "collective" or "common-mode" dephasing always means dephasing generated by the SUM of the two path-index operators, k + l, written out as above. A part of the two-qutrit literature uses the same words for another map (damping pattern diag(gamma, 1, 1, 1, gamma, 1, 1, 1, gamma): Jaeger and Ann; Ali; Zhang, Ali and Ma), under which bound-entangled states are known; none of its results is imported here, and "no bound entanglement" below is a property of this gate family (product input, nonzero amplitudes, diagonal gate, complete twirl over k + l), never of dephased two-qutrit states in general ([L1], [G]). In arXiv:2609.10697 "common" and "differential" name the mean and the difference of two independent local rates: not the generator used here. In Wu, arXiv:2406.10832, "common mode" names the two arms of ONE interferometer: [2406.10832v4 p1] "it does not lead to contrast loss or position localisation decoherence due to common mode cancellation" (a uniform acceleration noise dephases; it does not open the loop). Again not the generator used here (defect D11 of [S]).

Design points (all linear unless stated; (D, s, tau) in um, um, s; exact rationals from the pin).

design	role	invariant
two paths (450, 250, 5/2)	the committed geometry of record (Bose et al. design; [E1], [W])	Delta = -211/672 (second difference)
two paths (40, 10, 1)	the 2026 white-paper benchmark, **not a design of record** (closest approach 30 um, superposition size 10 um)	Delta = -211/1000
three paths (50, 10, 1)	the registered three-path benchmark row, **not a design of record**; closest approach 30 um; it spans 20 um, twice the white paper's superposition size	J = 211/1400
three paths (60, 15, 1)	data row (the experiment scout's second row)	J = 211/750
three paths (40, 5, 1)	data row, own extension of [P]: the benchmark at EQUAL span (10 um) and equal closest approach	J = 211/5250
four paths, parallel, (50, 10, 1)	data row	J_par = -0.025267927205 rad
three paths (700, 250, 5/2) and (450, 125, 5/2)	data rows, own extensions of the committed geometry (equal spacing, respectively equal span; closest approach 200 um)	J = 1055/3648 and J = 9495/66976

Here Delta = phi(0,0) + phi(1,1) - phi(0,1) - phi(1,0) = 2 g(0) - g(-1) - g(1) and J = [g(-2) - g(2)] - 2 [g(-1) - g(1)]. The white paper's benchmark sentence: [2610.07422v1 p7] "For the benchmark used below, we choose" a superposition size of 10 um and a centre distance of 40 um, with m = 10^-14 kg and tau = 1 s on its next page. Wording item (b) of [O]: J is "about" its third-derivative term -2 s^3 V''' tau/hbar only in the sense that the exact Newtonian invariant at s/D = 1/5 is 625/504 = 1.24007 times that term.

Platform sentence (NOT CHECKED as a three-path scheme). Lane [P]-literature checked "a spin-1 centre gives three equally spaced paths" only as a composition of two printed facts: the NV centre is a spin-1 system with a Stern-Gerlach force linear in S_z (Wan et al., arXiv:1511.02738), and [2312.05170v1 p11] "An arbitrary spin embedded in a mesoscopic mass undergoes splitting in 2j + 1 paths when a magnetic gradient is applied". No printed NV protocol populates the middle state as a third path; every scheme read uses two spin states. The word "native" is not licensed. Checked since at hand grade only (the steelman leg [S], its section 4.1; typed HAND: no script of record tested the physics, and the arithmetic was re-derived for this note, record [F]). The white paper's register is the pair of the +1 and -1 spin states ([2610.07422v1 p5] "A subsequent microwave spin flip reverses the acceleration"); a spin-1 three-path register adds the unforced state 0 as its middle path. Three things then stop cancelling, all on the inner pairs of paths: (a) the kinetic phase of a forced path against the unforced one, 8 m s^2/(3 hbar T) for a bang-bang loop, that is 2.528 x 10^10 rad at (10^-14 kg, 10 um, 1 s); (b) the zero-field splitting, 1.803 x 10^10 rad in 1 s at 2.87 GHz, whose printed slope near room temperature (-74.2(7) kHz/K between 280 and 330 K, as [S] read it in the abstract of arXiv:0911.3938) makes 0.1 rad a matter of 2.14 x 10^-7 K (at the cryogenic temperatures that the designs assume the slope is smaller: NOT READ); (c) closure in a curved field, where the two forced paths close on each other to first order and not on the unforced one. For the two-path loop on the +1 and -1 states the amplitude noise of the gradient cancels by symmetry, in print: [2406.10832v4 p11] "field noise does not cause dephasing effect via the spatial path integral phase" (Wu, for a gradient without a bias field). Whether a pulse sequence restores these cancellations while keeping one spacing history is NOT DERIVED by anyone. The platform sentence stays NOT CHECKED as a scheme, now with three first-order obstacles named (defect D5 of [S]).

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4. Theorem R (the common-mode lemma), with its proof

Grade and status. A corollary of a three-line lemma (a Hermitian matrix with unit diagonal and unimodular entries is positive semidefinite iff it has rank one) and a two-line polynomial identity; never a finding. The proof below was written and sealed by the pilot BEFORE its run to twelve paths (sha256 of the sealed file `0723271c7e5ef94c7994f1469766e873287adc51aaa45787d5b016ced3104a90`), tested step by step by the pilot's machine rows, and then checked line by line by the gate, which found no gap and scored prediction P-R2 of [O] TRUE; the gate also re-derived the block structure and the lattice identity by its own code. Notation for this section: V is the space of real tables alpha(k) + beta(l) + gamma(k + l); T = V + 2 pi Z^N is the set of tables that are **LU-trivial up to sector constants** (local phases, and one phase per sector k + l); "protected" carries the rider block of section 1 (complete common-mode dephasing, the path-diagonal model, matched mass times spacing, nothing against local dephasing).

4.1 Statement and proof (verbatim from the sealed proof of the pilot, its sections 0 to 10)

0. Statement

Setting. Registers A (paths k = 0..n_A-1) and B (paths l = 0..n_B-1), any sizes >= 1. Input a_k b_l with all amplitudes nonzero. A diagonal gate with an ARBITRARY real phase table phi(k,l) (the family g(l-k) is a special case). Then the complete twirl over the common-mode generator k + l: rho_tw = sum_S P_S |Psi><Psi| P_S, Psi(k,l) = a_k b_l e^(i phi(k,l)). V = {alpha(k) + beta(l) + gamma(k+l)}, T = V + 2 pi Z^N (the tables that are LU-trivial up to sector constants).

Theorem R (general form). The following are equivalent: (a) rho_tw is separable; (b) rho_tw is PPT; (c) phi lies in T; (d) every generator of L_inv^full = V^perp cap Z^N vanishes mod 2 pi on phi. L_inv^full is free of rank (n_A-2)(n_B-2) (zero if a register has fewer than three paths) with basis the window invariants W_(k,l)(phi) = phi(k,l+1) - phi(k+1,l) + phi(k+1,l+2) - phi(k+2,l+1) - phi(k,l+2) + phi(k+2,l), 0 <= k <= n_A-3, 0 <= l <= n_B-3. In particular no table is PPT without being trivial: the twirled family contains no bound entanglement, for any sizes.

Corollary L (the family, linear geometry, n_A, n_B >= 3). For phi(k,l) = g(l-k): (a) iff (b) iff g in span{1, m, m^2, (-1)^m} + 2 pi Z^M ("quadratic up to sector constants") iff J_c(g) = 0 mod 2 pi for every c in [-(n_A-3), n_B-3], where J_c = [g(c-2) - g(c+2)] - 2 [g(c-1) - g(c+1)]. These n_A + n_B - 5 invariants are a Z-basis of L_ppt = L_inv = L_quad; at n_A = n_B = n the rank is 2n - 5 and at n = 3 the single invariant is J = J_0.

Corollary P (the family, parallel geometry, n_A, n_B >= 3). For phi(k,l) = h(|l-k|): (a) iff (b) iff h in span{1, r^2, (-1)^r} + 2 pi Z^M. L_ppt = L_inv = L_quad has rank max(n_A,n_B) - 3 (n - 3 at equal sizes: zero at three paths, one generator at four).

Corollary 2 (two paths). If n_A = 2 or n_B = 2, every table is trivial and rho_tw is separable for every table. For (2, n_B) with n_B >= 4 the word "quadratic" is then wrong (there are trivial family tables outside the quadratic span): the statement that holds for all sizes is the general form, with "trivial" in place of "quadratic".

1. Lemma 1 (block structure of the partial transpose)

rho_tw has entries Psi(k,l) conj(Psi(k',l')) when k + l = k' + l' and 0 otherwise. Transposing B, (rho_tw^Gamma)*((k,l),(k',l')) = Psi(k,l') conj(Psi(k',l)) [k + l' = k' + l], which vanishes unless k - l = k' - l'. So rho_tw^Gamma is a direct sum over d of blocks B_d on l_d = {k : 0 <= k <= n_A-1, 0 <= k-d <= n_B-1} (l = k - d), with (B_d)*(k,k') = v_k conj(v_k') H_(kk'), v_k = a_k conj(b_(k-d)) != 0, H_(kk') = exp(i [phi(k, k'-d) - phi(k', k-d)]). H is Hermitian, has unit diagonal and unimodular entries, and B_d = D H D^dagger

with D = diag(v_k) invertible. Hence B_d is positive semidefinite iff H is, for every choice of nonzero amplitudes (Sylvester's law of inertia).

2. Lemma 2 (rank one)

Let H be Hermitian with unit diagonal and unimodular entries. Then H >= 0 iff H_ik = H_ij H_jk for all i, j, k iff rank H = 1. Proof. If the cocycle identity holds, put u_i = H_i1; then H_ij = H_i1 H_1j = u_i conj(u_j), so H = u u^dagger >= 0 of rank one. Conversely, if H >= 0 every principal 3x3 minor is >= 0; with p = H_ij, r = H_jk, q = H_ik the minor is 1 + 2 Re(p r conj(q)) - |p|^2 - |q|^2 - |r|^2 = 2 Re(p r conj(q)) - 2, and |p r conj(q)| = 1, so p r conj(q) = 1, i.e. q = p r. A rank-one Hermitian matrix with unit diagonal is u u^dagger with unimodular u_i, which satisfies the identity. QED. (Blocks of size 1 or 2 are always positive semidefinite.)

3. Lemma 3 (PPT as lattice conditions)

For three points (k_i, l_i), l_i = k_i - d, of one block, arg(H_12 H_23 conj(H_13)) is the hexagon functional of C-5 evaluated on phi: [phi(k1,l2) - phi(k2,l1)] + [phi(k2,l3) - phi(k3,l2)] - [phi(k1,l3) - phi(k3,l1)]. By Lemmas 1 and 2, rho_tw is PPT iff every hexagon functional vanishes mod 2 pi on phi, iff every element of the lattice L_ppt^full they generate does.

4. Lemma 4 (trivial tables are separable)

Let M >= n_A + n_B - 1, omega = e^(2 pi i/M), Z_A = diag(omega^k), Z_B = diag(omega^l). For any vector Phi, (1/M) sum_(j<M) (Z_A^j x Z_B^j) |Phi><Phi| (Z_A^j x Z_B^j)^dagger = sum_S P_S |Phi><Phi| P_S, because (1/M) sum_j omega^(j(S - S')) = [S = S'] for 0 <= S, S' <= n_A + n_B - 2. If phi = alpha + beta + gamma mod 2 pi then P_S Psi = e^(i gamma(S)) (U_A x U_B) P_S (psi_A x psi_B) with U_A = diag(e^(i alpha)), U_B = diag(e^(i beta)); the sector phases cancel in P_S |Psi><Psi| P_S, so rho_tw = (U_A x U_B) [sum_S P_S |psi_A psi_B><psi_A psi_B| P_S] (U_A x U_B)^dagger = (1/M) sum_j |p_j><p_j| with the product vectors p_j = (U_A Z_A^j psi_A) x (U_B Z_B^j psi_B). QED.

5. Lemma 5 (duality)

Let V be a subspace of R^N spanned by integer vectors and L = V^perp cap Z^N. Then phi is in V + 2 pi Z^N iff lambda(phi) is in 2 pi Z for every lambda in L. Proof. "Only if" is clear. L is saturated (if c lambda is in L with lambda integer and c a nonzero integer, then lambda is in V^perp), so Z^N / L is torsion free, hence free, and L is a direct summand: a basis lambda_1..lambda_r of L extends to a basis of Z^N, and the inverse of that unimodular matrix supplies integer vectors w_j with lambda_i(w_j) = [i = j]. If lambda_i(phi) = 2 pi z_i with integers z_i, put z = sum_i z_i w_i; then every lambda_i kills phi - 2 pi z. Since V is rational, V^perp is spanned by its integer points, i.e. by L, so phi - 2 pi z lies in (V^perp)^perp = V. QED.

6. Proposition 6 (the lattice identity, all sizes)

Write e_(k,l) <-> x^k y^l, so that integer functionals are integer polynomials P(x,y) with deg_x <= n_A-1, deg_y <= n_B-1. (i) P is orthogonal to the row indicators iff P(x,1) = 0, to the column indicators iff P(1,y) = 0, to the antidiagonal indicators iff P(t,t) = 0. So L_inv^full = {P : P(x,1) = 0, P(1,y) = 0, P(t,t) = 0}. (ii) P(x,1) = 0 gives P = (y-1) P_1 in Z[x,y] (division by the monic y - 1 over Z[x]). Then P_1(1,y) = 0, so P_1 = (x-1) P_2. Then (t-1)^2 P_2(t,t) = 0, so P_2(t,t) = 0 and P_2 = (x-y) Q (division by x - y, monic in x, over

Z[y]). Degrees: deg_x Q <= n_A-3 and deg_y Q <= n_B-3 (the leading coefficients in x, respectively y, of the three factors are units, so degrees add). Conversely every such Q gives an element. Hence the polynomials x^i y^j (x-1)(y-1)(x-y), 0 <= i <= n_A-3, 0 <= j <= n_B-3, are a Z-basis of L_inv^full; its rank is (n_A-2)(n_B-2). (iii) The hexagon of the consecutive points (k,l), (k+1,l+1), (k+2,l+2) is x^k y^(l+1) - x^(k+1) y^l + x^(k+1) y^(l+2) - x^(k+2) y^(l+1) - x^k y^(l+2) + x^(k+2) y^l = x^k y^l (y - x)(1 + xy - x - y) = - x^k y^l (x-1)(y-1)(x-y), a basis element up to sign (the functional W_(k,l) of the statement is this hexagon). (iv) Every hexagon lies in L_inv^full: on alpha(k) the six terms cancel in pairs; likewise on beta(l); on gamma(k+l) they cancel because k_i + l_j = k_j + l_i on a diagonal. By (iii) L_ppt^full contains a basis of L_inv^full; by (iv) it is contained in it. So L_ppt^full = L_inv^full. QED.

7. Proof of Theorem R

(a) implies (b): the Peres criterion. (b) implies (d): Lemma 3 and Proposition 6. (d) implies (c): Lemma 5 with the rational subspace V. (c) implies (a): Lemma 4. The rank and the basis are Proposition 6 (ii), (iii). QED.

8. Proof of Corollary L

Let iota send g to its table, (iota g)(k,l) = g(l-k), and let the push send a functional lambda to lambda o iota, i.e. x -> 1/t, y -> t on polynomials. By Theorem R, g gives a PPT state iff mu(g) = 0 mod 2 pi for every mu in the push of L_inv^full, which is generated by the pushes of the basis: -x^k y^l (x-1)(y-1)(x-y) goes to -t^(l-k) (1/t - 1)(t - 1)(1/t - t) = - t^(c-2) (t-1)^3 (t+1), c = l - k; as a functional this is -[g(c+2) - 2 g(c+1) + 2 g(c-1) - g(c-2)] = J_c(g). As (k,l) runs over 0..n_A-3, 0..n_B-3, c runs over [-(n_A-3), n_B-3]. On the other hand a functional p(t) = sum_m lambda_m t^m on the window [-(n_A-1), n_B-1] is orthogonal to 1, m, m^2 iff p(1) = p'(1) = p''(1) = 0 and to (-1)^m iff p(-1) = 0, i.e. iff (t-1)^3 (t+1) divides p in Z[t, 1/t] (monic divisor), i.e. iff p = sum_j q_j t^j (t-1)^3 (t+1) with integers q_j, j in [-(n_A-1), n_B-5]. This is the same set of generators, so L_ppt = L_inv = L_quad, free of rank n_A + n_B - 5 on the J_c. L_quad is by definition the saturated annihilator of the rational subspace span{1, m, m^2, (-1)^m}; Lemma 5 turns "L_quad vanishes mod 2 pi on g" into "g in span{1, m, m^2, (-1)^m} + 2 pi Z^M". (The four functions are tables in V: m^2 = 2k^2 + 2l^2 - (k+l)^2, m = l - k, (-1)^m = (-1)^(k+l).) QED.

9. Proof of Corollary P

The fold f sends e_m to e_|m|; the parallel lattices are the folds of the linear ones, so by Corollary L it suffices to show f(L_quad^lin) = L_quad^par, the saturated annihilator of span{1, r^2, (-1)^r} on r = 0..max(n_A,n_B)-1. Inclusion: for lambda in L_quad^lin and an even table e(m) = E(|m|), (f lambda)(E) = lambda(e), and 1, m^2, (-1)^m are even. Surjectivity: let mu be in L_quad^par. Put lambda_0 = mu_0; for r >= 2 put the whole of mu_r on m = +r if +r is in the window, else on m = -r (sign s_r = +1 or -1); let delta = - sum_(r>=2) s_r r mu_r and lambda_(+1) = (mu_1 + delta)/2, lambda_(-1) = (mu_1 - delta)/2. Then sum_m m lambda_m = 0, and the sums against 1, m^2, (-1)^m equal those of mu against 1, r^2, (-1)^r, which vanish. Integrality: mu_1 + delta = sum_(r odd) mu_r mod 2, and sum_r mu_r = 0 = sum_r (-1)^r mu_r give 2 sum_(r odd) mu_r = 0. So lambda is in L_quad^lin and f lambda = mu. Lemma 5 gives the stated form of the locus. The rank is the number of window points minus 3 (the three functions are independent on at least three points). QED.

10. Proof of Corollary 2

> If n_A = 2: alpha = 0, gamma(0) = 0, gamma(l+1) = gamma(l) + phi(1,l) - phi(0,l), beta(l) = phi(0,l) - gamma(l) solve alpha(k) + beta(l) + gamma(k+l) = phi(k,l) for every table; so V is the whole table space, every table is trivial, and Lemma 4 applies. (Consistently, no diagonal carries three points and L_ppt^full = 0 = L_inv^full.) For the family on (2, n_B) the window has n_B + 1 points and the quadratic span has dimension min(4, n_B + 1): for n_B >= 4 it is a proper subspace although every g is trivial. QED.

4.2 What the machines checked (the pilot and the gate, then the rigor leg; own routes)

- **Pilot (own route: the full table space through the polynomial basis, Hermite normal forms, Smith invariants).** 19 lattice rows with 3,332 hexagon functionals, exact integers: L_ppt = L_inv with index 1 for two n-path registers, n = 2 to 12, both geometries, and for the mixed registers (2,3), (3,4), (3,5); prediction P-R1 of [O] TRUE. Torsion search for exotic tables (PPT and not trivial), exhaustive, exact integers: 8,988,410 tables, 0 exotic. State level with no lemma assumed, exact inertia in Q(sqrt 3, i): 928 state rows, 0 disagreements, unequal amplitudes included. Mutation controls: a doubled or deleted generator, a doubled condition and a flipped sign are each caught.
- **Gate (own lattice code, own field Q(zeta_12), inertia by Descartes' rule).** L_ppt^full = L_inv^full, index 1, rank (n_A - 2)(n_B - 2) on every registered row and on (13,13), (14,14), (13,14); state level, 7,924 torsion tables and 40 non-torsion tables: PSD iff trivial, 0 exotic; four classical channels of its own construction on a product input give no negative eigenvalue of the partial transpose, twirled or not, while the twirled gate is NPT iff the table is not trivial.
- **Rigor leg [R] (third reader; own routes: the integer kernel of the generators against the Hermite form of all hexagon functionals; at the state level explicit certificates in both directions).** The lattice identity with index 1 on all 91 pairs of register sizes from 2 to 14, in both geometries, with rank pins at its own two tokens; no exotic table among its own state rows, unequal amplitudes and mixed registers included; its registered break attempts found no exotic table and, outside the theorem's letter, no PPT entangled state under partial noise (a realignment test: data, not a proof); it read the proof line by line and found no gap. It did not repeat the enumerations of the two legs above (their counts were regenerated at the finalization: Appendix B).
- **Kill criterion KR-2 of [O] (an exotic table): NOT FIRED.** The one wording item its letter asks to see first: with a two-path register against at least four paths every table is PPT and LU-trivial, hence separable by an explicit mixture, and NOT quadratic (rows (2,4) and (2,5) of the table below). That is "separable by another route", not bound entanglement, and it lies outside the family of two n-path registers. Consequence kept in every sentence of this note: "quadratic" is the right word exactly when both registers have at least three paths; the statement for all sizes says "LU-trivial up to sector constants". The note never prints "two paths against more paths: separable iff quadratic".

4.3 The invariant table

Ranks of the invariant lattice L_ppt = L_inv (exact; regenerated). "Full" is the full table space (any phase table phi(k,l)); "linear" and "parallel" are the two families of section 3; in brackets the rank of the quadratic annihilator L_quad.

registers	kind	full	linear: rank (L_quad)	parallel: rank (L_quad)	trivial locus
(2,2)	two n-path registers	0	0 (0)	0 (0)	quadratic
(3,3)	two n-path registers	1	1 (1)	0 (0)	quadratic
(4,4)	two n-path registers	4	3 (3)	1 (1)	quadratic

registers	kind	full	linear: rank (L_quad)	parallel: rank (L_quad)	trivial locus
(5,5)	two n-path registers	9	5 (5)	2 (2)	quadratic
(6,6)	two n-path registers	16	7 (7)	3 (3)	quadratic
(7,7)	two n-path registers	25	9 (9)	4 (4)	quadratic
(8,8)	two n-path registers	36	11 (11)	5 (5)	quadratic
(9,9)	two n-path registers	49	13 (13)	6 (6)	quadratic
(10,10)	two n-path registers	64	15 (15)	7 (7)	quadratic
(11,11)	two n-path registers	81	17 (17)	8 (8)	quadratic
(12,12)	two n-path registers	100	19 (19)	9 (9)	quadratic
(2,3)	mixed registers (registered)	0	0 (0)	0 (0)	quadratic
(3,4)	mixed registers (registered)	2	2 (2)	1 (1)	quadratic
(3,5)	mixed registers (registered)	3	3 (3)	2 (2)	quadratic
(2,4)	own extension row of [P]	0	0 (1)	0 (1)	PPT, LU-trivial and NOT quadratic: the wording item of section 4.2
(2,5)	own extension row of [P]	0	0 (2)	0 (2)	PPT, LU-trivial and NOT quadratic: the wording item of section 4.2
(4,5)	own extension row of [P]	6	4 (4)	2 (2)	quadratic
(4,7)	own extension row of [P]	10	6 (6)	4 (4)	quadratic
(3,12)	own extension row of [P]	10	10 (10)	9 (9)	quadratic

Explicit generators (each a Z-basis; the lattices are saturated):

- **Full table space, any sizes:** the window invariants W_(k,l)(phi) = phi(k,l+1) - phi(k+1,l) + phi(k+1,l+2) - phi(k+2,l+1) - phi(k,l+2) + phi(k+2,l), 0 <= k <= n_A - 3, 0 <= l <= n_B - 3. As polynomials (cell (k,l) written x^k y^l) they are -x^k y^l (x-1)(y-1)(x-y): the three factors are the three ways a table can be trivial (a function of l, of k, of k + l).
- **Linear geometry:** J_c(g) = [g(c-2) - g(c+2)] - 2 [g(c-1) - g(c+1)], c = -(n-3)..(n-3): 2n - 5 invariants; they annihilate exactly span{1, m, m^2, (-1)^m}. At three paths the single invariant is **J = J_0 = [g(-2) - g(2)] - 2 [g(-1) - g(1)]**, a third difference of the potential across the register (the Newtonian J is 1.2400 times its third-derivative term at s/D = 1/5).
- **Parallel geometry:** K_c(h) = [h(|c-2|) - h(c+2)] - 2 [h(|c-1|) - h(c+1)], c = 1..n-3: n - 3 invariants; they annihilate exactly span{1, r^2, (-1)^r}. K_c is the fold of J_c (J_0 folds to zero, J_(-c) to -K_c), and K_1 = [h(1) - h(3)] - 2 [h(0) - h(2)] is the invariant J_par of the four-path parallel rung. Typed COROLLARY (Corollary P with an explicit basis; a hand item of this assembly, checked against the engine's lattices for n = 3 to 12 with rank pins modulo the two tokens of this leg: check A-5 of [B]).

Reading. What complete common-mode dephasing erases from the certificate is exactly the part of the phase table that is quadratic in the path index, up to sector constants; what it cannot erase is the non-quadratic part, measured by the invariants. The second difference that the two-path experiment reads is a quadratic datum: it is erased. The whole bilinear (Gaussian-order) coupling is quadratic at every size:

erased. At three paths in a line one number survives, the third difference J; in the parallel geometry the table is even in l - k, three paths have no invariant at all, and four are needed.

"In a line" is an alignment requirement, and the cited multi-path designs set the line aside (defect D6 of the steelman leg [S], its section 3.5; data, re-certified by intervals: record [F]). For two identical registers whose superposition axis is tilted by an angle th against the line of centres the table is still a function of l - k, with g(m) proportional to 1/sqrt(D^2 + 2 D m s cos th + m^2 s^2), so Theorem R and Corollary L apply unchanged. At (50, 10, 1) the invariant is 0.9949 of its value in the line at 2 degrees, 0.9688 at 5, 0.8794 at 10, 0.5751 at 20 and 0.2140 at 30; it vanishes at 36.68 degrees and again at 90 degrees (the parallel geometry of Corollary P); the count of the fixed witness at perfect local coherence grows by the factors 1.06, 1.28 and 21.3 at 5, 10 and 30 degrees. A null on the sector that complete common-mode dephasing spares therefore has a geometric zero that no law produces (section 8.5). The multi-path designs that this note cites do not use the linear geometry: [2101.08086v2 p7] "discard the linear set-up in the remainder of the simulations" and [2312.05170v1 p8] "we will only consider the parallel interferometric geometry"; in the parallel geometry three paths have no invariant at all (the table above).

4.4 Two paths: what the lemma says and what it does not (wording line W1 of [L1])

"A two-path pair is left separable" is a statement about the COMPLETE twirl only. At every finite common-mode strength, with perfect local coherence and zero mismatch, the two-path state stays entangled, with a margin that dies like C^2 exp(-sigma^2) (hand item H-C1 of [P]; section 5.5, where the closed form is CITED). Three printed sentences must be read beside the lemma, and none contradicts it:

- [quant-ph_0507027v2 p5] "This single non-vanishing element is enough to prevent complete disentanglement at any temperature and for any coupling strength." (Roszak and Machnikowski, for the two-path gate state at entangling phase pi under a common reservoir of finite strength: the finite-strength complement of the two-path half, not its negation.)
- [2007.15029v2 p11] "technical noises do not change the degree of entanglement, but can only affect the value of a particular entanglement witness" (Toros et al., for their effective entanglement phase; it is not a statement about an entanglement measure of the noisy state: in the complete limit of their own printed matrix only the coherence between the two mixed branches survives and the state is separable for every value of the phases, a hand reading of [P] whose algebra is the gate's check V-4d).
- Lemma 3 of arXiv:2611.05044 v1: for a bilinear coupling with local rates and a correlated rate, when the square of the coupling plus the square of the correlated rate exceeds the product of the local rates the generator [2611.05044v1 p22] "L is non-separable and generates entanglement". A fully shared Markovian dephasing therefore never stops a bilinear coupling from entangling at short times; Theorem R says that the complete twirl leaves a quadratic table separable. Finite rate against the complete limit: the two statements stand side by side ([L34], section 4.1 (ii)).

A fourth printed use of the same object answers another question: Feng and Vedral, arXiv:2611.12229 v2, appendix H 2, print that their dynamics test of the conditional phase tolerates shared classical phase noise: [2611.12229v2 p25] "Correlated local dephasing is still compatible with the classical path-local benchmark." for two paths the printed answer to "what can shared noise not erase" is a premise-conditional signature of the conditional phase, and the house's answer is that the certificate is erased.

4.5 What is printed, said first

- **The principle** is Datta and Miao's (arXiv:2104.04414; Gaussian oscillators; section 2): cited first, never claimed.
- **Two and three paths follow from one 2005 paper by substitution.** Li and Xu, arXiv:quant-ph/0504228 v1 (Mod. Phys. Lett. B 19, 1803; journal version NOT READ): their eq. (6), the stationary concurrence of two qubits under collective dephasing, vanishes identically on the two-path gate state for any amplitudes and any table; their criterion for two spin-1 systems, that the stationary state is entangled if a printed cubic [quant-ph_0504228v1 p4] "has a negative root, or at least one of the following two inequalities are satisfied", gives on the three-path gate state a negative root exactly when the window invariant is not 0 modulo 2 pi (checked exactly by [L1]; the verdict needs only the sign of the printed constant term). Neither half is stated there, and the criterion is one-directional. For the VALUE of the root, repair R2 of [G], verbatim: The root of the characteristic cubic of the 3 by 3 block on which Li and Xu's criterion (12) rests; their printed quadratic coefficient (squared populations, arXiv v1; journal NOT READ) does not reproduce it, the trace does. (With the printed coefficient the negative root at J = 211/1400 is -0.0227982, with the trace it is -0.0095240.)
- **The technique is printed.** The block structure of the partial transpose of a state that commutes with a total charge: Cornfeld, Goldstein and Sela, arXiv:1804.00632, [1804.00632v2 p3] "This commutativity elicits a block matrix decomposition"; circulant decompositions: Chruscinski and Kossakowski, arXiv:0705.3534. The step "a trivial table gives a separable state" rests on a printed fact, that common-mode dephasing is a mixture of correlated local unitaries: [1602.08973v2 p10] "We know the map is separable, hence it cannot create entanglement" (Carnio, Buchleitner and Gessner) and, for the two-path gravitational pair, Hebib, arXiv:2511.11879: [2511.11879v3 p3] "it cannot generate quantum correlations or entanglement between the two systems".
- **What remains the house's statement**, at NOTE grade ([L1], verbatim): the theorem in setting S for all sizes, the "only if" direction for every size, the invariant lattice and its ranks, the closed form N_3(J) and the threshold lemma H-V3, the certified cells that are not printed. Never "new" for the instances at two and three levels: "restated in gate terms, with the converse and every size added".

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5. Closed forms

All of this section sits in the path-diagonal model with uniform amplitudes; "protected" carries the rider block of section 1. Every closed form here is typed COROLLARY or CITED; none is a finding.

5.1 Three paths in a line: the circulant block and the protected negativity

After the complete twirl the partial transpose of the three-path state is a direct sum of blocks of sizes 3, 2, 2, 1, 1 (Lemma 1 of section 4.1); blocks of size 1 or 2 are positive semidefinite, so everything happens in the block d = 0. In a diagonal gauge that block, times 9, is the circulant with first row (1, e^(i theta), e^(-i theta)), theta = J/3, whose eigenvalues are 1 + 2 cos(theta + 2 pi m/3), m = 0, 1, 2, on the vectors (1, w^m, w^(2m))/sqrt 3, w = e^(2 pi i/3). For J reduced to (-pi, pi] and not zero exactly one of them is negative: 1 - c with

> **c = 2 cos((pi - |J|)/3), protected negativity N_3(J) = (c - 1)/9 = (2 cos((pi - |J|)/3) - 1)/9.**

Typed COROLLARY (hand items H-B2 of the judge, H-R1 and H-R10 of [P], H-V2 of [G]); equivalently, N_3 is minus the least root of the characteristic cubic of the block, x^3 - x^2/3 + (2/729)(1 - cos J) = 0, which is the cubic of Li and Xu with the trace (section 4.5). Two laws that the experiment scout of [O] had found numerically are its corollaries: slope 1/(9 sqrt 3) at small J, maximum 1/9 at |J| = pi. At the

benchmark, J = 211/1400: N_3 = 0.009524074220 (it regresses the judge's 0.009524074221 by rounding).

5.2 The threshold in local coherence: a lemma for every J (lemma H-V3 of the gate), with its proof

Statement of record (the gate's registration, sealed before its scripts existed), verbatim:

> H-V3 (the "iff" of cell C3 as a lemma for every J; NEW relative to the pilot's hand item H-R10, which worked J = pi only). For 0 < J <= pi, theta = J/3, t0 = 1/c: rho(t0) = p_+ tau_+ + p_- tau_-, where tau_s is the average over w of the twirled product state with moment phases (u, v, w), u = a_1 + b_1, v = a_2 + b_2, w = a_1 + b_2 (a_1 = alpha_0 - alpha_1, a_2 = alpha_1 - alpha_2, b_1 = beta_1 - beta_0, b_2 = beta_2 - beta_1 for product vectors with uniform moduli), in the gauge where the five target moments are E e^(iu) = E e^(iv) = t e^(i theta), E e^(i(u+v)) = t e^(-i theta), E e^(iw) = E e^(i(u+v-w)) = t e^(-i theta/2). The singular block pins (u, v) to (0, 0) (weight p_+) and (2 pi/3, 2 pi/3) (weight p_-) with p_- = 2 sin(theta)/(sqrt 3 c), p_+ = 1 - p_-; the first moments z_s = E[e^(iw) | s] are forced: p_- z_- = (2 t sin(theta/2)/sqrt 3) e^(-i pi/3), p_+ z_+ = t e^(-i theta/2) - p_- z_-. Then |z_-| = 1/(2 cos(theta/2)) <= 1/sqrt 3 and 1 - |z_+|^2 has the sign of (4/3) sin(3 theta/2) cos(theta/2 + pi/6) > 0. So rho(t0) is a finite mixture of twirled product states (each first moment realised by one point mass plus three equally spaced points), hence separable; by convexity with the identity, rho(t) is separable for every t <= 1/c. The three-path protected state with uniform amplitudes is entangled iff lam > c^(-1/2), for every J: a corollary of a short lemma, with no separable ball. At J = 211/1400: p_- = 0.053408 (two-digit hand arithmetic), the pilot printed 0.053407.

Lemma T. Three paths in a line, uniform amplitudes, complete common-mode dephasing, local coherence lam on each register (RULE-N), J reduced to (-pi, pi] and not zero, c as in 5.1. The state is entangled iff lam > c^(-1/2). (At J = 0 modulo 2 pi it is separable for every lam, by Theorem R.)

Proof. Write t = lam^2 and take J > 0 (J < 0 is the complex conjugate state).

Step 0 (local dephasing acts as white noise here; hand item H-R10 of [P]). A coherence that survives the twirl joins (k,l) and (k',l') with k + l = k' + l' and (k,l) different from (k',l'), hence with k different from k' AND l different from l': it carries the factor lam^2. So rho(t) = t rho_tw + (1 - t) 1/9.

Step 1 (NPT above the threshold). By 5.1 the least eigenvalue of the partial transpose of rho(t) is (1 - t c)/9, negative iff t c > 1.

Step 2 (separable at t = 1/c: two pinned configurations). Take product vectors with uniform moduli, phases alpha_k on A and beta_l on B. Twirled, each gives populations 1/9 and five coherences of modulus 1/9 whose phases are u between |01> and |10>, v between |12> and |21>, and, inside the sector k + l = 2, w between |02> and |11>, u + v - w between |11> and |20>, u + v between |02> and |20>; the three angles u, v, w are free. A mixture of such states is separable (Lemma 4 of section 4.1), and it equals rho(t) iff its five moments equal the five coherences of rho(t). Local unitaries bring those to the symmetric gauge: E e^(iu) = E e^(iv) = t e^(i theta), E e^(i(u+v)) = t e^(-i theta), E e^(iw) = E e^(i(u+v-w)) = t e^(-i theta/2), theta = J/3 (J is the only gauge-invariant phase). Now the moment matrix E[x x^dagger] of x = (1, e^(-iu), e^(-i(u+v))) is the circulant of 5.1 with its off-diagonal entries multiplied by t; at t = 1/c it kills the vector n = (1, w, w^2), so the mean of |<n|x>|^2 vanishes and every point of the mixture satisfies 1 + conj(w) e^(-iu) + w e^(-i(u+v)) = 0. Three unit numbers that sum to zero are the three cube roots of unity in some order: **(u, v) = (0, 0) or (2 pi/3, 2 pi/3)**, with weights p_+ and p_-. The moment E e^(iu) = p_+ + p_- w = t e^(i theta) fixes

> p_- = 2 sin(theta)/(sqrt 3 c), p_+ = 1 - p_- (both positive for 0 < theta <= pi/3).

Step 3 (the first moments are forced, and they fit). Only the conditional first moments z_+ and z_- of e^(iw) enter the two remaining equations, p_+ z_+ + p_- z_- = t e^(-i theta/2) and p_+ conj(z_+) + w^2 p_- conj(z_-) = t e^(-i theta/2). Their solution is p_- z_- = (2 t sin(theta/2)/sqrt 3) e^(-i pi/3), p_+ z_+ = t e^(-i theta/2) - p_- z_-, and **two inequalities** hold:

> |z_-| = 1/(2 cos(theta/2)) <= 1/sqrt 3, p_+^2 (1 - |z_+|^2) = t^2 (4/3) sin(3 theta/2) cos(theta/2 + pi/6) > 0.

A first moment z inside the unit disc is realised by a point mass of weight |z| at the phase of z and three equally spaced points of weight (1 - |z|)/3 each. So rho(1/c) is an explicit mixture of 2 x 4 = 8 twirled product states: separable.

Step 4 (below the threshold). For t < 1/c, rho(t) = (t c) rho(1/c) + (1 - t c) 1/9 is a convex combination of separable states. QED.

Certificates. The gate proved the six identities exactly at 120 rational points (identically true), the full 9 by 9 identity at five designs with residuals below 1e-58, and the two inequalities on a grid of 60 values of J; this assembly re-derived them by an own route (exact symbolic identities, and the five designs to fourteen decimals: check A-3 of [B]). At the benchmark: p_- = 0.05340763280715, |z_-| = 0.50015778311967, |z_+| = 0.95860051238449. Typing, repair R4 (v) of [G]: the "iff" is a COROLLARY of this lemma for every J; the cited separable ball that the pilot's first certificate used is no longer needed. Scope: uniform amplitudes; the registered local model (one factor lam per register whatever the path distance; the force-type variant is section 6); complete common-mode dephasing at matched spacing.

5.3 The fixed protected-sector witness and its margin

W = Q^Gamma (the partial transpose of Q), Q the projector onto (u_0 |00> + u_1 |11> + u_2 |22>)/sqrt 3 with u the unit phases of the eigenvector of 5.1. Since tr(W s) = tr(Q s^Gamma) and s^Gamma is positive for every separable s, **tr(W s) >= 0 for every separable state of the two registers, whatever the noise: the separable bound is sound with no model premise** (hand item H-W1 of [P]; the path-diagonal model enters the predicted SIZE of a positive and the reading of a null, never the soundness of a positive). Under RULE-N the margin m_3 = -tr(W rho) is

> m_3 = (c t (2 b + b^4) - 3)/27, t = lam^2, b = exp(-sigma^2 eps^2 / 2),

and (c (2 b lam^2 + b^4 lam^8) - 3)/27 if the independent noise is of force type (the outer pair of paths then loses the fourth power); at b = 1 it is (t c - 1)/9 = N_3 at t = 1. It depends on the common-mode rms phase sigma and on the mismatch eps only through their product (hand item H-C2 of [P], confirmed against the engine at every point used; the gate's own closed form agrees). The witness is read in seven settings (the path basis, and two interference settings for each of the three pairs of paths).

Pair-dependent damping (defect D8 of [S]; a COROLLARY of the linearity of the trace, checked against the definition of W: record [F]). If the two inner coherences of the block d = 0 (between |k l> and |l k> for neighbouring paths) carry a damping factor x_i and the outer one a factor x_o, then m_3 = (c (2 x_i + x_o) - 3)/27. RULE-N is x_i = t b, x_o = t b^4; force-type independent noise is x_i = lam^2 b, x_o = lam^8 b^4. With local coherences lam_i on the inner pairs and lam_o on the outer pair (complete common-mode dephasing, matched spacing) the benchmark witness is positive iff c (2 lam_i^2 + lam_o^2) exceeds 3: lam_i above 0.9389 when lam_o = 1, lam_o above 0.8735 when lam_i = 1; the inner pairs are the expensive ones. A source midway between the registers (clarification (i) of section 1) dephases the two inner coherences only; the benchmark witness then stays positive only while the rms phase on them is below 0.5020 rad at lam = 1 and below 0.3561 rad at lam = 49/50. By the steelman leg's reading, the local model of RULE-N (one factor per register whatever the path distance) is the convention of Tilly et al. and the short-wavelength limit of Braccini et al., and the force-type row is their long-wavelength limit; both are in print for independent noise only.

5.4 Four paths in the parallel geometry

The block d = 0 of the twirled partial transpose is of rank one; the blocks d = 1 and d = -1 are 3 by 3 with triple phase J_par = K_1 = [h(1) - h(3)] - 2 [h(0) - h(2)]; the others are positive. Hence twirled negativity (2 cos((pi - |J_par|)/3) - 1)/8 and NPT threshold in local coherence (2 cos((pi - |J_par|)/3))^(-1/2), the same function as at three paths (hand item H-C4 of [P], confirmed by certified inertia: two eigenvalues at the closed form). Separability below that threshold is NOT BUILT for 4 by 4 systems, so no "iff" is printed for this rung.

5.5 Two paths under partial common-mode noise at matched spacing: a CITED closed form

Printed. Wang, Pedernales and Plenio, arXiv:2611.05044 v1 (2026-11-03), Supplemental Material: the master equation (A30) for the two-path pair (the gate generated by the product of the two path operators, local dephasing at rate Gam on each mass, a correlated term at rate Gam_AB) and, for the uniform product input, [2611.05044v1 p14] "the concurrence at time t reads as follows:" eq. (A31),

> C(t') = (1/2) max{0, e^(-2 Gam t') sqrt(4 sin^2(2 lam_NG t') + e^(-4 Gam t') sinh^2(4 Gam_AB t')) - |1 - e^(-4 Gam t') cosh(4 Gam_AB t')|}

(transcribed in ASCII by lanes [L1] and [L34] from the PDF text and a page rendering; t' is that paper's time).

Dictionary D (exact). RULE-N at zero mismatch damps the one-flip coherences by lam a, the coherence between the like branches by lam^2 a^4 and the coherence between the mixed branches by lam^2, with a = exp(-sigma^2/2); (A30) damps them at rates 2 Gam, 4 (Gam + Gam_AB) and 4 (Gam - Gam_AB). Hence e^(-2 Gam t') = lam a, e^(-4 Gam_AB t') = a^2, 2 lam_NG t' = Delta/2. In the budget rule RULE-B of section 6.1 this is f = Gam_AB/Gam.

Own derivation of the identity (hand item H-N1 and check A-2 of [B]: exact algebra, then certified intervals; lane [L34] asked for it before any entry). In the gauge with phase +Delta/4 on the like branches and -Delta/4 on the mixed ones the state commutes with the simultaneous flip of both path labels, so four times its partial transpose is a direct sum E + O of two 2 by 2 blocks, with t = lam^2 and C = |sin(Delta/2)|:

> E = ((1 + t, 2 lam a cos(Delta/2)), (2 lam a cos(Delta/2), 1 + t a^4)), O = ((1 - t, -2 i lam a sin(Delta/2)), (2 i lam a sin(Delta/2), 1 - t a^4)).

E is positive semidefinite for all parameters, because (1 + t)(1 + t a^4) - 4 t a^2 = (1 - t a^2)^2 + t (1 - a^2)^2. So everything is in O: **the state is NPT iff 4 t a^2 C^2 > (1 - t)(1 - t a^4)**, with margin (minus the least eigenvalue of the partial transpose wherever the state is NPT, which is all that is used here: it is a quarter of minus the lower eigenvalue of O; where the state is PPT the least eigenvalue can sit in E instead, as at the benchmark phase, local coherence 9/10 and sigma = 3/10, where it is 0.00768, in E, against 0.01590 for O; remark R-a of [R])

> m_2 = (1/4) [sqrt(t^2 (1 - a^4)^2/4 + 4 t a^2 C^2) - (1 - t (1 + a^4)/2)],

and the printed (A31) equals 2 m_2 under D wherever it is positive: identically. For two qubits PPT is separability (Peres and Horodecki, cited), so the least sigma at which the state is separable is given by e^(-sigma_0^2) = y*, the positive root of t (1 - t) y^2 + 4 t C^2 y - (1 - t) = 0.

The cited rows (cell C5 of the pilot, two paths, zero mismatch, local coherence 49/50). The pilot's brackets are certified inertia brackets of its own engine (regenerated); the root is this note's enclosure of

the printed closed form; it lies inside both brackets and equals the lanes' float locators to at least twelve decimals.

design	certified bracket of [P] (NPT end, PPT end; rad)	root sigma_0 of the printed closed form (rad)	type
(40, 10, 1), benchmark, not a design of record	[0.710119724, 0.710120201]	0.710119811032728	CITED (arXiv:2611.05044 v1, eq. (A31), under D); bracket COMPUTED
(450, 250, 5/2), committed geometry	[1.001051903, 1.001052856]	1.001051916545234	CITED, as above

At perfect local coherence (t = 1) the condition reads 4 a^2 C^2 > 0: the two-path state is NPT at EVERY finite sigma when C is not zero, with margin m_2 = a^2 C^2 / (R + (1 - a^4)/2), R = sqrt((1 - a^4)^2/4 + 4 a^2 C^2), of order C^2 exp(-sigma^2); it is separable only in the complete limit. This is hand item H-C1 of [P], now also CITED: the closed form reproduces its ten certified margins to their seven printed digits (4.619216e-03 at sigma = 1 down to 1.540036e-13 at sigma = 5 for the benchmark; 9.943116e-03 down to 3.394950e-13 for the committed geometry). At Gam_AB = 0 the same paper's eq. (A24) gives the two-path threshold without common-mode noise, sqrt(1 + C^2) - C: 0.855798920368957 at the committed phase (the battery anchor r* of [O]) and 0.900224820400 at the benchmark phase; it is cited, not claimed.

What is not printed there ([L34]): the complete, fully correlated limit and the separability of its state (the paper excludes equal rates from its asymptotic formula and never takes the limit); unequal amplitudes; mismatch at finite noise; three or more paths with a non-bilinear phase table (the invariant J, the protected negativity, the lattice statement); any witness or shot count; anything of the law column. **Typing under the assembly rule of [B]:** SCOOPED-CELL is typed for these entries (the two brackets above and hand item H-C1 with its ten values) on the entry reading; the six two-path brackets with mismatch and every three-path row stay COMPUTED; on the group reading the word is not typed. The other outcome words of section 12 (the lemma, the ladder, and the word on whether the protection pays) and prediction P-R4 rest on Theorem R, on the three-path cells and on witness margins and counts, none of which is printed (riders R5 and R11: sections 6.1 and 6.2).

5.6 The inverse: what a protected-sector null bounds, inside the model

lam = (2 cos((pi - |J|)/3))^(-1/2) is solved by |J| = pi - 3 arccos(1/(2 lam^2)) for 1/sqrt 2 < lam <= 1. So in the path-diagonal model, at matched spacing, a protected three-path state with certified local coherence lam that shows no entanglement has |J| below that number: 0.47720 of the Newtonian J = 211/1400 at lam = 49/50 and 0.23414 of it at lam = 99/100 (COROLLARY; certified by bracketing with the forward form: check A-7 of [B]; the gate's own enclosures agree).

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6. The certified cells: column C at both design points

Model of every cell: RULE-N of section 3, uniform amplitudes; "protected" carries the rider block of section 1. Every sign of an eigenvalue is a certified interval inertia; thresholds are brackets (certified on both sides); shot counts are exact integer ceilings. The pilot's cells were regenerated for this note (Appendix A) and had been re-certified by the gate's own routes: all 64 rows of the shot-count grid, all 90 rows of the budget family and all 36 threshold brackets, digit for digit or bracket for bracket. The rigor leg [R] then confirmed every one of them a third time by its own routes (its section 3.4). **The benchmark designs are the white paper's representative point, "not a design of record"; the committed geometry is the design of record.**

6.1 Does the protection pay at the white paper's stated allowance? No (cell C6; the outcome word)

CITED. The white paper (Bose, Mazumdar, Toros, Zhou et al., arXiv:2610.07422 v1) allocates one number, Gamma_max = 3/100, [2610.07422v1 p8] "as a conservative allowance for RAN-induced phase variance": a per-interferometer variance of the phase between two paths 10 um apart (dephasing factor exp(-Gamma/2); rms phase 0.173205 rad), stated at independence: [2610.07422v1 p6] "For this benchmark calculation, we assume equal, independent local dephasing in the two interferometers". It prints no split into shared and independent noise, no correlation and no mismatch, and names what is missing: [2610.07422v1 p6] "Correlated dephasing, such as that arising from shared apparatus motion, requires evaluating the witness using the corresponding joint noise model."

The registered split (RULE-B of [P]; the pilot's definition, sealed before any readout; not the white paper's). The noise phases of the two interferometers are jointly Gaussian with equal variances Gamma and correlation f in [0, 1]; for two paths this is exactly RULE-N with sigma^2 = f Gamma and lam = exp(-(1 - f) Gamma/2). Status line from lane [L34]: the correlation f is the printed ratio Gam_AB / Gam of 2611.05044 eq. (A30) and the printed coefficient of 2610.22097 eq. (27); the white paper prints none. The two ends f = 0 and f = 1 are the two printed two-path models (the white paper's eq. (6); Grossardt, arXiv:2007.05762, eq. (27), and Toros et al., arXiv:2007.15029, eqs. (53) to (59)): the noise MODEL of the two-path rows is CITED, the counts are COMPUTED. Mismatch 1/1000 (the pilot's value; the paper prints none). **LOCAL-G** is the sensitivity row in which the independent part is a force noise, so that the outer pair of a three-path register loses the fourth power; for an acceleration noise it is the faithful row, and the word below survives it and is stronger there.

The shot-count convention (RULE-SC, on the E1-M pattern of [E1-M]; a convention, sealed before the engine existed, and it travels with every count). N shots per setting, worst-case shot noise, 5 sigma, exact ceiling: N is the least integer with m^2 >= 25 kappa/N for a certified margin m. Two paths: the BEST witness of the state at hand at every noise level, variance constant kappa = 3/16. Three paths: ONE fixed protected-sector witness (section 5.3), seven settings, kappa = 7/36. The comparison is tilted against the protection on purpose.

The cell, at the stated allowance (Lambda = 1), two paths (40, 10, 1) against three paths (50, 10, 1):

f (share of the variance that is common)	0	1/4	449/1000	1/2	3/4	9/10	99/100	999/1000	1
N_2 per setting (best witness)	2,370	2,366	2,357	2,354	2,334	2,318	2,307	2,306	2,306
N_3 per setting (fixed protected-sector witness)	136,907	103,900	85,362	81,395	65,381	57,902	54,001	53,632	53,591
N_3, LOCAL-G (force-type independent noise)	719,057	266,142	153,211	135,710	81,255	62,723	54,415	53,673	53,591

PROTECTION-DOES-NOT-PAY at the stated allowance for every registered f: the three-path count is certified above the two-path count in every column (the engine's own budget point, f = 1, agrees: 2,306 against 53,591). Typing (repair R4 (i)): decided at the sealing of RULE-B by hand item H-B2 of [P] (in the gate's words: the protected margin is at most 0.00952 against a two-path margin near 0.045, and the word needs only their ratio above 0.982); the counts are COMPUTED. Rider R11, verbatim: the fixed protected-sector witness against the best two-path witness, per setting, under RULE-SC; it is not a comparison of the best three-path certificate with the best two-path certificate at the stated allowance (NOT COMPUTED; the noise-free three-path negativity is 8.83 times the protected one). On total shots at f = 1 the three-path design needs 7 N_3 = 375,137 against 3 N_2 = 6,918 (the sealed secondary convention, whose three two-path settings are NOT DERIVED) or 9 N_2 = 20,754 (nine local Pauli

settings; repair R8: the variance constant 3/16 assumes fifteen separately estimated Pauli products, and with nine shared settings the bound needs covariance terms: NOT DERIVED); the word holds for any two-path setting count up to fifteen (7 x 53,591 = 375,137 against 15 x 2,306 = 34,590). No verdict rests on a total-shot reading. Data rows at the allowance: with three paths at (60, 15, 1) (variance scaled by 9/4) N_3 runs from 58,241 (f = 0) to 15,814 (f = 1), still above N_2; at equal span, (40, 5, 1) (variance scaled by 1/4), from 1,645,514 to 737,034.

Three data rows of the adversarial legs at the allowance (re-certified for this note by own intervals and by the engine, record [F]; none forms a verdict, and the outcome word above keeps its letter). (1) *The continuum of splits (remark R-g of [R]).* For EVERY split f between 0 and 1, registered or not (mismatch 1/1000, as in the cell), the best two-path witness has a margin above 0.0444, while the fixed witness of the protected sector ("protected" as in the rider block of section 1) has a margin of at most 0.0095240742; so the three-path count exceeds the two-path count on the whole interval of splits: there too the protection does not pay in the sense of rider R11 (a fixed witness against a best one, per setting, under RULE-SC), although the word of record is typed on the registered splits only. (2) *The datum behind "NOT COMPUTED" in rider R11 (remark R-h of [R]; item H-ST13 of [S]).* At the allowance the least eigenvalue of the partial transpose of the three-path STATE gives a best-witness margin from 0.0632 (f = 1) to 0.0759 (f = 0): above the two-path margin (0.0450 to 0.0444) at every registered split, and below the margin of the same register used as a two-path register on its outer paths (0.0888 to 0.1102). It uses coherences that common-mode noise does not spare, and RULE-SC defines no variance constant for a general witness of two three-path registers: no count is formed, and the comparison of the best certificates stays NOT COMPUTED as a count. (3) *The equal-resource comparator (defect D4 of [S]).* The registered (50, 10, 1) register can be used as a two-path register on its two outer paths, an argument that the design literature prints ([2101.08086v2 p12] "state including only the outer spin states and hence recreating a qubit state which performs better"). That pair has Delta = -422/875 and needs 386 (f = 0) to 594 (f = 1) shots per setting at the allowance, so against it the fixed witness (rider R11 again: fixed against best) costs 90.2 (f = 1) to 354.6 (f = 0) times more per setting, where the comparison of record, against the benchmark pair, gives 23.2 to 57.7. On the white paper's own hardware (one centre, the same product of gradient and time) the three-path register is the equal-span row (40, 5, 1): 319.6 to 694.3 times the two-path count per setting at the allowance; the registered three-path row presupposes twice that product (HAND, [S] section 4.2).

Where it would pay (data of equal standing; Lambda = total variance over the allowance, on the grid 1, 3, 10, 30, 100, 300, 10^3, 10^4, 10^5, 10^6).

f	Lambda at which the three-path count is below the two-path count	Lambda at which a three-path positive exists at all
0	never	1
1/4	never	1, 3
449/1000	never	1, 3
1/2	never	1, 3
3/4	never	1, 3, 10
9/10	never	1, 3, 10
99/100	30, 100	1, 3, 10, 30, 100
999/1000	30, 100, 300, 10^3	1, 3, 10, 30, 100, 300, 10^3
1	30, 100, 300, 10^3, 10^4, 10^5, 10^6	1, 3, 10, 30, 100, 300, 10^3, 10^4, 10^5, 10^6

The three-path count first falls below the two-path count at Lambda = 30, and on the registered grid of f only at f = 99/100 and above (at f = 9/10 never; nothing is certified between those two values: remark R-

d of [R]); at Lambda = 30 and f = 1: 169,303 against 53,592. For the registered f up to 9/10 the independent part destroys the protected state first. Reading, with its riders (R5, R11; reworded by text RW-10 of [S]): under this registered split the three-path design would pay only for an experiment whose uniform shared noise misses the allowance by more than an order of magnitude in variance while its independent noise meets it, with a 20 um register and force histories matched at the registered 1/1000; it is not a way to relax the white paper's requirement by the eight orders that separate it from tabletop isolation: at the registered mismatch 1/1000 the fixed witness ends near 289 rad of common-mode rms phase (section 6.4, with its rider RW-3), about three orders above the allowance's 0.173 rad.

6.2 Prediction P-R4, printed only with its riders (cell C4)

P-R4 of [O]: "Under Gaussian common-mode phase noise of rms sigma, mismatch 10^-3 and local coherence 0.98, there is a sigma* below 3 rad above which the three-path design reaches a 5 sigma witness in fewer shots than the two-path design at equal closest approach (30 um)." Registered rule: two paths (40, 10, 1) against three paths (50, 10, 1), sigma on the grid j/20 (j = 0..60) and 5, 10, 30 rad; sigma* is the least grid value from which the three-path count per setting stays below the two-path count (infinite where the two-path state is certified PPT). Rider R5 follows the table and belongs to every number in it.

sigma (rad)	0	0.5	0.55	0.6	0.65	0.7	0.75	3	30
N_2 per setting	2,696	65,484	136,632	348,850	1,408,627	62,659,974	none (state certified PPT)	none (state certified PPT)	none (state certified PPT)
N_3 per setting	215,731	215,734	215,734	215,735	215,735	215,736	215,737	215,826	225,523

*P-R4: TRUE on the registered primary reading, sigma = 3/5 rad.** Rider R5 of [G], which travels with every printing of P-R4 or of sigma*: the registered three-path row spans 20 um, twice the white paper's superposition size, and at equal span (40, 5, 1) there is no certified positive at local coherence 49/50; the window is 3/5 to 205.6 rad at mismatch 1/1000 (certified here on the continuum up to 30 rad by monotonicity, and to 205.6 rad because the two-path state stays PPT); the count is 80 times the noise-free two-path count; at the stated allowance the word is the other one (precisions of the two adversarial legs, wherever this text is printed: its "noise-free two-path count" is the count at zero common-mode noise and local coherence 49/50 (defect D-3 of [R]; section 6.2), and its 205.6 rad is a number of the registered mismatch 1/1000, the pilot's value and not a design's (rider RW-3 of [S]; section 6.4), and its "80 times" is the count while the product of rms phase and mismatch is negligible: along the window the count grows with that product, to 225,523 at 30 rad, and without bound toward the end of the window (closing audit, item B-1 of Appendix D; section 6.2)). In numbers (R5):

- The two-path state stops being entangled at sigma in [0.709764481, 0.709764957] rad, because local coherence 49/50 and common-mode noise together cross its PPT boundary; the three-path count stays flat because the protected coherences do not see sigma at matched spacing (at the registered mismatch 1/1000 the count is flat only while the product of sigma and the mismatch is negligible: the table above has 215,731 at 0 and 225,523 at 30 rad, and the count grows without bound toward the reach, where the margin vanishes; the fixed witness stays positive up to sigma = 205.617432 rad; rider RW-3 of section 6.4 says what that number is and what it is not; closing audit, item B-1 of Appendix D).
- (R5) The flat count is 80.0 times the two-path count at zero common-mode noise and local coherence 49/50 (215,731 against 2,696; the gate's phrase calls that count noise-free: defect D-3

of [R]). A pair with no noise at all needs 1,691 shots per setting, and the flat count is 127.5 times that; against the two-path register on the outer paths of the same 20 um register (409 shots per setting at zero common-mode noise and local coherence 49/50, the condition of the 2,696; defect D4 of [S]) it is 527.4 times. From sigma* on the three-path witness needs fewer shots per setting. Between sigma* and the two-path threshold of the previous item both designs still certify (on the grid the two-path count grows from 348,850 at 0.6 rad to 62,659,974 at 0.7 rad); from that threshold to the reach of the fixed witness the three-path design is the only one of the two that certifies anything. The gate keeps one sentence of [P] in these words: "above sigma* the three-path design is the only one that certifies anything" is true inside the registered model, grid and convention and must keep R5 and R11 beside it. A precision of this note: that sentence is exact from the two-path threshold on, not from sigma*; R11 applies to it as well (a fixed witness against a best witness). It never certifies cheaply.

- (R5) sigma* = 3/5 rad is 12 times the white paper's allowance in variance (0.36 against 0.03); at the allowance itself the outcome word is the other one (section 6.1).
- Total shots (R5, R8): sigma* = 13/20 with three two-path settings (NOT DERIVED), 3/5 with nine (variance constant NOT DERIVED for shared settings), 11/20 with fifteen separately estimated products (data).
- Data rows (R5). Three paths at (60, 15, 1): N_3 = 31,420, sigma* = 9/20. Three paths at EQUAL span, (40, 5, 1): no certified positive at local coherence 49/50 (margin -0.001933527179), hence no sigma*. Every "three paths" number of this note that is favourable to the protection is a number of the wider, 20 um design. The equal-span row is the three-path register on the white paper's own hardware (one centre, the same product of gradient and time: its +1 and -1 paths 10 um apart, the unforced path between them); the registered row presupposes twice that product (HAND; defect D4 of [S]).
- Committed geometry (data; own extension rows of [P]; no prediction scored): N_2 = 1,054 at sigma = 0; two-path PPT threshold in [1.000550270, 1.000551224] rad; three paths at (700, 250, 5/2): N_3 = 29,237, sigma* = 7/10; at (450, 125, 5/2): N_3 = 274,024, sigma* = 9/10 (riders R5 and R11 apply unchanged).

6.3 The price in local coherence (cell C3, prediction P-R3) and cell C7

Complete common-mode dephasing at matched spacing; local coherence lam on each register. Threshold lam_NPT = (2 cos((pi - |J|)/3))^(-1/2) (sections 5.1 and 5.2); NPT side certified by an inertia bracket at plus and minus 1e-12; separable side by Lemma T (the pilot's first certificate, an explicit mixture plus a cited separable ball, located the same weights p_-).

design	invariant	entangled iff lam exceeds	weight p_- at the threshold (Lemma T)	type
three paths (50, 10, 1), benchmark, not a design of record	211/1400	0.959713817036	0.05340763280715	closed form: COROLLARY; the "iff": COROLLARY of Lemma T (repair R4 (v)); bracket COMPUTED
three paths (60, 15, 1), data	211/750	0.929360005213	0.09339003233102	as above
three paths (40, 5, 1), data: equal span	211/5250	0.988639780258	0.01511939583409	as above
three paths (700, 250, 5/2), data: committed geometry, equal spacing	1055/3648	0.927650134973	0.09564038472465	as above

design	invariant	entangled iff lam exceeds	weight p_- at the threshold (Lemma T)	type
three paths (450, 125, 5/2), data: committed geometry, equal span	9495/66976	0.961937635007	0.05047264442671	as above
two paths (40, 10, 1), no common-mode noise (reference)	Delta = -211/1000	0.900224820400	not applicable	sqrt(1 + C^2) - C: CITED (section 5.5); two qubits: PPT is separable (cited)
two paths (450, 250, 5/2), committed geometry (reference)	Delta = -211/672	0.855798920369	not applicable	the anchor r* of [O], a regression; CITED (section 5.5)

P-R3 of [O]: both clauses TRUE (the threshold of the benchmark row lies in (0.93, 0.97) and has a closed form). Reading: the protected three-path state stays entangled only while the local coherence of each register is above 0.9597, the unprotected two-path benchmark while it is above 0.9002: the protected state needs MORE local coherence, as the ceiling rider says, and at equal span more still (0.9886). Cell C7 (the third clause of P-R7, a regression): at the benchmark angle -211/1000 the committed thresholds are F_ent = 0.904728143850 and v_DI = 0.994501181800 (C = 0.105304401989), which regress the experiment scout's printed 0.904728 and 0.994501.

6.4 Partial common-mode noise and mismatch (cell C5)

Two paths: the least sigma at which the state is PPT (bracket: certified NPT below, certified PPT above; rad).

design	eps 1/1000, lam 49/50	eps 0, lam 49/50 (CITED row)	eps 1/1000, lam 1	eps 1/100, lam 1
two paths (40, 10, 1), benchmark	[0.709764481, 0.709764957]	[0.710119724, 0.710120201]	[3.028190613, 3.028192520]	[2.270484924, 2.270486832]
two paths (450, 250, 5/2), committed	[1.000550270, 1.000551224]	[1.001051903, 1.001052856]	[3.143857956, 3.143859863]	[2.407722473, 2.407724380]

The two columns at zero mismatch are the CITED rows of section 5.5 (roots of a printed closed form); the columns with mismatch are COMPUTED and are not printed anywhere found. At zero mismatch and perfect local coherence the two-path state is NPT at every finite sigma (section 5.5): "the two-path pair is left separable" is a statement about the complete twirl; at finite common-mode noise the two-path certificate dies by its margin, or by any local dephasing or mismatch.

Three paths: the reach in the product sigma times eps (lower end of each certified bracket; values at eps = 1/1000; the lower ends at eps = 1/100 differ from these by less than 1e-6 in every one of the 13 pairs; section 5.3 predicts equal products).

reach in sigma times eps	(50, 10, 1)	(60, 15, 1)	(40, 5, 1)	(700, 250, 5/2)	(450, 125, 5/2)
fixed witness positive up to, lam = 1	0.28978931	0.39006543	0.15159814	0.39513037	0.28134741
fixed witness positive up to, lam = 49/50	0.20561743	0.33019458	none	0.33603735	0.19379443
state NPT up to, lam = 1	0.45692285	0.55209912	NOT RUN	NOT RUN	NOT RUN
state NPT up to, lam = 49/50	0.24337500	0.41754785	NOT RUN	NOT RUN	NOT RUN

Printing (remark R-b of [R]): the entries of this table are the eight-digit roundings that the log prints of the lower bracket ends; for instance 0.28978931 stands for the certified end 0.289789307, and the root of the closed form of section 5.3 is 0.2897894848, so the witness is positive at the printed value.

Reading (reworded by text RW-11 of [S]). With a mismatch eps of the two spin-force histories (the masses drop out: clarification (ii) of section 1) the fixed witness tolerates a common-mode rms phase of about 0.28/eps at the benchmark. At the registered eps = 1/1000 (the pilot's value, not a design's) and perfect local coherence that is 289.789307 rad against 3.028190613 rad for two paths (a factor 95.6 in phase); at local coherence 49/50, 205.617432 rad against 0.709764481 rad (a factor 289.6): two to two and a half orders of tolerable phase. At eps = 1/100 it is 28.978943 rad against 2.270484924 rad (a factor 12.7) at perfect local coherence, and 20.561737 rad against 0.7065 rad (a factor 29.1) at local coherence 49/50: one to one and a half orders (the two-path threshold at mismatch 1/100 and local coherence 49/50 is a cell that the finalize leg added: certified NPT at 0.706569916 rad and PPT at 0.706569993 rad, by an own route and by the engine). Never the eight orders that separate the white paper's requirement from tabletop isolation.

Rider RW-3 (steelman leg [S], items HS-3 and D1; it belongs to every printing of 205.6 rad and of the window of P-R4, beside rider R5 of the gate). 205.6 rad is a number of the registered mismatch 1/1000, not of a design. The benchmark's closest approach of 30 um exists only behind a shield, which the white paper names ([2610.07422v1 p5] "separated by a thin membrane, ideally a superconducting shield") and which the model of this note does not contain. In the linear geometry, the only one with a three-path invariant, the paths of a register sit at different distances from that shield; a common acceleration moves the shield against the free masses, and the phase between each mass and the shield turns that motion into a phase on the two inner coherences of the block d = 0, with the opposite sign on the two registers (clarification (i) of section 1; section 5.3). A static hand estimate, typed HAND (premises: a plate exactly midway, so that the paths of each register sit at 15, 25 and 35 um from it; the Casimir term alone, with the phase rate that the screening design prints, eqs. (23) and (24) of arXiv:2307.07536, proportional to the inverse fourth power of the distance; diamond of 10^-14 kg, with relative permittivity 5.7 and density 3510 kg/m^3 as the steelman leg takes them; masses free along the axis for 1 s; the constant-spacing model; nothing integrated): an effective mismatch of 0.0118 for this channel alone, and with it a reach of 30.1 rad at local coherence 49/50 where the registered mismatch gives 205.6 rad. The steelman leg's band-averaged estimate is lower still and is not re-derived here ([S] section 3.6). For translated registers in one field whose gradient varies over a length L_c the mismatch is D/L_c (HAND): 1/20 at one millimetre, 1/100 at five. No cell is re-scored by this rider; the bounded reproduction that KR-4 of [O] reserves for the shield-mediated channel of arXiv:2604.22593 would replace the estimate and is the owner's call.

6.5 Data rows: what the twirl costs, and the parallel four-path rung (cells T-d and C8)

row	value	type
noise-free negativity of the three-path gate state, (50, 10, 1)	0.084110405688498 (three negative eigenvalues, each enclosed; none further)	COMPUTED
protected negativity, (50, 10, 1): (2 cos((pi - J)/3) - 1)/9	0.009524074220937; the noise-free value is 8.831347146 times this	COROLLARY
the same at (60, 15, 1)	0.125036513262222 and 0.017532888182016; ratio 7.131541134	COMPUTED, COROLLARY
four paths, parallel, (50, 10, 1): J_par = K_1	-0.025267927205899 rad	COMPUTED
its twirled negativity (2 cos((pi - abs J_par)/3) - 1)/8	0.001819100231 (two eigenvalues at the closed form, certified)	COROLLARY
its NPT threshold in local coherence	0.992802067058 (bracket)	COMPUTED; separability below it NOT BUILT, so no "iff"

6.6 The committed geometry of record

The design of record has two paths, (450, 250, 5/2), Delta = -211/672. Its anchors regress by two legs: F_ent = 0.864790122 (the record's 0.864790); the state-level threshold (1 + cos Delta)/2 = 0.975554698 of [E1-M] (the record's 0.975555 by rounding); N_min = 1105; the path-dephasing threshold r* = 0.855798920 (CITED: section 5.5); F_sep = 0.444286581 (the record's 0.444287 by rounding). The record has no three-path member at the committed geometry; the rows (700, 250, 5/2) and (450, 125, 5/2) above are own extension rows of the pilot, data only. The law column reads nothing there (section 8.4).

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7. What a positive and a null on the protected sector certify

"Protected" carries the rider block of section 1 throughout. The statements of record are Theorems W and W' of [W] (`06_witness_theorem.md`, sha256 `94b3ea235bad8d1d904b045ea0ef5eecf7ed4b3629adfbc86e57b90ee8ea8eaf`), transcribed below from their lines through a fixed character map (em dash to " - ", en dash to "-", Greek letters spelled out, the double arrow to "iff", the rightwards arrow to "->", the not-equal sign to "!=", the section sign to "section ", the prime to an apostrophe, the superscript plus to "+", the infinity sign to "infinity", the logical-and sign to "and", and the accented letters of two surnames to their base letters: the complete map, defect D-4 of [R]); nothing in them is amended here.

7.1 A positive

Soundness comes for free; the theorem does not. The fixed witness of section 5.3 is W = Q^Gamma with Q positive, so tr(W s) >= 0 for EVERY separable state s of the two registers, whatever the noise and whatever the model: a certified negative value is a statistic with a sound separable bound, which is what the head sentence of clause 1 of Definition 1 of [W] asks for. A classical channel acting on a product input outputs a separable state and so cannot produce it (prediction P-R7, clause 2, a COROLLARY; exercised exactly on 48 channel rows by the pilot and on 48 rows of four channels of the gate's own construction: section 7.3).

Theorem W, verbatim (head, the two readings, and the closing sentence):

> **Theorem W.** Assume W1-W6 at the BMV regime of section 1.1, and suppose the protocol returns a certified positive result (Definition 1, all three clauses). Then the degree of freedom that coupled the two labs is **non-classical**, in the following exhaustive sense. Exactly one of two structural readings holds - they agree on all mass-side data and differ only in the S1-S3 signatures (Lemma M Theorem 3 [V]):

> **(I) Mediation reading.** The channel satisfies Definition M with mediator M (the gravitational degrees of freedom). Then M is non-classical:

> **(II) Direct-gate reading.** The channel fails Definition M: constrained-direct-gate semantics - **no mediating degree of freedom at all**. The Newtonian sector as pure constraint bookkeeping: no independent initial data (the field remains exactly in vacuum [V, 04]), a direct diagonal A-B term after reduction, operator-Schmidt rank exactly 2 at all times, no dead time (entanglement from t = 0+ at every separation, rate = coupling [V, Lemma M T5]) - the c_M -> infinity point of the timing family. This reading **concedes the certified entanglement** and is analyzed in section 1.5.

> **In either reading:** the observed entanglement is attributable to no classical carrier - (I) certifies a non-classical mediator; (II) asserts a nonseparable quantum interaction with no carrier - and every classical-gravity model in the corpus (Aziz-Howl c-number, Diosi static-background, Schrodinger-

> Newton/mean-field, Struyve separable-potential, Sienicki channel, decohered-field, KTM, Oppenheim CQ within the scoped class) is refuted on its own formulation (Corollary W2). What a positive result **cannot** do, from mass-side data alone, is discriminate (I) from (II) [V, polaron identity; principled by Lemma M: the readings differ only in S1-S3, which BMV does not measure].

How it applies to the protected sector: a CONDITIONAL corollary (repair R3 of [G]; the word "verbatim" of [O] section 5.2 (ii) and of prediction P-R7, clause 1, is struck). Verbatim from the gate: the conclusion of Theorem W follows for a certified protected-sector positive once W1 to W6 and clauses 2 and 3 of Definition 1 are re-established at the benchmark regime for a three-path register; clause 1 is met through its head sentence (H-W1: W = Q^Gamma with Q positive, so the separable bound holds with no model premise). Why not more: the printed theorem is scoped to the committed two-path regime of [W] section 1.1 (450 um, 250 um, closest approach 200 um), and clause 1 of Definition 1 lists two statistics, neither of which is this witness. **NOT CHECKED, and owed by the experiment before the sentence may be used:** clause 2 of Definition 1 (the electromagnetic budget at a closest approach of 30 um), clause 3 (per-run statistics), and premises W1 to W6 for a multi-path platform; the readout of a three-path register in seven interference settings is not a design of record, and the platform sentence is NOT CHECKED as a three-path scheme (section 3). Also NOT CHECKED (defect D10 of [S]): the sound separable bound is that of the ideal operator W; the bound of the estimator assembled from seven measured settings under a miscalibration of those settings is not derived (true of every multi-setting witness), and the witness value is the small difference of two terms, 0.0789 of the larger one at the benchmark and 0.0225 at equal span (both at perfect local coherence), against 0.1528 for the two-path benchmark witness at the allowance with all of it shared (the split f = 1, where the local coherence is perfect as well). The phases of the witness must absorb the static phases of the inner pairs of paths and need them stable over the campaign ([S] section 4.3; section 7.2 says what a wrong phase does to a null); the electromagnetic budget of clause 2 includes the shield of rider RW-3 (section 6.4); and each run needs a three-outcome, single-shot readout of both registers after a pair-selective rotation, NOT CHECKED for any platform.

What a positive would then certify, and no more: "non-classical" in the sense of Definition 2 of [W], in the form of Theorem W's disjunction, for the channel between the two registers. It is a statement about a channel and its detection. It says nothing about the mediator beyond that disjunction, and neither of the two slogans at the head of the ban list of section 1 follows from it.

7.2 A null

Theorem W', verbatim (the sensitivity clause, the theorem, its two branches in their opening sentences, and its closing line):

> **Sufficient sensitivity:** decoherence budget independently certified by single-mass controls leaves 1.1563 resolvable with margin; multi-point (d, tau) scan excludes the measure-zero accident Delta = 2pin; fair sampling (W6).

> **Theorem W'.** At sufficient sensitivity, a null certifies **unconditionally**: the gravitational channel imparts no second-difference phase - branch phases compose additively at the probed regime (entanglement iff Delta != 2pin is exact [V, 01]). Interpretation is then conditional on **linear phase additivity** (MV's own caveat, [V]-flagged, U9):

- > **Under it:** the null refutes the conjunction {quantized gravity and linearized coupling and QM for matter at these scales}.
- > **Without it:** the null is uninformative about mediator quantumness (nonlinearly composing quantum mediators also predict null). Not closable by the experiment itself.

> **A null never certifies "gravity is classical"** - all corpus classical models predict null, but so do modified-coupling quantum models; affirming the consequent is unavailable.

It does NOT transfer to the protected sector. Its unconditional clause is about the second-difference phase; the protected sector does not see the second difference at all: that is the content of Theorem R. What a protected-sector null certifies instead:

- **Inside the path-diagonal model**, at certified local coherence lam: at matched spacing (sigma times eps negligible) only that |J|, reduced to (-pi, pi], is below pi - 3 arccos(1/(2 lam^2)) (section 5.6): below 0.4772 of the Newtonian J at lam = 49/50 and below 0.2341 of it at lam = 99/100. With a mismatch the bound is weaker (defect D-1 of the rigor leg [R]: the draft attached the matched-spacing bound to the whole reach of section 6.4): a null of the fixed witness then bounds |J| only below the root of 2 cos((pi - |J|)/3) = 3/(lam^2 (2 b + b^4)), b = exp(-(sigma eps)^2/2), that is below 0.5075, 0.5991 and 0.9713 of the Newtonian J at sigma times eps = 1/20, 1/10 and 1/5 for lam = 49/50, and below the Newtonian J itself at the edge of the reach. And it bounds |J| at all only if the witness is read with the phases of the state (a precision of the finalize leg [F], with [S] section 4.3): the phase step of the fixed witness per path is J/3 - a up to a third of a turn, with a = g(1) - g(-1) a first difference of the phase table, which another law changes; a witness phased for the Newtonian table and read on another table can be null whatever |J| is (on the Newtonian state itself an offset of 1/2 rad per path already gives a negative margin: -0.0115 for the sign of the offset that the certification of record ran, and a more negative value for the other sign). The bound therefore needs those phases scanned, or the block reconstructed from the two quadratures of each pair of paths. A null is a statement modulo 2 pi (repair R1; section 8.5), and it has a geometric zero under tilt (section 4.3).
- **Outside the model** (gradient noise, imperfect closure, unequal spacing histories, an unmatched mass times spacing beyond the reach): nothing.
- A null of either kind, on either sector, never feeds a classicality sentence about gravity ([W], the last line quoted above).

7.3 The teeth: classical channels cannot fake a positive (the mandatory control P-R-ctrl of [O])

Reading fixed before any script: "PPT in every sector" means that every block of the partial transpose of the twirled output is positive semidefinite; it never means a postselected sector (a single postselected sector of a product state is entangled: the pilot's check S2-h5 and the gate's own check). Pilot: three measure-and-prepare channels (a sharp which-path record of A fed forward; unsharp path monitoring fed forward; both registers measured in the Fourier basis with product re-preparation) and one two-way LOCC protocol, on four phase tables and three amplitude sets, exact arithmetic: 48 channel rows, 96 inertias, no negative eigenvalue, twirled or not; the gate itself, as the control of opposite sign, is NPT after the twirl exactly on the non-trivial tables. Gate: four channels of its own construction (twelve mutually unbiased effects with re-preparation and feed-forward; a Fourier measurement of B fed back to A; twelve correlated local unitaries with unequal weights; three rounds of weak measurements in both directions): 48 exact inertias, no negative eigenvalue. The test has teeth in both directions and did not fire (KR-ctrl of [O]: not fired).

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8. The law column (static, tree-level, one registered family; every bound CITED)

Riders on every line of this section: the potentials are static and tree-level; the family is the registered one (Newton; one Yukawa term; the Stelle form); torsion-balance bounds are cited, never re-derived; the benchmark is the white paper's representative point, not a design of record; the plot of the two-path phase against the range is OCCUPIED (van Manen, Blankenstein and Mazumdar, arXiv:2609.22501; Matsui, arXiv:2610.25994, whose trilemma is cited and never re-derived), and the house's delta is the ladder of two readings and the null rule, and it is small. Everything here is a property of closed forms on one static family, never a finding about gravity, and nothing here bears on quantization. "Protected" carries the rider block of section 1.

8.1 The window that the cited limits leave open (CITED; two readings of one measurement)

Source of record. Lee, Adelberger, Cook, Fleischer and Heckel, PRL 124, 101101 (2020), arXiv:2002.11761 v1: one added Yukawa term, V = V_N [1 + alpha exp(-r/lambda)], limits on the modulus of alpha at 95 percent confidence, with the PRINTED sentence [2002.11761v1 p5] "We find that any gravitational-strength Yukawa interaction must have" a range below 38.6 um.

one Yukawa term of strength	modulus curve of the 2020 figure, CURVE-READ by [P] (own re-read of [L2])	sign-resolved limit, TABLE (thesis), from [L2]	bracketing printed rows of the thesis table
-4/3 (member M1: the Stelle form with the scalar decoupled)	open below 35.742 um, plus or minus 0.5 (35.7358 um)	**open below about 24.7584 um**	22 um: -2.41 (allowed); 25 um: -1.27 (excluded)
-1 (member M2: the Stelle form at equal ranges)	38.6 um PRINTED; curve read 38.584 um (38.5757 um)	about 26.9533 um	25 um: -1.27; 28 um: -0.886
+1/3 (member M3: the spin-2 term decoupled)	open below 54.01 um, plus or minus 0.7 (53.9899 um)	about 56.9763 um	56 um: 0.35 (allowed); 63 um: 0.251 (excluded)

Riders, all of them. (i) The sign-resolved limits were read by lane [L2] in the first author's dissertation (its Table 11.4: [thesis p145] "We placed positive and negative exclusion limits,"), from a web-archive copy; the journal's Supplemental Material answered HTTP 401 and is NOT READ. (ii) [L2] tested that the dissertation's limit set is the published one (its modulus column agrees with the published curve at all 45 tabulated ranges inside the drawn span; it crosses 1 at 38.6298 um against the PRINTED 38.6 um). (iii) An edge between two printed rows is a log-log interpolation, a LOCATOR; its certificate is the two bracketing rows. (iv) One Yukawa term only: the two-term Stelle form with unequal ranges (member M4) is NOT BOUNDED JOINTLY by any reading here, and the cited literature says the same: [2610.25994v1 p20] "so a quantitative constraint requires fitting both terms". (v) Lane [L2] is one leg at scout grade, without an adversarial verifier; the pilot's cells and the gate's ruling used the modulus curve, and by the assembly rule no cell is re-scored: both windows are printed side by side. (vi) Lower end: the published curve is drawn down to 5.84 um, where it reaches a modulus of 10^6. (vii) Kill criterion KR-3 of [O] (a new bound closing the strength-4/3 ranges above 10 um, or the benchmark superseded): NOT FIRED ([L2], [L34]); the sign-resolved item NARROWS the window and is the 2020 measurement's own table, not a newer bound; levitated-sphere, Casimir-shielded and micro-cantilever searches exclude nothing at strengths 4/3 and 1/3 ([L2]).

8.2 The law table (cell L1): each reading as a ratio to its Newtonian value

Readings: rho_2, two paths at (40, 10): the second difference; rho_3, three paths at (50, 10): the invariant J; rho_4p, four paths in the parallel geometry at (50, 10): J_par. With x = exp(-10 um/range) the Yukawa ratio polynomials are exact, because every distance is a multiple of 10 um: **A_2 = 6x^5 - 15x^4 + 10x^3, A_3 = 14x^3 - 21x^4 + 14x^6 - 6x^7**, both increasing strictly from 0 to 1 (A_2' = 30x^2(1-x)^2, A_3' =

42x^2(1-x)^3(1+x)). Member M1: rho = 1 - (4/3) A(l2); M2: rho = 1 - A(l); M3: rho = 1 + (1/3) A(l0). All entries COMPUTED from the exact polynomials (rho_4p by intervals); rows marked (a) are data rows of this assembly at lane [L2]'s locators and bracketing rows (check A-6 of [B]; rho_4p NOT RUN there).

member	range (um)	rho_2	rho_3	rho_4p	status under the cited limits
M1	5.84	0.941334933	0.919416977	0.994647720	the lower end of the drawn curve
M1	10	0.648581624	0.544504294	0.900045834	inside both windows
M1	15	0.299806635	0.152614588	0.670165698	inside both windows
M1	20	0.074956874	-0.063487934	0.445667115	inside both windows
M1	22 (a)	0.012443663	-0.117304490	NOT RUN	inside both windows (a printed row of the thesis table: allowed)
M1	24.7584 (a)	-0.055553000	-0.172149649	NOT RUN	the sign-resolved edge (LOCATOR)
M1	25	-0.060674731	-0.176110746	0.270434346	inside the modulus-curve window; excluded for the negative sign (printed row)
M1	30	-0.144121276	-0.236845912	0.140978743	inside the modulus-curve window only
M1	35.742	-0.203543661	-0.275009854	0.033937133	the modulus-curve edge (CURVE-READ)
M1	38.6	-0.224135690	-0.287022381	-0.007205056	outside both (the edge that [O] used)
M2	10	0.736436218	0.658378221	0.925034375	inside both windows
M2	20	0.306217655	0.202384049	0.584250336	inside both windows
M2	25 (a)	0.204493951	0.117916940	NOT RUN	inside both windows (a printed row: allowed)
M2	26.9533 (a)	0.176510474	0.096860482	NOT RUN	the sign-resolved edge (LOCATOR)
M2	28 (a)	0.163513503	0.087449460	NOT RUN	inside the modulus-curve window; excluded for the negative sign (printed row)
M2	30	0.141909043	0.072365566	0.355734058	inside the modulus-curve window only
M2	38.6	0.081898232	0.034733214	0.244596208	the PRINTED modulus edge
M3	10	1.087854594	1.113873926	1.024988542	inside both windows
M3	20	1.231260782	1.265871984	1.138583221	inside both windows
M3	30	1.286030319	1.309211478	1.214755314	inside both windows
M3	40	1.308167564	1.322947352	1.256259516	inside both windows
M3	54.01	1.321042741	1.329341858	1.286589773	the modulus-curve edge (CURVE-READ)
M3	56 (a)	1.322096966	1.329791933	NOT RUN	beyond the modulus-curve edge; allowed for the positive sign (printed row)
M3	56.9763 (a)	1.322570587	1.329989654	NOT RUN	the sign-resolved edge (LOCATOR)
M3	63 (a)	1.324977961	1.330948019	NOT RUN	excluded (printed row)

(The full grid, and twelve points of the two-range member M4, which no cited bound constrains jointly, are in the regenerated log of record.) Reading. At the benchmark the two-path reading runs from 0.94 of its Newtonian value at the lower end of the drawn curve to -0.20 at the modulus-curve edge of member M1; on the sign-resolved window it runs from 0.94 to about -0.055, and still crosses zero inside the window (section 8.3). On M2 it stays between 1 and 0.08 up to the PRINTED 38.6 um (down to 0.176 at the sign-resolved locator); on M3 it runs from 1 up to 1.32 at the modulus-curve edge (truncated digits). That is the sense in which the benchmark reads the law at order one where the committed geometry

reads nothing; the sentence of [O], "between +1 and about -0.2 times its Newtonian value", holds on the modulus-curve window and reads "between +1 and about -0.05" on the sign-resolved one.

8.3 Zeros, the ladder, and prediction P-R5

cell	value	certificate	type
zero of rho_2 on M1	l2 = 22.4514012387 um	single root of A_2 = 3/4 (Sturm count), certified bracket; inside both windows	COMPUTED
zero of rho_3 on M1	l2 = 18.1640474040 um (a rounding: the gate's enclosure is 18.16404740397856; repair R7)	the same for A_3 = 3/4; inside both windows	COMPUTED
zero of rho_4p on M1	l2 = 38.064348382 um, OUTSIDE both windows; no sign change on the grid inside them	certified sign change; uniqueness NOT DECIDED (grid data)	COMPUTED, data
M2 and M3	no zero of rho_2 or rho_3 at any finite range	A_2 and A_3 increase strictly from 0 to 1	COROLLARY
rho_3 at the zero of rho_2; rho_2 at the zero of rho_3 (M1)	-0.127642329613; 0.144714574963	interval evaluation at the certified roots	COMPUTED
least value over ALL ranges of the larger modulus (M1)	0.068865094372 at 20.177951524 um	single root of A_2 + A_3 = 3/2 and monotonicity; inside both windows	COMPUTED
P-R5, the benchmark pair, one Yukawa term	the zero curves alpha = -1/A_2 and alpha = -1/A_3 never meet at any finite range	A_2 - A_3 = 2x^3 (x-1)^3 (3x+2) exactly	COROLLARY of a polynomial identity
P-R5, the same apparatus: the nine two-path sub-registers of the three-path register, each against J	no meeting point for any of the nine	nine identities of the same kind (below)	COROLLARY (repair R4 (iv); first typed COMPUTED by Sturm counts)
on the zero curve of one reading, the other reading over the open part of that curve, modulus-curve reading	at least 0.051413112993 in modulus (rho_3 on the zero curve of rho_2) and 0.048899060091 (rho_2 on the zero curve of rho_3), both attained at the far end, 38.584 um	A_3/A_2 strictly monotone; a conservative six-point reading of the cited curve	COMPUTED lower bounds
the same on the sign-resolved limits (a)	the open part ends near 25.0955 um and 26.0162 um (LOCATORs of [L2]); there the other reading is -0.1081 and 0.0928, at strengths -1.2547 and -1.1189	this note's enclosures at the lane's locators as printed (check A-6 of [B])	COMPUTED at a CITED locator; data

The ten identities (each difference of ratio polynomials equals c x^3 (x-1)^3 (x - x0) with x0 outside (0,1); regenerated from the engine's own polynomials and equal to the gate's list; "A(i, j) x B(k, l)" is the two-path sub-register on paths i, j of one register and k, l of the other):

pair: reading A against the three-path invariant	A - A_3 equals
benchmark (40,10) two paths	(6) x^3 (x-1)^3 (x - (-2/3))
A(0, 1) x B(0, 1)	(6) x^3 (x-1)^3 (x - (-7/3))
A(0, 1) x B(1, 2)	(21) x^3 (x-1)^3 (x - (-2/3))
A(0, 1) x B(0, 2)	(126/11) x^3 (x-1)^3 (x - (-11/9))

pair: reading A against the three-path invariant	A - A_3 equals
A(1, 2) x B(0, 1)	(6) x^3 (x-1)^3 (x - (-2/3))
A(1, 2) x B(1, 2)	(6) x^3 (x-1)^3 (x - (-7/3))
A(1, 2) x B(0, 2)	(6) x^3 (x-1)^3 (x - (-11/9))
A(0, 2) x B(0, 1)	(6) x^3 (x-1)^3 (x - (-11/9))
A(0, 2) x B(1, 2)	(126/11) x^3 (x-1)^3 (x - (-11/9))
A(0, 2) x B(0, 2)	(63/8) x^3 (x-1)^3 (x - (-11/9))

P-R5 of [O]: TRUE, in the strong form: no meeting point exists at all, so the cited curve is not needed for the verdict. **LADDER-INDEPENDENT**, typed COROLLARY (repair R4 (iii)): the two rungs read different functionals of the registered family. The former words "no blind point in the cited window" are replaced, verbatim from repair R1 of [G], by: no common zero of the two ratio functions at any finite range; joint aliased nulls exist for one Yukawa term at strengths from about 190 (23.3 um) upward and are bounded only by the cited curve, whose vertices are not in the record. Since the gate, lane [L2] has put the curve's vertices and the signed table into the record and typed every alias (section 8.5).

What the ladder is worth, said plainly. The zero curves approach each other as the range grows (A_2 = 1 - O((1-x)^3), A_3 = 1 - O((1-x)^4)), so the reading that is not nulled is smallest at the far edge of the open region: 0.0488 and 0.0514 of its Newtonian value on the modulus-curve reading (truncated digits of the two lower bounds above; defect D-2 of [R]), which is the size of the record's 5 percent phase-model precision, and 0.0928 and 0.1081 in modulus at the sign-resolved locators (the last row above). On member M1 the least separation is 0.0688. On the zero curves the strength is negative, with a modulus from 1.03 to 22.7 on the modulus-curve reading (truncated digits of the two end values): the sign-resolved limits are the relevant ones there. Two different geometries are compared in the benchmark pair. The judge's "0.0689263 at 20.18 um" is the value at a grid point (rho_2 = 0.068795611, rho_3 = -0.068926266 there); the certified minimum is 0.068865094.

The ladder exists inside the two-path rung (item HS-5 and text RW-5 of [S], defect D7; exact identities and counts re-certified, record [F]). The nine two-path sub-registers of the (50, 10) register have six distinct ratio polynomials, and all fifteen pairs of them differ by c x^3 (x-1)^3 or by c x^3 (x-1)^3 (x - x0) with x0 outside (0, 1): no two of them share a zero of 1 + alpha A at any finite range, so two two-path geometries of one apparatus already read different functionals of the law. One of the nine, A(1, 2) x B(0, 1), is the benchmark reading itself. At the zero of that reading on member M1 (l2 = 22.4514 um) the protected sector ("protected" as in the rider block of section 1) needs 3,203,752 shots per setting at perfect local coherence (protected negativity 0.001231794738), while the other sub-registers of the same register need 26,136 (the outer pair) to 254,440: 12.5 to 122.5 times fewer (truncated ratios; RULE-SC; no noise). The protected rung is the one that survives a uniform shared force, not the one that a second reading of the law needs; a second two-path geometry is the multi-point scan that the sensitivity clause of Theorem W' already asks for (section 8.5).

8.4 The committed geometry of record (cell LC): SATURATED

Two paths at (450, 250): rho - 1 = -0.012468724 (M1 at 35.742 um), -0.014144118 (M2 at 38.6 um), +0.020525992 (M3 at 54.01 um). Every modulus is certified below 5/100, the phase-model precision of the record: typed **SATURATED** (the rule was registered before the readout). The figures of row R21 of [T] at 52 um regress (-5.3442 and -7.1256 percent) but 52 um lies outside the cited single-term windows. So the law column is a statement about the benchmark only; at the geometry of record it reads nothing.

8.5 The null rule, with the alias rider (repair R1 of [G])

The rider. A witness null is a statement modulo 2 pi: the sensitivity clause of Theorem W' sets aside "the measure-zero accident" of a phase equal to a multiple of 2 pi by a scan, and Theorem R says "vanishes modulo 2 pi". Every row below is for readings reduced to (-pi, pi], and "the law lies on the zero curve" always reads on the zero curve or on an alias curve |1 + alpha A_2| = 29.78 |k|, |1 + alpha A_3| = 41.69 |k'|. The fixed-strength members have no alias: the members M1, M2, M3 have readings at most 4/3 of Newtonian in modulus and no alias. For one Yukawa term of free strength the gate located 464 joint aliased nulls with multiples up to 40 (both witnesses exactly null, neither reading zero); the one of least strength inside the pilot's six-point reading sits at 23.319478 um with alpha = +192.8662, where the two-path phase is 2 pi times -5 and the protected invariant 2 pi times 4 (certified brackets). Lane [L2] typed all of them against the published limits:

typing of the 464 joint aliased nulls (lane [L2])	EXCLUDED	OPEN	on the margin	not covered
1: the modulus curve of 2020 (CURVE-READ)	336	86	2	40 below the drawn span
2: the lowest of the eleven curves of the figure	368	94	2	0
3: the signed table (TABLE (thesis))	357	65 (47 of positive, 18 of negative strength)	2	40 below 5 um

Drafted wording of [L2] for this rider (drafted, not entered anywhere): joint aliased nulls with multiples up to 40 are excluded by the published 2020 limits at every range above about 12 um; those left open lie between about 6.4 and 11.9 um at strengths of about 800 and more in modulus; 40 further ones lie below the drawn and tabulated ranges and are not covered (CURVE-READ and TABLE (thesis); one Yukawa term). The least-strength alias above is excluded under all three typings. Status after the adversarial legs: the rigor leg re-derived typing 3 row by row and only recounted typings 1 and 2; the finalize leg regenerated all three typings from an isolated copy of the lane's script (log and table identical to the lane's) and re-typed typings 1 and 2 from the lane's transcribed vertices by an own interpolation, with every one of the 464 labels reproduced. The vertices themselves are a transcription of a published figure by one scout leg; no adversarial leg has opened the figure.

The rule (which combination excludes which law; every row inherits sections 7 and 8.1; sensitivities are the closed forms of sections 5.1, 5.2 and 5.5).

observed, at sufficient sensitivity, readings reduced to (-pi, pi]	certifies (licensed wording)	on the registered family	needs
N-1: a two-path witness null with an extracted angle consistent with 0	no second-difference phase at the probed regime, modulo 2 pi (Theorem W', unconditional clause)	the law lies on the zero curve of rho_2 or on an alias curve: M1 at l2 = 22.4514 um; one Yukawa term on alpha = -1/A_2 or on an alias; Newton, M2 and M3 are excluded where resolvable	the two-path record's sensitivity
N-2: a protected-sector null	inside the path-diagonal model only, and only for a witness read with the phases of the state: the modulus of J, reduced, is below the size that the certified local coherence and the certified product sigma times eps resolve (section 5.6 at	the law lies near the zero set of rho_3 or on an alias curve: M1 at l2 = 18.1640 um; alpha = -1/A_3 or an alias	local coherence above 0.9597 to resolve the Newtonian J; sigma times eps below 0.2056 at lam = 49/50; the tilt of section 4.3 excluded

observed, at sufficient sensitivity, readings reduced to (-pi, pi]	certifies (licensed wording)	on the registered family	needs
	matched spacing; section 7.2 with a mismatch)		
N-3: an extracted two-path angle (the E1-M route)	rho_2 directly, within the 5 percent phase model, modulo 2 pi	a band around one level curve of rho_2	as [E1-M]
N-4: a two-path null AND a protected-sector positive	the second difference vanishes modulo 2 pi and the third difference does not	M1 at 22.45 um: rho_3 = -0.127642330, J of modulus 0.019237523 rad, protected negativity 0.001231794738	local coherence above 0.994502591
N-4': a protected-sector null AND a two-path positive	the mirror row	M1 at 18.16 um: rho_2 = 0.144714575, Delta of modulus 0.030534775 rad, two-path margin 0.007633397	local coherence above 0.984849736
N-5: both null, each at a sensitivity that resolves the other's row	no member of the registered fixed-strength family (Newton, M1, M2, M3) inside the cited windows of section 8.1 (remark R-e of [R]: on M2 both readings tend to zero as the range grows, 0.0818 and 0.0347 of Newtonian at 38.6 um); for one Yukawa term of free strength a joint alias remains (the table above)	on M1 some reading is at least 0.0688 of Newtonian at every range; on the zero curves of one term at least 0.048 (modulus-curve reading)	local coherence above 0.997020224 (M1) to 0.997772282 (zero-curve edge) for the protected reading, 0.992761188 to 0.994854479 for the two-path reading

A joint null would therefore not be a statement about the fixed-strength family; what it would say is Theorem W' for the two-path sector and section 7.2 for the protected one, never more. The sensitivities in the last column are the honest price of the ladder: the confirming reading at the other's null needs a local coherence above 0.984 in the least demanding row (N-4') and above 0.997 in the most demanding one (N-5 at the zero-curve edge), in truncated digits. Row N-4 has a cheaper twin inside the two-path rung ([S] section 6): a two-path null at one geometry and a two-path positive at another (section 8.3). And for a bench the larger ambiguity of a null on the sector that common-mode dephasing spares is not an alias of the law but geometry and apparatus: tilt (section 4.3), the mismatch and the phases of the witness (section 7.2), the shield (rider RW-3 of section 6.4).

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9. The Gaussian reading (prediction P-R6 of [O]): a typed row on one exact finite example, never headline

Cited first. Datta and Miao, arXiv:2104.04414 v2 (read in full by the pilot): for two oscillators the gravitational coupling, to second order in the relative coordinate, shifts the frequency of the differential mode and leaves the common mode alone: [2104.04414v2 p1] "The gravitational interaction affects only the differential one" and [2104.04414v2 p1] "In principle, this squeezing is equivalent to quantum entanglement between the masses."

The example (registered by the pilot before any computation; exact rational arithmetic; units hbar = m = 1 and trap frequency 1; an exact finite example, not a physical size). H = (p_+^2 + q_+^2)/2 + (p_-^2 +

q_-^2/4)/2, that is local terms 5/16 (q_A^2 + q_B^2) and the bilinear coupling 3/8 q_A q_B; both oscillators start in the vacuum; time pi; a white force on the common mode adds n times the identity to the common-mode covariance and nothing to the differential mode. Output covariance in (q_+, p_+, q_-, p_-): diag(1/2 + n, 1/2 + n, 2, 1/8).

registered n	0	1/2	1	3/2	2	10	1000
partial transpose violates the uncertainty relation (NPT; exact inertia)	yes	yes	yes	no	no	no	no
explicit separable decomposition	no	no	no	yes	yes	yes	yes
Var(p_-) (vacuum value 1/2)	1/8	1/8	1/8	1/8	1/8	1/8	1/8
Var(q_+) Var(p_-) (separable bound 1/4, cited)	1/16	1/8	3/16	1/4	5/16	21/16	2001/16
channel CC-A reproduces the whole covariance matrix	invalid	invalid	invalid	yes	yes	yes	yes

- **Clause 1: TRUE on the registered example.** For n at least n* = 3/2 the two masses are separable (an explicit Gaussian mixture of displaced product states) while the differential momentum stays 6.0206 dB below the vacuum at every n; below n* the state is NPT. Here a FINITE common-mode noise already removes the certificate.
- **Clause 2: TRUE on the registered example.** Channel CC-A: each mass evolves alone under (p^2 + q^2/4)/2 for the time pi (a local unitary; no interaction, no communication), then both are displaced by shared classical Gaussian randomness on the common mode. For every registered n at least 3/2 its output covariance EQUALS the coupled and noisy one; both states are zero-mean Gaussian, so the states are equal.
- **Typing (repair R4 (ii) of [G]):** exhibited by construction (H-G1), machine-checked exactly; CC-A is local unitaries plus shared randomness with local squeezers that no named model supplies.
- **What this licenses, and no more.** On this example the squeezing of the differential mode is a diagnostic of a coupling model and not a certificate that excludes classical channels; the statistic with a sound separable bound is the product Var(q_+) Var(p_-) (bound 1/4, cited), and common-mode noise acts on its first factor. It does exclude the cited paper's own printed alternative on this example: with both modes at one shifted frequency the differential variance would be 0.429889402538, not 1/8 (data). The cited paper's sentence on squeezing and entanglement is a statement without the shared noise and is not contradicted.
- **Printed neighbours of single clauses ([L34]; no posting prints both).** Miki, Matsumura and Yamamoto, arXiv:2311.00563: [2311.00563v1 p10] "The gravity-induced squeezing appears when the thermal (classical) noises are dominant." (their squeezing is a ratio of the differential to the common mode, not a variance below the vacuum). Miki et al., arXiv:2210.13169 (a common-mode condition beside the differential one for entanglement); Matsumoto et al., arXiv:2507.12899 (a common-mode acceleration-noise requirement for the Gaussian certificate).
- **A hand reading of [L34], typed HAND, no verdict:** by Lemma 3 of arXiv:2611.05044 a bilinear coupling with no local Hamiltonian stays entangling at any finite fully correlated rate; the example loses its certificate at finite noise because its oscillators rotate the noise into both quadratures.
- **Riders.** One example, one time, vacuum inputs, matched oscillators; nothing here is a statement about the feasibility of the cited proposal. Hand item H-G2 of [P] (the printed frequency shift against a second-order expansion, a factor 2 in delta squared) is NOT CHECKED against the journal version and carries no verdict.

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10. Column M (assembled, never headline): what stays invisible to matter, and a control universe

This section assembles the records of two other legs. It contains no finding and no sentence about the quantization of the mediator; a size in it is never a certificate of anything; a protected-sector reading (sections 4 to 8; rider block of section 1) is a statement about a channel and its detection and never about the mediator.

10.1 The mediator column (mandate M-R7 of [O]; source [M])

Grade of the source: assembly grade, one leg, no adversarial verifier; every row is DRAFTED for the block of [T] and NOT ENTERED; no status word of [T] moves (SATURATED stands). "The committed geometry" in this subsection is the two-path design of record; the 2026 benchmark was NOT EVALUATED by [M] (no ramp profile of record). The column at a glance, verbatim from [M]:

cell	size at the committed geometry	type
SAT-1, as committed	(25/63) x = 8.378812e-14 rad (PIN) = [8.37719e-14, 8.37758e-14] rad (CODATA); peak at 0.667 ps, exactly 0 from 2.33 ps	SATURATED (carried); regression COMPUTED
SAT-1 ceiling in Planck units (R-3)	the discrepancy is strictly below 2 x for every positive 4-tuple of separations and 2 x is approached, never attained; 2 x = 4.222921e-13 (PIN) = [4.22210e-13, 4.22230e-13] (CODATA); collinear family max(2u^2/(1+u), u/(1+u)) < 1; 0.1 rad would need m = 0.5020 m_Planck (1.09e-8 kg) read inside 2.33 ps	COROLLARY (two lines); numbers COMPUTED
SAT-2, as committed	d/c = 0.667 ps at 200 um; last light-crossing 2.33 ps	SATURATED (carried); regression COMPUTED
SAT-3, as committed	1.3e-57 quanta per run in the record's E/(hbar omega_bar) convention ([1.293e-57, 1.294e-57]); exact N_Delta = [5.390568e-58, 5.390811e-58]	SATURATED (carried); regression COMPUTED
SAT-4, as committed	0 in every mass-side statistic (the polaron identity)	SATURATED (carried); CITED from the record, not regressed
SAT-5, as committed	tau_pi = 1680 pi/211 s = [25.013628, 25.013629] s	CITED from the record; regression COMPUTED
SAT-6, as committed	witness visibility [0.86479012, 0.86479013]; Bell-test visibilities 0.98799697 and 0.956106	CITED from the record; first number regressed, the two others transcribed
the commutator (Bargmann) phase, as printed (R-B1, R-B2)	B = (X|G_PJ|Y); B^2 <= 16 Gamma_X Gamma_Y; proxy maximum 3.2e-60 ([3.1571e-60, 3.1572e-60])	CITED (2610.35253, read in full); regression COMPUTED
the same at the committed protocol	exactly 0	COROLLARY of print; COMPUTED exactly
its printed ceiling at the committed geometry (K-LEVER)	4 sqrt(Gamma_X Gamma_Y) = [2.695284e-58, 2.695406e-58]	COMPUTED (CITED bound)
a delayed variant of the committed sequence (data)	sup over the delay [2.128649e-58, 2.128746e-58] at a delay of 0.110469 s; budget ratio [0.78976817, 0.78976818]	COMPUTED
the 1/c^4 datum, its scope (R-DQ6-3)	collinear layout only; exact value -27/14,000,000,000 in the toy's units, of which +27/4,000,000,000 from the four junctions; relative size [4.2081e-47, 4.2082e-47]	COMPUTED

cell	size at the committed geometry	type
the transverse 1/c^2 term (R-DQ6-3)	[3.72985774087e-4, 3.72985774088e-4] in the toy's units; relative size [2.5501e-24, 2.5502e-24]	COMPUTED
the autonomy signature S3 (the duty cell)	**S3 = [8.070113e-26, 8.070114e-26]** (K-LEVER; relic benchmark state, phase-averaged; exponent form identical to seven digits; trace distance on the committed product input [2.435372e-26, 2.435373e-26])	**COMPUTED** from CITED functionals with CITED benchmark inputs
S3, thermal preparation at 1 K	[6.334762e-48, 6.335048e-48]	COMPUTED from CITED functionals
S3, vacuum reference	Gamma_vac(worst pair) = [2.695284e-58, 2.695406e-58]	COMPUTED (it is SAT-3 in exponent form)
S3, local-recoil convention	0 at quadrupole order	COROLLARY of print
S3, the record's lever rule	relic response 3.99e-19 at L = 1 m, 3.99e-13 at L = 1 km	COROLLARY; COMPUTED
companion: a deterministic classical wave (never part of S3)	a local phase of at most 8.001e-8 rad per 1e-21 of strain (between 6.6332e13 and 8.0009e13 rad per unit strain)	COMPUTED

> **In three sentences.** With the autonomy cell filled, every discriminator of the saturation map carries a number and none moves a status word of doc 30a section 6. The new sizes are a ceiling of 4.22e-13 for the light-crossing transient, exactly 0 for the commutator phase at the committed protocol (at most 2.13e-58 for a delayed variant) and 8.07e-26 for the autonomy response (3.99e-13 under the record's most pessimistic lever). The autonomy response is one that a classical stochastic field reproduces exactly.

Three of the five items that [M] reports first (its items 2 to 4), verbatim:

> 2. **The duty cell's kind of number is in print for a proxy.** arXiv:2607.20867 Table 1 prints the non-vacuum response of one interferometer for a proxy of the BMV geometry (a one-sided Gaussian excursion with m d^2, d = 250 um, tau = 0.5 s): Gamma_inf = 2.63e-27 beside Gamma_vac = 8.71e-61. The cell below is that printed functional evaluated at the committed geometry, the committed quintic sequence and the record's own convention for SAT-3. It is a COMPUTED size from CITED formulas, not a new kind of result.
> 3. **The duty cell is convention-bound, and the convention matters by many orders.** The record's SAT-3 number is taken about the midpoint of the pair (lever d). In that convention S3 = 8.07e-26. If each mass recoils against its own trap centre the branches are mirror-symmetric and every modulus response vanishes at quadrupole order (print: 2607.20867 Eq. (41)). Under the record's own worst-case lever rule (a branch-correlated recoil at 1 km) the same response is 3.99e-13: above the adversary's 1e-20 line, still twelve orders below one, and still no certificate. All three are stated in section 3.4.
> 4. **At the committed protocol the commutator phase is exactly zero, not "about 1e-60".** The printed figure 3.2e-60 is the maximum over delays of a Gaussian proxy. The committed sequence is simultaneous and time-symmetric, for which the source itself prints that the phase vanishes; own exact arithmetic confirms it (K5(0) = 0). In the record's lever convention the printed budget ceiling at the committed geometry is 2.70e-58 and a delayed variant of the committed sequence reaches at most 2.13e-58 (section 3.2).

Riders of [M] that travel with these rows: the SAT-1 ceiling bounds the discriminator between an instantaneous gate and anything retarded, which is not the discriminator between a mediator and its absence ("The mediator question opens only at the Planck mass" would over-read it and is not licensed); the autonomy response S3 is a signature of autonomy, never of quantization (every registered preparation is Wigner-positive and is reproduced by a classical stochastic field: Matsui, arXiv:2610.35253, Theorem 1); its inputs are printed benchmarks, not measurements; the identification of S3 with that response is a registration of [M], not a committed definition. Status after the rigor leg [R]: it confirmed by own routes every size of this subsection that has a stated formula, the duty cell S3 included; it did not reproduce SAT-4 (no number), the Bell-test visibility 0.956106 (a transcription of the committed record), the local-recoil zero, or that the cited functionals are printed as registered: those rows are transcriptions, typed as [M] typed them, and carry no verdict of this note. One digit of [M]'s own report (a delay ratio that this note does not print) is on the owner's list (remark R-i of [R]).

10.2 The control universe (mandate M-R6 of [O]; source [C], with its verifier [CV]: PASS WITH REWORDINGS, applied here)

The dictionary rider, verbatim from [CV]; it is fused into every sentence of this subsection:

> **The dictionary rider, fused into every sentence below.** "The control universe" is the Turaev-Viro theory TV_k (T_k = SU(2)_k times its mirror) read as exact 2+1 quantum gravity: Euclidean signature, positive cosmological constant, compact gauge group, finite level, point masses as punctures (a spinless mass is the object (a,a); the chiral rows concern the objects (a,0)), which-path labels as external qubits. It has no local degrees of freedom and no Newtonian limit; the same state sum is a string-net phase. The Z_2 double (the toric code) used for the lattice rows is the abelian flat-connection model; it is not a member of the TV_k family (TV_1 is the double of the semion theory), and its rows travel as "on the Z_2 double". No sentence here is a sentence about gravity in 3+1 dimensions. A null there never feeds a classicality claim, in that universe or in ours; a Clifford-hull row never feeds one either; no sentence says that gravity needs a mediator or does not.

Four sentences, each a conjunction of corollaries with its encoding. S-C1 and S-C4 were CONFIRMED by [CV] and are verbatim from [C]; S-C2 and S-C3 carry the replacement texts of [CV] (its rewording items 11 and 3), marked.

> - **S-C1 (the null; U-1, Z-1).** In the control universe two which-path interferometers in disjoint labs end in a product state at every level and for all masses, whenever the fusion register is itself a product across the lab cut (for instance every pair created inside its lab): an exact null, by isotopy invariance and locality, in a quantum theory. On the Z_2 double every one of the 1,554 unlinked placements is null and every one of the 702 linked placements gives one ebit.

> - **S-C2 (the linked gate; constraint presentation), as reworded.** Masses whose worldlines link are acted on by a gate that is a function of the closed braid alone, CZ(Delta_c) at fixed total charge, entangling iff 4r does not divide E (68,050 of 71,070 rows to k = 40); it fails Definition M at M1, M2 and M4/S2 and is a direct gate with no mediator in the sense of Definition M, outside Theorem W's premises; its onset is a step at closure, with no rate. In the lattice presentation the edge where the worldlines cross is a local quantum memory. Scope clause of [CV]: the clauses M1 and S3 hold as worded for worldlines confined to a disc.
> - **S-C3 (the traced gate), as reworded.** With the pair charge not read, the which-path channel is ((1+M_ab)/2) identity + ((1-M_ab)/2) CZ at every level, in T_k and (with M_ab^2) for spinless masses in TV_k: inside the hull of Clifford unitary channels, and entangling iff M_ab is not 1 for chiral masses and iff M_ab is neither 1 nor -1 for spinless masses, whose scalar is M_ab^2: the

> simple current with an odd partner gives the Clifford CZ in the chiral case and the identity in the spinless one (420 label pairs with k <= 40).

> - **S-C4 (the fixed-charge gate; D-1 to D-5).** At fixed charge the gate is Clifford up to local unitaries exactly on a divisibility set: 2r | E for chiral masses (4,539 of 71,070 rows to k = 40), 2r | c(c+2) - c'(c'+2) for spinless masses, where it is the identity on every spinless joint charge.

The one sentence this column takes from the control universe. [O] section 9 words it as a universe "in which the experiment is exactly null and in which Theorem W's second reading is inhabited". The first half is S-C1. The second half must not travel bare; in the reworded form of [CV] (its item 6) it reads: and in which, with the constraints imposed, a carrier-free gate of the kind reading (II) names exists for linked worldlines, outside the theorem's premises. The antecedent of Theorem W (a certified positive for labs in disjoint regions) is never met there, so the theorem never fires and that universe cannot type its disjunction; the null there never feeds a classicality claim, which is the point of the exhibit.

What no row licenses, verbatim from [C] (CONFIRMED and adopted by [CV]):

> **Not licensed, by any row.** Any sentence about 3+1 gravity. "Gravity needs no mediator" in any form: the printed sentence of Carney (section 6) is about entanglement generation in d = 2+1, and in 3+1 the same paper argues for radiative degrees of freedom under its assumptions. Any classicality reading of the null or of a Clifford-hull row. "Reading (II) is inhabited" bare: what is inhabited is a gate that fails Definition M by slaving, for linked worldlines, outside the theorem's premises; a certified positive at separation typed as a direct gate does not exist there. Any law for the three-body scalar cells. Any closed-universe sentence beyond the registered cells.

Rows that carry no sentence here (a DEFECT or NOT-REPRODUCED verdict of [CV], or a refutation reported by [C] itself): the registered sentence U-1 as first worded (false without its register proviso; S-C1 carries the proviso); BR-1 on the cells (iii) of the three-body exhibit (refuted as worded, reported first by [C]); the cited theorem behind row D-6 and row 5 of the drafted rider R-M-2 (read by neither leg; only its regression was reproduced); and the F-symbol battery rows of [C] as such (not reproducible on the verifier's engine by construction; every gauge-invariant consequence was confirmed). The three-body exhibit, the closed-universe cells and the drafted riders for Papers A, B, F and M stay in [C] and `relative_control/DRAFT_RIDERS.md` with the replacement texts of [CV]; none is entered anywhere and none is reproduced here.

Status after the rigor leg [R]. It confirmed by an own integer census and an own count of linking parities every number that the four sentences above print (71,070; 68,050; 4,539; 420; 1,554 and 702), with one remark (R-c; data, recounted for this note, record [F]): the census of 71,070 rows runs over the labels 0 <= a <= b <= k and includes 860 rows with the vacuum label, which are trivially Clifford and not entangling; with both labels nontrivial there are 70,210 rows, 3,679 of them Clifford and 2,160 not entangling. It did not reproduce the stabilizer simulation or any row of the SU(2)_k engines: those stand on [C] and on its own verifier [CV] alone.

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11. Must-cite block (in the order the note uses it; cite-and-distinguish on every export)

1. **Datta and Miao, arXiv:2104.04414** (Quantum Sci. Technol. 6, 045014 (2021)): the principle, for Gaussian oscillators (the interaction depends on the relative position alone; noise on the common mode can be rejected). First; never claimed.

2. **Wang, Pedernales and Plenio, arXiv:2611.05044 v1** (2026-11-03), Supplemental Material, eqs. (A24), (A30), (A31) and Lemma 3: the two-path pair under local plus correlated dephasing with a closed-form concurrence; the two-path entries of cell C5 at matched spacing and hand item H-C1 are evaluations of it (CITED rows, section 5.5). Distinguish in one line: finite time and a bilinear gate there; the complete twirl and the non-quadratic part of the phase table here.
3. **Li and Xu, arXiv:quant-ph/0504228** (Mod. Phys. Lett. B 19, 1803 (2005); journal version NOT READ): the partial-transpose blocks for two qubits and two spin-1 systems under collective dephasing; the two-path and the three-path instance follow from its eqs. (6) and (12) to (15) on the gate state; the verdict needs only the sign of the printed constant term, the value needs the trace (repair R2, section 4.5).
4. **Printed two-path shared-noise models and sentences, each distinguished from the complete-twirl statement (section 4.4):** Grossardt, arXiv:2007.05762 (the earliest model found); Toros, van de Kamp, Marshman, Kim, Mazumdar and Bose, arXiv:2007.15029 (the model, and a sentence on the degree of entanglement); Hebib, arXiv:2511.11879 (the common-mode generator in Lindblad form; a clock-anchored size of ONE common-mode source, which is not the acceleration noise of shared apparatus motion and must not be quoted as its size); Rahman, Noman, Zhu, Peng and Haddadi, arXiv:2610.22097 (a spatially correlated dephasing matrix; the registered correlation 449/1000 stands for the one in its illustration); Roszak and Machnikowski, arXiv:quant-ph/0507027 (the finite-strength sentence); Feng and Vedral, arXiv:2611.12229, appendix H 2 (a dynamics test that tolerates shared classical phase noise). And the printed reason why shared noise is not thereby common-mode: Bulling, Steiner, Pedernales and Plenio, arXiv:2604.22593 (the shield between the two masses; its displacement as a correlated fluctuation with opposite signs on the two masses; clarification (i) of section 1 and rider RW-3 of section 6.4).
5. **The technique and the abstract form:** Cornfeld, Goldstein and Sela, arXiv:1804.00632 (imbalance blocks of the partial transpose); Chruscinski and Kossakowski, arXiv:0705.3534 (circulant decompositions); Steinhoff and de Oliveira, arXiv:0906.1297; Johnston and MacLean, arXiv:1807.06897; Singh and Nechita, arXiv:2010.07898; Carnio, Buchleitner and Gessner, arXiv:1502.01286 and arXiv:1602.08973 (collective dephasing as a separable map; the asymptotic state as a sum over sectors); Yu and Eberly, arXiv:quant-ph/0209037 (robust and fragile states under collective dephasing).
6. **Vocabulary guard (another map under the same words; section 3):** Jaeger and Ann, arXiv:0709.2703; Ali, arXiv:0911.0767 and arXiv:1810.01123; Zhang, Ali and Ma, arXiv:2611.04568.
7. **The benchmark, the designs, the platform:** Bose, Mazumdar, Toros, Zhou et al., arXiv:2610.07422 (the white paper: the benchmark, independent dephasing, the joint noise model named as missing); the multi-path designs Tilly et al., arXiv:2101.08086; Braccini et al., arXiv:2312.05170; Li, Ling and Yu, arXiv:2412.20462; Zampieri and Bose, arXiv:2610.38840 (registers, local noise models, removable phases and parity blocks); Wan et al., arXiv:1511.02738 (the spin-1 centre). For the bench riders added at the finalization: Schut et al., arXiv:2307.07536 (the screening design; its printed Casimir phase rate is the input of the hand estimate of rider RW-3); Wu, arXiv:2406.10832 (a uniform acceleration noise dephases without contrast loss; the amplitude noise of the gradient cancels in the two-path loop; "common mode" there names the two arms of one interferometer); arXiv:0911.3938 (the temperature slope of the zero-field splitting; abstract grade, through [S]).
8. **The law column:** Lee, Adelberger, Cook, Fleischer and Heckel, PRL 124, 101101 (2020), arXiv:2002.11761 (the bound; every reading labelled CURVE-READ, PRINTED or TABLE (thesis)); J. G. Lee's dissertation, University of Washington (2020), Table 11.4 (the sign-resolved limits); Kapner et al., PRL 98, 021101 (2007); Tan et al., PRL 124, 051301 (2020); Matsui, arXiv:2610.25994 (the scope sentence on fitting both terms; its trilemma is cited, never re-derived); van Manen,

Blankenstein and Mazumdar, arXiv:2609.22501; Murata, Fujiie and Suzuki, arXiv:2605.18212 (the state of the bounds; it restricts itself to positive strengths); Kumar, Krisnanda, Arumugam and Paterek, arXiv:2206.12897 (Quantum 7, 1008), the nearest printed neighbour of the ladder's first sentence, for continuous variables and without noise: [2206.12897v3 p1] "We show that entanglement in such experiments is sensitive to initial relative momentum only when the system evolves into non-Gaussian states, i.e., when the potential is expanded at least up to the cubic term."

9. **The Gaussian row:** Miki, Matsumura and Yamamoto, arXiv:2311.00563; Miki et al., arXiv:2210.13169; Matsumoto et al., arXiv:2507.12899.
10. **Column M:** Matsui, arXiv:2610.35253 and arXiv:2607.20867; Mitrakos, Papageorgiou, Perche and Christodoulou, arXiv:2601.03214; Parikh, Wilczek and Zahariade, arXiv:2010.08208; Carney, arXiv:2108.06320; Ashtekar, arXiv:2211.01525; Balasubramanian, Fliss, Leigh and Parrikar, arXiv:1611.05460.
11. **In-house:** [W], [E1], [E1-M], [T]; the design of record is that of Bose et al., arXiv:1707.06050 (PRL 119, 240401).

Standing: every quotation event owes the literature re-sweep first; the citers of arXiv:2104.04414, 2610.07422, 2609.22501, 2610.25994, 2611.12229 and 2611.05044 were re-dated at the start and at the end of the assembly leg, at the end of the steelman leg and at the start of the finalization, each time with no difference by value from lane [L34] (Appendix B). Occupancy grade of the absence claims: net grade and claim grade ([L1] prices a printed statement of the two-path half in words, somewhere in the older journal literature, at about 15 percent).

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12. Typed outcomes, predictions and kill criteria

Outcome words of [O] (each by its own letter; not exclusive). "Protected" carries the rider block of section 1.

word	status	basis
NOTE-BY-LEMMA	TYPED	Theorem R holds by both legs (to n = 12 as registered, to 14 by the gate's lattice code, for every n by the checked proof); every cell is typed
EXOTIC-ROW	not typed	no table that is PPT and not trivial in the registered scope; the two-path wording item of section 4.2
LADDER-INDEPENDENT	TYPED, a COROLLARY	ten polynomial identities (section 8.3), with the alias rider of repair R1
PROTECTION-DOES-NOT-PAY	TYPED	section 6.1; decided at the sealing of RULE-B by a hand item, counts COMPUTED (repair R4 (i)); rider R11 travels with it: it compares the fixed protected-sector witness against the best two-path witness, per setting, under RULE-SC; it is not a comparison of the best three-path certificate with the best two-path certificate at the stated allowance (NOT COMPUTED; the noise-free three-path negativity is 8.83 times the protected one); the companion prediction is a different sentence and carries rider R5 (next table)

word	status	basis
SCOOPED-CELL	TYPED on the entry reading, for two entries of cell C5 and hand item H-C1 only; not typed on the group reading; not typed by the gate (the lanes ran after it)	section 5.5; the assembly rule of [B]; the reading of the word is the owner's to confirm

Predictions of record.

prediction	judge's forecast	outcome (as the gate scored it)
P-R-ctrl (battery and classical-channel teeth)	mandatory	PASS by two legs (sections 6.6 and 7.3)
P-R1 (the lemma to n = 12; mixed registers)	85 percent	TRUE
P-R2 (a proof for every n, checked by the verifier)	65 percent	TRUE (section 4)
P-R3 (the price in local coherence; a closed form)	80 and 55 percent	TRUE in both clauses (sections 5.2 and 6.3)
P-R4 (a sigma* below 3 rad)	60 percent	TRUE on the registered primary reading, sigma* = 3/5 rad; printed only with rider R5 of [G]: the registered three-path row spans 20 um, twice the white paper's superposition size, and at equal span (40, 5, 1) there is no certified positive at local coherence 49/50; the window is 3/5 to 205.6 rad at mismatch 1/1000 (certified here on the continuum up to 30 rad by monotonicity, and to 205.6 rad because the two-path state stays PPT); the count is 80 times the noise-free two-path count; at the stated allowance the word is the other one (precisions of the two adversarial legs, wherever this text is printed: its "noise-free two-path count" is the count at zero common-mode noise and local coherence 49/50 (defect D-3 of [R]; section 6.2), and its 205.6 rad is a number of the registered mismatch 1/1000, the pilot's value and not a design's (rider RW-3 of [S]; section 6.4), and its "80 times" is the count while the product of rms phase and mismatch is negligible: along the window the count grows with that product, to 225,523 at 30 rad, and without bound toward the end of the window (closing audit, item B-1 of Appendix D; section 6.2)) (section 6.2)
P-R5 (the zero curves do not meet in the open region)	75 percent	TRUE, in the strong form (section 8.3)
P-R6 (the Gaussian reading, two clauses)	70 and 50 percent	TRUE in both clauses on the registered example; exhibited by construction (section 9)
P-R7 (near-corollaries)	99 percent	clause 3 TRUE; clause 2 a corollary; clause 1 a CONDITIONAL corollary with "verbatim" struck (section 7.1)

After the two adversarial legs: the rigor leg [R] scored the predictions and the outcome words of the two tables above by its own routes as printed here (its section 5), with three exceptions that it names itself (clause 1 of P-R7 was not re-opened: repair R3 of the gate was taken as ruled; the 48 + 48 classical-channel rows of P-R-ctrl were not repeated: one own channel served as a control; for SCOOPED-CELL it confirmed the identity behind the two CITED rows and left the reading of the word to the owner) and with its defect D-3 on one phrase of rider R5; the steelman leg [S] re-scored no cell (its quantities are data) and found the leading word robust against a harder comparator (section 6.1). Neither leg changed a row.

Kill criteria of record.

criterion	status
KR-ctrl	not fired (two legs)
KR-0 (the lesson's leading branch)	HOLDS: a note, not a front. The status line "a closed form, a certified number under a registered convention, or a cited number" is wider than KR-0's letter ("a closed form or a cited number"); the gate keeps it and says so (repair R6), and so does this note
KR-1 (printed)	NOT FIRED by its letter ([P], [L1], [L34]); two cell entries and one hand item are CITED rows (section 5.5)
KR-2 (exotic)	NOT FIRED
KR-3 (the window)	NOT FIRED, with one NARROWS item ([L2]; section 8.1)
KR-4 (budget)	within budget; no bounded reproduction was run
KR-5 (wording bans)	the document check of this note (Appendix B)
KR-6 (trigger mid-run)	NOT FIRED ([L34]; re-dated at the start and the end of the assembly leg, at the end of the steelman leg and at the start of the finalization)
KR-7 (the co-runner)	not fired: the co-runner's verifier types its outcome NOT-DECIDED-BELOW-CAPS; no SURVIVOR (its record is `relative_preq/`, read by this leg for this line only; that typing is the state of the co-runner's record on 2026-11-11: program QSG-NC (doc 35) has since decided, on stated classes, the reading that this typing left open, and `34_relative.amendment_1.md` gives the two riders that travel with the co-runner's sentence since 2026-11-12); not this note's subject

13. What this note does not say; the sentence S-R as amended by the two adversarial legs

It does not say:

- that the mediator is quantized, or that it is not. A positive on either sector certifies the disjunction of Theorem W under its premises and nothing beyond; a null on either sector never feeds a classicality sentence; a protected-sector reading is a statement about a channel and its detection.
- that anything here is a discovery or a new principle. The principle is Datta and Miao's; the two-against-three contrast follows from Li and Xu by substitution; the two-path closed form under partial shared noise is Wang, Pedernales and Plenio's; the block technique is printed. Theorem R is a corollary of a three-line lemma.
- that the pair is shielded from noise in general. "Protected" means only (the rider block of section 1): against complete common-mode dephasing, in the path-diagonal model, at matched mass times spacing; nothing against local dephasing, gradient noise, imperfect closure or unequal

spacing histories. Noise that the two registers merely share is not thereby common-mode (clarification (i) of section 1).

- that a three-path design is the better experiment. At the benchmark, under the stated allowance, the protection does not pay (riders R5 and R11); the comparison of the best three-path certificate with the best two-path certificate is NOT COMPUTED as a count (the margins are data: section 6.1).
- that the white paper's acceleration-noise requirement can be relaxed by eight orders, or by any amount at the allowance.
- that three paths are native to, or feasible on, any platform (NOT CHECKED as a scheme; three first-order obstacles are named at hand grade in section 3, and the cited multi-path designs set the linear geometry aside: section 4.3).
- that the three-path reading should be kept as a fallback, or as a second reading of the law: the draft's summary said so and the record does not carry it (item HS-8 of [S]); the ladder exists inside the two-path rung (section 8.3).
- that Theorem W applies verbatim to the protected sector (a conditional corollary), or that Theorem W' transfers to it (it does not).
- anything about gravity from the law column: it is a property of closed forms on one static, tree-level family; the bounds are cited; the two-range Stelle form is not bounded jointly; a joint null does not exclude one Yukawa term of free strength (aliases).
- anything about the mediator from the protected sector; column M is assembled from other legs, contains no finding, and no sentence about the control universe is a sentence about gravity in 3+1 dimensions.
- that any cell holds beyond its registered model, grid and conventions (RULE-N, RULE-B, RULE-SC), or that the Gaussian row is more than one example.
- that it is posted, released or externally quotable. It is Final v1.1 for the binding order of [O]: in-house, after its two adversarial legs and after the closing audit of [O] (item 0.9), whose ruling is neither a licence nor a posting; whether a quotation event follows is the owner's call, and it owes a fresh literature re-sweep at its date; licensing is never posting; release is the owner's call.

S-R (the sentence of this note as amended by its two adversarial legs. The steelman leg ruled its clauses on Theorem R, on the factor 23.2 to 57.7, on the local coherence, on what a positive and a null certify and on the prior art LICENSED as drafted (items HS-1, HS-2, HS-4, HS-6, HS-7 of [S]) and supplied the rewordings of its clauses on the window of P-R4, beside rider R5, and on the law (HS-3, HS-5); the rigor leg supplied the precision on "80 times" (defect D-3 of [R]) and the wording of the null clause (defect D-1), to which the finalize leg added the clause on the phases of the witness (F-P1, with [S] section 4.3). The closing audit of [O] supplied the clause on the count along the window (item B-1 of Appendix D) and the wording of the range of ratios with the word "truncated" (item B-2). It travels whole, with every rider in it. The texts of the closing audit have had no further reading; licensing is never posting.)

> "In the path-diagonal model of two matter-wave interferometers with n paths each and matched mass times spacing (in a Stern-Gerlach interferometer: matched spin-force histories), complete common-mode dephasing leaves the gravitationally gated state separable iff it is PPT iff the phase table is LU-trivial up to sector constants (quadratic up to sector constants when both registers have at least three paths): a corollary of a three-line lemma, proved for every n; a two-path pair keeps nothing in the complete limit, three paths in a line keep a state governed by one third-difference invariant J unless J vanishes modulo 2 pi, and the parallel geometry needs four paths. At the 2026 white-paper benchmark, which is not a design of record, under the white paper's stated noise allowance and for every registered split of it into shared and independent noise, this protection does not pay: the fixed

> protected-sector witness needs 23.2 to 57.7 times more shots per setting than the best two-path witness (truncated ratios; a fixed witness against a best witness under a registered convention; best against best is not computed); from a common-mode rms phase of 3/5 rad (12 times the allowance in variance), at local coherence 49/50 and the registered mismatch 1/1000 (the pilot's value, not a design's), the fixed three-path witness needs fewer shots per setting than the best two-path one, and from the two-path threshold near 0.709 rad up to 205.6 rad at that mismatch (a shield midway in the linear geometry alone caps the reach near 30 rad on a static hand estimate of the steelman leg) the three-path design is the only one of the two that still certifies, at 80 times the two-path count at zero common-mode noise and the same local coherence while the product of rms phase and mismatch is negligible (527.4 times the count of the two-path register on the outer paths of the same register; the count is not flat along that window: it grows with that product, to 225,523 shots per setting at 30 rad, and without bound toward the end of the window, where the margin of the fixed witness vanishes), for a register that spans 20 um, twice the white paper's superposition size (at equal span there is no certified positive at that coherence). The protected state needs more local coherence than the unprotected pair (0.9597 against 0.9002), not less. The two rungs read different functionals of a static, tree-level law (on the scalar-decoupled Stelle member their ratios to Newton vanish at 22.45 and 18.16 um and have no common zero at any finite range; neither have any two different two-path sub-registers of one apparatus (fifteen exact identities of the same kind), and at the zero of the benchmark reading a second two-path geometry reads the law with 12.5 to 122.5 times fewer shots per setting than the protected sector: the protected rung is the one that survives a uniform shared force, not the one that a second reading needs; witness nulls are statements modulo 2 pi), inside a window that the cited modulus curve puts below 35.742 um and the published sign-resolved limits below about 24.7 um. A certified positive on the protected sector certifies the disjunction of Theorem W once its premises W1 to W6 and clauses 2 and 3 of Definition 1 are re-established for a three-path register, and nothing about the mediator beyond it; a null certifies, inside the model only and for a witness read with the phases of the state, a bound on |J| that weakens with the mismatch, and never feeds a classicality sentence. The principle is Datta and Miao's (arXiv:2104.04414); at two and three paths the statement follows from Li and Xu (2005) by substitution; the two-path closed form under partial shared noise is Wang, Pedernales and Plenio's (arXiv:2611.05044); the two-path coherence that a shared jitter leaves untouched is printed by Toros et al. (arXiv:2007.15029), and in the complete limit it alone leaves a separable state. Shared noise is not thereby common-mode: a shield midway acts with opposite signs on the two registers (Bulling, Steiner, Pedernales and Plenio, arXiv:2604.22593)."

One-line summary for the collaboration (reworded: item HS-8 and text RW-8 of [S]; the advice of the draft's last clause is withdrawn). *If the uniform-force noise that falls on both of your interferometers alike were complete, a two-path pair would keep no certificate at all and a three-path pair would keep exactly one number, the third difference J of the potential across the register; but at your own benchmark and allowance the three-path certificate costs 23.2 to 57.7 times more shots per setting than the two-path one (truncated ratios; a fixed witness against your best witness), it needs more local coherence (0.9597 against 0.9002) and a register twice as wide as your superposition size, and it would pay only if the uniform shared noise missed the allowance by more than an order of magnitude in variance while the independent noise met it: so run the two-path experiment. A three-path register is not shown to be usable on any platform (at hand grade: on a spin-1 centre its middle path gives up the symmetry that protects the two-path loop), and a second two-path geometry reads the short-distance law at fewer shots per setting than the three-path sector does. What the three-path sector adds is one third-difference number that a uniform shared force cannot erase; its nulls are statements modulo 2 pi and never feed a classicality sentence.*

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14. Assumptions and caveats ledger; open items; hand-offs

1. **The path-diagonal model carries everything, and "protected" always carries the rider block of section 1.** Equally spaced branches with one spacing history, closed at readout; noise as a uniform force. Gradient noise, imperfect closure and unequal spacing histories are not common-mode phases, and the lemma says nothing about them. The experimentalist steelman of M-R5 ran on exactly this ([S] section 3): by its ruling the model does not survive the bench as a description of a shielded Stern-Gerlach pair, in respects that it names and sizes as data; the two that matter most are the shield (rider RW-3 of section 6.4) and the middle path of a three-level register (section 3), and none of its sizes is entered here as a number except where a section says so. It also sized the tilt of the line (section 4.3), gradient noise, closure in a curved field, unequal spacing histories and the amplitude noise of the gradient ([S] sections 3.2 to 3.5 and 3.7), all outside the model, and it found one point for the note: the block d = 0 is untouched by every shared noise of the form f(k) + f(l), not only by a uniform force (the clarifications of section 1).
2. **Uniform amplitudes** in every cell; the theorem itself holds for any nonzero amplitudes.
3. **RULE-B is the pilot's definition of a split that the white paper does not print.** Its local model for three paths (one factor lam per register whatever the path distance) flatters the protection; the force-type row LOCAL-G is printed beside it and is the faithful row for an acceleration noise; the outcome word survives it. A budget splits by source, not by one correlation (steelman leg, HAND, [S] section 3.9): the inertial motion of one rigid apparatus that carries both traps is the corner f = 1; the same motion re-enters through the shield with the opposite sign on the two registers, for which RULE-B has no column; what the white paper puts into its intrinsic rate is local. The interior columns of the cell describe no apparatus, and the outcome word does not depend on any of this.
4. **RULE-SC is a convention.** One fixed three-path witness against the best two-path witness (rider R11); the nine-setting two-path total is NOT DERIVED (repair R8); no verdict rests on a total-shot reading.
5. **The registered three-path benchmark row spans 20 um** (rider R5); at equal span the protected witness has no certified positive at local coherence 49/50.
6. **Two readings of the window** (section 8.1): the sign-resolved limits come from a dissertation table read by one scout leg; the journal supplement is NOT READ; the sign-resolved limits of the other experiments are NOT READ.
7. **Aliases** (section 8.5): any null is a statement modulo 2 pi; for one Yukawa term of free strength the aliases left open by the published limits are short-ranged and strong.
8. **Reading grades of the literature legs.** Most texts were read at claim grade (a net over every page with context windows), a few in full; the journal version of Li and Xu is NOT READ; the absence of a printed path-encoded statement is a net-grade and claim-grade absence. Neither adversarial leg changed this. The rigor leg opened no paper, thesis or published curve: it checked the arithmetic done on the lanes' transcriptions and the trace of each cited figure, and typed every source reading NOT-REPRODUCED, which in its sealed vocabulary carries no judgment of the cell. The steelman leg read the design papers for the bench (its grades are in [S] section 9), and its nets in the bench vocabulary found no posting of the path-encoded fact. Every CITED row of the law column therefore stands on lanes [P]-literature and [L2] alone; every quotation of this note was re-found on its stated page by the document check of the finalization (Appendix B).
9. **NOT BUILT, NOT DECIDED or NOT COMPUTED:** separability below the four-path parallel threshold; uniqueness of the zero of rho_4p; a joint bound on the two-range Stelle form; the best three-path certificate at the allowance as a count (its margin is data: section 6.1); partial common-mode noise and shot counts on the parallel rung. Data of the rigor leg that is NOT ENTERED in any row (no claim was added after the legs): the two-range member M4 has no joint zero of the two

readings at finite ranges (its remark R-f, an exact resultant), and its break attempt under partial noise found no PPT entangled state outside the theorem's letter (a realignment test, not a proof of separability).

10. **NOT CHECKED:** clauses 2 and 3 of Definition 1 and premises W1 to W6 for a multi-path platform; the platform sentence as a three-path scheme; hand item H-G2; the mediator column and lane [L2] by any adversarial leg beyond their arithmetic (the rigor leg confirmed the sizes of column M that have a stated formula and the arithmetic on lane [L2]'s transcriptions; it opened no source); the bound of the witness estimator under a miscalibration of its settings, and a three-outcome single-shot readout (section 7.1); every hand estimate of the steelman leg as physics (typed HAND).
11. **Hand-offs to the two adversarial legs (M-R5), as the draft listed them; both legs have run and answered them ([R]; [S] sections 3.8 and 3.9).** Certificate-rigor leg: every number by own routes (the regenerated logs and the checks of [B] are the inputs; the two CITED rows and Lemma T first). Experimentalist steelman, from [G]: RULE-B's local model; the fixed witness against a best three-path witness; the nine-setting variance; the span of the registered row; the alias curves as a real ambiguity of any null; hand item H-G2. From [L1]: Lemma 3 of arXiv:2611.05044 (what a finite-time, bilinear-gate reader expects); the single visibility law of arXiv:2511.11879 against its own generator (a hand reading, never a correction issued here); the Roszak and Machnikowski sentence. From [L34]: the printed sizes of shared noise (a clock-anchored common-mode sector nine to eleven orders below the local rates; correlated rates for two collapse models), which bear on what share of a real budget is shared at all; Feng and Vedral's test as the printed alternative use of a run under shared noise.
12. **For the owner's fold (nothing applied by the assembly leg or by the finalize leg; no committed document was touched):** item 0.6; the draft rows and riders of [M], [C] and [CV]; the lessons drafted by the lanes; README, memory and the status files. One procedure item of [G], repair R10 (a development log for every script of record; rule files written after their addendum), is listed here for the house procedure and applied to no committed document; the assembly leg kept its own development logs under `relative_note/dev/`, the finalize leg under `relative_note/finalize/dev/`. From the two adversarial legs and the finalization (record-side lines drafted in `relative_note/finalize/REPAIRS_LEDGER.md`, nothing entered anywhere): whether the bounded reproduction that KR-4 of [O] reserves for the shield-mediated channel of arXiv:2604.22593 is wanted now (it would replace the static hand estimate of rider RW-3); remark R-i of [R] for the report of [M]; remark R-f of [R] for row N-5; the reading of SCOOPED-CELL (item 0.3).

Appendix A. Numbers provenance

Rule of this note (registration of [B], rule AR-1): no number of columns C or L was typed by hand. The note is generated from a template by `relative_note/n_build.py`; each placeholder is filled from a log REGENERATED for this note by an isolated, sha-verified copy of the pilot's scripts (`relative_note/n_regen.py`; copies in `relative_note/regen/`), or from the log of this leg's own assembly checks (`relative_note/n_checks.log`); the document check re-finds every long number of the final text in those files. Regeneration result: all 17 regenerated logs (9 of the pilot, 8 of the gate's stages; the gate's stage v7, an audit of file times, cannot be repeated from a copy and was NOT RUN) equal their logs of record line for line once time stamps, wall times and per-row seconds are masked. Printed digits are prefixes of the digits the logs print.

The finalization changed the draft only through the count-asserted patcher `relative_note/finalize/f_patch.py`, which fills each number that it adds from the record of its certifications (`relative_note/finalize/f_certs_record.json`); the regeneration was repeated from a fresh isolated copy (17 of 17 logs identical to the logs of record after the same masks: `relative_note/finalize/f_regen.log`).

numbers	section	regenerated source (log of record it equals)	grade
Delta = -211/672, -211/1000; J = 211/1400; N_3; F_ent, the state-level threshold, N_min = 1105, r*, F_sep; the lattice ranks	3, 5.1, 6.6	`regen_s1_battery.log` (`relative_pilot/s1_battery.log`)	exact rationals; interval enclosures (regressions of the record)
classical-channel rows	7.3	`regen_s2_teeth.log`; the gate's `regen_verifier_MR2__v2_states.log`	exact inertias
invariant table; counts of lattice rows, torsion tables, state rows	4.2, 4.3	`regen_s3_theorem_r.log`; totals in `n_checks.log` (A-11); bases in `n_checks.log` (A-5)	exact integers
thresholds in local coherence; cell C7; the shot-count grid; sigma* (with rider R5); the budget point	6.1 to 6.3	`regen_rc_cells.log`; `regen_cells__committed_run__rc_cells.log`	certified inertia brackets; exact ceilings
the budget family; where it would pay; thresholds under partial noise and mismatch; reaches; the twirl cost; the parallel rung	6.1, 6.4, 6.5	`regen_cells__cx_column_c.log`; `regen_cells__fx_followup.log`	COMPUTED
the law table; zeros; ladder; committed row; null-rule numbers	8.2 to 8.5	`regen_cells__lx_column_l.log`; `regen_cells__fx_followup.log`	COMPUTED from exact polynomials
the Gaussian table	9	`regen_cells__g6_gaussian.log`	exact rationals
Lemma T weights and moments; the ten identities; the alias count and the least-strength alias; the wrong root of the printed cubic; the zero to fourteen decimals	5.2, 8.3, 8.5, 4.5	the gate's `regen_verifier_MR2__v3_closed.log`, `__v5_law.log`, `__v8_addendum.log`; `n_checks.log` (A-3, A-4)	the gate's own routes, regenerated; this leg's regressions
roots of the printed two-path closed form; H-C1 by the closed form	5.5	`n_checks.log` (A-2)	exact algebra; certified intervals
rows marked (a) of the law table; the other reading at the sign-resolved ends of the zero curves	8.2, 8.3	`n_checks.log` (A-6, A-6b)	this note's enclosures at lane [L2]'s printed locators; data
the inverse threshold	5.6, 7.2	`n_checks.log` (A-7)	certified brackets
totals and ratios in prose (7 N_3, 9 N_2, 15 N_2; the ratios of counts and of reaches)	0.1, 6.1, 6.2, 6.4	`n_checks.log` (A-8, A-9)	exact arithmetic on regenerated integers; ratios truncated

numbers	section	regenerated source (log of record it equals)	grade
window edges, thesis rows, alias typings	8.1, 8.5	`relative_lanes/L2_short_range.md` and `L2_short_range_probe/r4_alias.log`, `r6_pointers.log`; `relative_pilot/literature/LITERATURE.md`	CITED: PRINTED, CURVE-READ, TABLE (thesis), LOCATOR, as labelled
every number of section 10	10	verbatim blocks of `relative_mediator/MEDIATOR_COLUMN.md`, `relative_control/CONTROL_REPORT.md`, `relative_control/verify/VERIFIER_REPORT.md`	as those legs typed them
every number added by the finalization (the null bound with a mismatch; the divisor of "80 times"; the continuum of splits; the best-witness margins; the outer-pair comparator; the sub-registers at the benchmark zero; pair-dependent damping; the shield estimate; mismatch 1/100; the platform arithmetic; tilt; the residue fractions; the census without the vacuum label)	0.8, 1, 3, 4.3, 5.3, 5.5, 6.1, 6.2, 6.4, 7.1, 7.2, 8.3, 10.2, 13	`relative_note/finalize/f_certs.log` (own interval and exact routes; the engine through a fresh isolated copy wherever it can produce the number) and `f_alias.log`	certified enclosures printed as truncations; exact integers and rationals; the rows typed HAND are arithmetic on their stated premises

Verbatim text. The blocks between the markers `V:` and `/V` are pulled by line number from the named files and re-derived by the document check; inline verbatim substrings are marked `VS:`; quotations of papers are marked [id vN pP] and were re-found by this leg on their stated pages of the texts that the quoting lanes had fetched. Texts of the two adversarial legs that are entered verbatim are marked `VS:` with their report as the named file; the quotations added at the finalization were re-found on their stated pages in the same way.

Appendix B. Build and verification record: the assembly of the draft, then the finalization

The first seven items are the record of the assembly leg, transcribed from the draft ("this leg", "this text" and "this note" in them are that leg and its draft). The items after them are the finalization's.

- **Order kept.** Canon pins before any act of this leg (a fixed list of 1401 committed files: the top-level documents, `analysis/`, the paper sources, the DQ6 scout and judge files), then the registration `relative_note/REGISTRATION_NOTE.md` (sha256 `4eb3c34f809afbda6f8c6d2a54006080123cd56e6f7942af01ede977d87e1a5b`, sealed 2026-11-12 00:02:48.272 CET), sealed before any script of this leg but the pins script existed; one dated addendum of new checks (`REGISTRATION_NOTE_ADDENDUM_1.md`, sha256 `b54a04cd332b0f9dcc881815b3d9989764cc9e127421cb118ffe38a4891d4ff3`, sealed 2026-11-12 00:20:29.509 CET) before the run of record of the assembly checks and before the build and check scripts existed; a second dated addendum (`REGISTRATION_NOTE_ADDENDUM_2.md`, sha256 `e14679a26e2fa369e2e8ee824ea363dfa5ab1d31b0e16ed3d1d57b511500f595`, sealed 2026-11-12 01:05:16.416 CET) at the start of the second sitting, before any repair of the template and before the second versions of the build and check scripts existed; a third

(`REGISTRATION_NOTE_ADDENDUM_3.md`, sha256 `33b405279b0a772e8e08853c56b5b578b8c817ef10478fd3de083c5f0c48525c`, sealed 2026-11-12 01:11:03.941 CET) before the second version of the checker and its control script existed, with two corrections of the table of that control and three further template repairs. Every script was hashed before its run of record (`scripts_prerun_*.sha256`).

- **Two sittings.** This leg ran in two sittings: the first (2026-11-11 23:51 to 2026-11-12 00:38 CET) was cut off by a usage limit after the registration, addendum 1, the regeneration, the assembly checks of record, the template and the generator; the second re-verified every sealed hash, sealed addendum 2 before any repair, repaired the template, and made the build of record, the document checks, the end-of-leg triggers, the pins and the manifest; nothing sealed in the first sitting was edited, re-sealed or re-run.
- **Triggers (M-R9 of [O]).** The four Adjudicator triggers and KR-6, re-dated like for like with lane [L34]'s end state: 53 items, 0 differences by value, 0 NOT RUN (2026-11-12 00:04:44.848 CET) at the start of the leg; the end-of-leg re-date is in `relative_note/n_triggers_end.log` and in the build manifest (it is taken after this text is frozen).
- **Regeneration (A-1, A-1b):** 17 of 17 logs identical after masking (`n_regen.log`).
- **Assembly checks (`n_checks.log`):** 20 checks, 19 passed. Reported first: the forecast of A-6 (0.9), that lane [L2]'s float pointers regress to their printed digits at the lane's four-decimal locators, is REFUTED by its letter for 2 of 12 pointers (the lane evaluated at unrounded locators); the new check A-6b (each pointer meets the range of the closed form over the rounding interval of its locator) holds for 12 of 12. Consequence: the rows marked (a) are this note's own enclosures at the locators as printed. A-2 certified the identity behind the two CITED rows; A-3 re-derived Lemma T; A-4 the ten identities; A-5 the explicit bases of the invariant lattices with rank pins modulo 9,800,027 and 9,800,053 (regression pins; nothing is decided by a prime).
- **Document checks.** The check of record is the last run in `relative_note/n_doc_check.log`, made on this final text: placeholders, numbers, verbatim blocks and substrings, quotations on their pages, the ban scan, ASCII and dashes, table shapes, hashes, the repair texts, the riders beside the outcome word (R11) and beside P-R4 (R5) per paragraph and per line, the lead of the note (title line, first paragraph, first item), the rider of "protected" per section (a pointer to the rider block of section 1), the head of the must-cite block; and the checker's own control, nineteen mutations of this text each of which must be caught (`relative_note/n_doc_teeth.log`). Its counts are properties of this very text and are kept in that log and in `relative_note/BUILD_MANIFEST.md`, not here.
- **Canon pins after the build:** in `relative_note/pins_after.txt` and the manifest. No committed document, engine or record was modified; nothing was written under another leg's directory or under `release/`; the only new files in the corpus are this note and `relative_note/` (outside the corpus: one variant paragraph appended to the house procedure file, after the checks of record); no process is left running; nothing was posted or sent.
- **The finalization (finalize leg; record [F] = `relative_note/finalize/`; 2026-11-12, from 02:45 CET).** Order kept: canon pins before any act (the same fixed list of 1401 committed files; equal, hash for hash, to the final pins of the assembly leg and of both adversarial legs); then the registration `REGISTRATION_FINALIZE.md` (sha256 `bac1cdf0857c2535a2bee469692836759821699e8e296c2d68e17bcc31f8caad`, sealed 2026-11-12 02:58:16.650 CET) with the fold rules, a disposition for every item of both legs, the rule that types the note Final or PROVISIONAL, the certifications and the checks, sealed before any script of the leg but its pins script existed, before any network query and before any change to the note; one dated addendum (`REGISTRATION_FINALIZE_ADDENDUM_1.md`, sha256 `61f606c7a34056ea9441a8cb47bd43dd74c19046e275708e9aa8440c6757f2ab`, sealed 2026-11-12

03:03:13.658 CET) with one item found by the leg's own reading, before the certification script existed.

- **Triggers at the start of the finalization** (2026-11-12 03:00:25.671 CET): 53 items, 0 differences by value against the steelman leg's end state (itself like for like with the assembly leg and lane [L34]), 0 NOT RUN. The end-of-leg re-date is taken after this text is frozen; it is in `relative_note/finalize/f_triggers_end.log`, the ledger and the manifest.
- **Regeneration (the NOT-REPRODUCED rows of the rigor leg that an engine of record produces):** a fresh isolated tree, every copied file equal to the pilot's and the gate's manifests; 17 of 17 logs identical to the logs of record after the same masks (`f_regen.log`), among them the enumeration counts of the pilot and the gate and the 48 + 48 classical-channel rows. Lane [L2]'s alias script from an isolated copy with its data files: log and table identical, and typings 1 and 2 reproduced label for label by an own interpolation on the lane's transcribed vertices (`f_alias.log`). The gate's stage v7 was NOT RUN, as before.
- **Certifications:** 64 of 64 checks passed in the run of record of `f_certs.py`, which equals its last development run line for line; every number that the finalization added regresses the value of the leg that first printed it at that leg's printed digits; the one cell without a precedent (the two-path threshold at mismatch 1/100 and local coherence 49/50) met its registered forecast. Four slips of the leg's own scripts (a rounding helper at an exact square, the base path of one manifest, one count assertion, one placeholder pattern of the patcher) were found in development runs and repaired before the runs of record; the development logs are under `relative_note/finalize/dev/`.
- **The patch:** the draft is kept byte-identical as `relative_note/finalize/34a_E1R_protected_sector_note.v0.9.md` (sha256 `cfedc9c86b0fc69c8285b691b4247b9af8b4cb716814722d1c42a794f320b55d`, the hash that both adversarial legs reviewed); this text is written from it by `f_patch.py` alone (count-asserted replacements, each tagged with the item it folds).
- **Checks of the final text:** the registered document checks of the assembly leg, unweakened, through a patched copy of its checker (`f_doc_check.py`); the finalization's own checks in `f_checks.py` (fold anchors; stale strings of the draft; invariance of the verbatim blocks, of the tables of section 12, of every long number and of every table row of the draft; the status lines; hashes; the character map); and a control of both by mutations (`f_teeth.py`). Their counts are properties of this very text and are kept in those logs, in the ledger and in the manifest, not here.
- **Canon pins after the finalization:** in `relative_note/finalize/pins_after.txt` and the manifest. No committed document, engine or record was modified; nothing was written under another leg's directory or under `release/`; the only files written in the corpus are this note, in place, and new files under `relative_note/finalize/` (outside the corpus: one variant paragraph appended to the house procedure file, after the checks of record); no process is left running; nothing was posted or sent.
- **The closing audit and Final v1.1 (record [K]; 2026-11-14).** Order kept: canon pins before any act; the registration `owner_blocks/B34/REGISTRATION_B34.md` (sha256 `88b055a8dcc519b473356dd4cfbd52cdc6d4dbd74651923bee16e6fdea7a83cb`) with the ruling rule, the hand items and the forecasts, sealed before any certificate script and before any query (two reading aids ran before it and are disclosed there); dated addenda before what they govern; own routes only, with no line of an earlier leg's code; prime tokens of the closing reserve as rank pins only. This text is written from Final v1.0 by `owner_blocks/B34/apply_B34.py` alone (replacements asserted to occur exactly once, each tagged with its item), applied by the main loop and not by the audit. The counts of the audit's checks and its pins are in [K], not here.

Appendix C. Repairs ledger of Final v1.0 (each repair: what, why, verdict-neutral or not, the evidence)

The full ledger, with the disposition of every item of both legs, the texts of the legs beside the texts entered and the record-side lines for the owner, is `relative_note/finalize/REPAIRS_LEDGER.md`. Item names: D-n and R-x are the defects and remarks of the rigor leg [R]; HS-n, RW-n and Dn are the headline sentences, replacement texts and defects of the steelman leg [S]; F-P1 is the finalize leg's own. "Neutral" means: no verdict, outcome word, truth value of a prediction, status of a kill criterion, type or printed digit of a certified cell changes (tested against the draft by the invariance check of the finalization). "Protected" carries the rider block of section 1 in every row. C-n is a certification of record [F] (`f_certs.log`).

item	what changed, and where	why	verdict-neutral	evidence
D-1 of [R]	section 7.2: the bound of a protected-sector null holds at matched spacing; with a mismatch the weaker root form, with three values; row N-2 of section 8.5 points to both	the draft's sentence was false as worded inside the reach	yes: no outcome word, prediction or count uses the bound	[R] section 2; C-1 (own intervals; the engine has opposite signs on the two sides of each printed bracket)
F-P1 (finalize leg, with [S] section 4.3)	sections 7.1 and 7.2, row N-2: a null bounds the modulus of J only for a witness read with the phases of the state	found while planning C-1: the engine's witness is always phased by the table of its own state	yes: it weakens what a null certifies; no cell of record meets the other case	addendum 1 of [F]; C-1b (direct computation: an offset of 1/2 rad per path nulls the witness on the Newtonian state)
D-2 of [R]	section 8.3: two "between" phrases replaced by the four truncated figures	each was false as an interval for one of its two numbers	yes: the table rows above them carry the exact figures	[R] section 2; C-2 (one figure printed truncated where [R] prints its rounding: it is a lower bound)
D-3 of [R]	a precision beside every printing of rider R5; section 6.2 and sentence S-R name the divisor	"noise-free" misnames a count taken at local coherence 49/50	yes: 80.0 is exact for what it divides; sigma* and the outcome word do not use it	[R] section 2; C-3 (by own closed form and by the engine)
D-4 of [R]	section 7: the complete character map	the stated map was incomplete (cosmetic)	yes	[R] section 2; C-4 (twelve classes; seven were unnamed, four more than [R] listed)
R-a, R-b of [R]	section 5.5: m_2 is the odd block's value, exact where the state is NPT; section 6.4: the reach table prints roundings	precision	yes	C-5, C-6
R-c of [R]	section 10.2: the census includes 860 rows with the vacuum label	data	yes	C-7 (own integer census)
R-d, R-e of [R]	section 6.1: "at least 99 percent" was a statement on the registered grid; row N-5: inside the cited windows	precision	yes	the regenerated tables
R-g, R-h of [R]; H-	section 6.1: the word on the continuum of splits; the	data that rider R11 names	yes: the word of record keeps its	C-10, C-11 (own enclosures and the engine)

item	what changed, and where	why	verdict-neutral	evidence
ST13 of [S]	best-witness margins behind "NOT COMPUTED"		letter; no count is formed	
HS-8, RW-8, D3, D9 of [S]	the one-line summary of section 13: the advice of its last clause withdrawn; "would pay only if"; the platform clause; four-digit thresholds	OVER-REACH: advice that the record does not carry	yes	[S] section 1; two deviations from text RW-8, both toward a certified figure (the four-digit truncations where it prints "0.96 against 0.90"; the certified comparison where it prints "the cheapest second reading")
HS-9, RW-9, D1, D2 of [S]	section 1: three clarifications for the bench, verbatim, with their algebra; sections 2, 5.3, 6.4, 11, 13	"shared" is not "common-mode"; the shield; force histories, not masses; the middle path	yes: riders	[S] sections 3.1 and 3.6; C-14 (the algebra, exactly)
HS-3, RW-3, D1 of [S]	section 6.4: rider RW-3, pointed to beside every printing of rider R5 and carried in S-R	205.6 rad is a number of the registered mismatch, not of a design	yes: no cell is re-scored	[S] section 3.6; C-15 (the static estimate re-derived and typed HAND; the band estimate is not entered)
HS-10, RW-10 of [S]	sections 2 and 6.1: "would pay only for" (verbatim in section 2)	"a fallback for" read as available	yes	[S] section 1
HS-11, RW-11, D2 of [S]	sections 2 and 6.4: the mismatch sentence; one new data cell at mismatch 1/100	the mismatch read as a mass tolerance and its value as a design number	yes: data	C-16 (a registered forecast, met; own route and the engine)
HS-5, RW-5, D7 of [S]	sections 2, 8.3, 8.5 and 13: the ladder exists inside the two-path rung; counts at the benchmark zero	the ladder was presented as a property of the two rungs	yes: LADDER-INDEPENDENT and prediction P-R5 are untouched	[S] section 6; C-13 (fifteen exact identities; counts by intervals and by the engine)
D4 of [S]	sections 6.1 and 6.2: the outer-pair comparator; the equal-span row is the white paper's own hardware	the comparator of record is the benchmark pair	yes: the word of record is unchanged, and stronger against that comparator	C-12
D5 of [S]	section 3: the platform sentence, three first-order obstacles typed HAND	it stopped at NOT CHECKED	yes: it stays NOT CHECKED as a scheme	[S] section 4.1; C-17
D6 of [S]	section 4.3: tilt, and the cited designs set the linear geometry aside; one line in section 8.5	"in a line" had no tolerance	yes: data	[S] section 3.5; C-18
D8 of [S]	section 5.3: pair-dependent damping; the source of the local model	no source and no pair-dependent form	yes: a corollary	C-14
D10, D11 of [S]	section 7.1: the estimator's bound, the residue fraction, the readout; section 3: one line in the vocabulary guard	NOT CHECKED items; vocabulary	yes	C-20

item	what changed, and where	why	verdict-neutral	evidence
the optional addition of [S] to S-R	S-R names the printed two-path coherence of Toros et al.	prior art	yes	its first half is entered with the wording of section 4.4; "the opposite conclusion" is not entered
section 3.9 of [S]	section 14 item 3: a budget splits by source	a rider on RULE-B	yes: the outcome word does not depend on it	[S] section 3.9 (HAND)
forecast records of [R] and [S]	item 0.2	the house rule on refuted forecasts	yes	items 0.1 of [R] and 0.1 to 0.4 of [S]

NOT-REPRODUCED rows of the rigor leg (its sealed vocabulary: no own route was completed; no judgment of the cell). Regenerated at the finalization from an engine of record and compared with the log of record: the enumeration counts of the pilot and the gate, the 48 + 48 classical-channel rows, the three alias typings (Appendix B). CITED, hence not regenerable by any engine: every reading of a paper, of the dissertation table and of the published curve, and every quotation: the type CITED stands, the pages were re-checked, and section 14 item 8 says that no adversarial leg opened the sources. Column M: the rows named in sections 10.1 and 10.2 carry no verdict of this note. The audit of file times of the earlier legs: NOT RUN.

What did not move. Every verdict word, outcome word and truth value of sections 0 to 12; every status of a kill criterion; every type and printed digit of a certified cell; the 21 verbatim blocks and every verbatim text of the gate; the tables of the draft row for row, except rows N-2 and N-5 of the null rule (a pointer and a window clause); Theorem R and its proof.

Record-side (drafted in the ledger file; nothing entered anywhere): remark R-f of [R] (member M4 for row N-5); remark R-i of [R] (one digit of the report of [M]); the bounded reproduction of arXiv:2604.22593 that KR-4 of [O] reserves (the owner's call).

Forecasts of the finalization (registered before the computations they predict; a refuted one is reported first; those about the checks of this very text and about the end-of-leg trigger re-date are read after the text is frozen and are in the ledger file and the manifest).

forecast	prior	outcome
T-start: no difference by value at the start-of-leg trigger re-date	0.9	TRUE (53 items, 0 NOT RUN)
F-1: 17 of 17 regenerated logs identical to the logs of record	0.92	TRUE
F-2: every regression of a leg's number holds at that leg's printed digits	0.85	TRUE (64 of 64 checks)
H-F12: the new two-path threshold at mismatch 1/100 lies in [0.69, 0.7098) rad, and the reach is 28 to 30 times it	0.85	TRUE
C-1b: an offset of 1/2 rad per path nulls the fixed witness on the Newtonian state	0.9	TRUE
F-4: the own re-typing reproduces the alias typings 1 and 2	0.7	TRUE (464 of 464 labels in both typings)

Appendix D. Repairs of Final v1.1 (the closing audit of [O]; each repair: what, why, verdict-neutral or not, the evidence)

Item names: B-n are the listed defects of the closing audit [K], R-n its precisions, CUR-1 a rider of a later date. "Neutral" as in Appendix C. "Protected" carries the rider block of section 1 in every row. The logs named are files of [K].

item	what changed, and where	why	verdict-neutral	evidence
B-1	sentence S-R of section 13, the precision beside each of the four printings of rider R5, and section 6.2: "80 times" is the count while the product of rms phase and mismatch is negligible; the count grows along the window and without bound toward its end	the clause was attached to the whole window of P-R4, as the bound of defect D-1 had been attached to the whole reach	yes: sigma*, the truth value of P-R4 and the outcome word do not use the count beyond the registered grid, and the table of section 6.2 already printed 225,523 at 30 rad	`b34_r2_cells.log`, checks E-1, E-4, E-5 (the closed form and the definition of the witness)
B-2	six "between" phrases replaced by "a to b" with the word "truncated": the lead, item 0.1, section 2 item 4, section 8.2, section 8.3 (twice), sentence S-R, the one-line summary (which now prints the figures of S-R)	each was false as an interval for its larger end value, the class of defect D-2 of [R]; the phrase on the sub-registers had been added by the finalization	yes: no digit of a cell changes	`b34_r2_cells.log` F-3; `b34_r34_law_misc.log` L-8, L-10, L-15; `b34_r5_text.log` T-6b
B-3	section 12: what the rigor leg scored by its own routes, with the three exceptions that its report names	the sentence said "every prediction and every outcome word"	yes: a statement about the coverage of a leg	[R] section 5; `b34_r5_text.log` T-7a
B-4	section 7.1: the residue fractions carry their conditions (perfect local coherence; the split f = 1)	"at the allowance" named no split	yes: data in a paragraph of NOT CHECKED items	`b34_r2_cells.log` K-14, K-15
B-5	section 6.2: the count 409 carries its condition	it followed the clause on a pair with no noise at all	yes	`b34_r2_cells.log` K-5
B-6	the one-line summary: "the three-path sector" where the finalization had written "the protected sector"; "at hand grade" before the clause on the middle path	the word had entered that line without its rider; the clause is hand grade	yes	`b34_r5_text.log` T-5e
R-1	section 7.2: the margin -0.0115 is that of one sign of the offset	the other sign gives a more negative margin; the witness is null for both	yes	`b34_r2_cells.log` K-2, K-3
R-2	section 1, reading conventions: one sentence on truncated bounds, thresholds and ranges	five bounds of section 7.2 and four thresholds of section 5.3 are truncations of certified values, in the direction that says slightly	yes: a convention; no figure changes	`b34_r2_cells.log` K-1, K-8, K-11

item	what changed, and where	why	verdict-neutral	evidence
		more than the certificate		
CUR-1	section 12, row KR-7: the co-runner's typing is dated, with a pointer to `34_relative.amendment_1.md`	that amendment, of 2026-11-12, dates the typing; no defect of Final v1.0 at its date	yes: the status "not fired" is unchanged	`b34_r5_text.log` T-10a to T-10c

What did not move. Every verdict word, outcome word and truth value; every status of a kill criterion; every type and printed digit of a certified cell; the 21 verbatim blocks and every marked verbatim text of the gate, the lanes and the steelman leg; every table row of Final v1.0 except row KR-7 (a date and a pointer); Theorem R and its proof; the must-cite order.

For the owner's fold (nothing applied by the audit). Doc 34 section 4.2 and the README entry quote sentence S-R and its figures as Final v1.0 printed them.

End of the E1-R note, Final v1.1. Licensing is never posting; release is the owner's call.