Public release copy. Source: `21a_E1M_magic_note.md`, Gravity Frontier Research, David Vávra, Independent Researcher, Prague. SHA-256 of the Markdown source: `49ff6efea2de2d3ae3a96b0dfe091ad972e0fe0ed074aeb445451adb0b5bdf06`. First published at vavradev.com/research on 2026-11-04. The text below is the published copy of the source (punctuation normalized for publication: no em dashes).

E1-M, A ledgered upgrade layer over the BMV entanglement witness: certifying, under stated added assumptions, that the gravitationally mediated gate was non-Clifford, magic injected at gate grade, from the conditional fringes a run already records

Final v1.0, 2026-08-24. Experiment-facing deliverable of the witness–magic program ([WM] §7.5(2)), finalized after two adversarial verifications (certificate-rigor, 96/102 checks with the six intended defect probes; experimentalist steelman), which re-derived every number of record independently and forced five substantive repairs: **(1)** the primary extraction is now route (a″), the Z-conditioned fringe difference on the witness's and E1's own settings, because at the symmetric BMV geometry (s_RR = s_LL) the Pauli-quadruple route (a′) cannot determine $\hat{\Delta}$ from equatorial data and its model-calibrated reading returns the calibration model whatever the true coupling (§4.1; (a′) is demoted to a consistency reading); **(2)** every sub-threshold visibility statement is now gated on the witness itself certifying, a 5σ witness at 10^4 shots per setting needs $F \geq 0.926$ on the white-noise benchmark, and the record 12.3σ / 9.7σ rows are angle-resolution statements at visibilities where that benchmark state is separable (§6); **(3)** the classical-channel null branch is corrected, a classical measure-and-act channel imprints the full Newtonian second difference in the conditional fringes, so the extraction alone is never a non-classicality certificate (§7); **(4)** the shot accounting is per setting and per contour point, the quadruple as dedicated settings costs ≈ 4.6 machine-days per contour point, ≈ 23 over the five registered contours, not "5–7 machine-days" once (§4.1, §6), and the measured $\hat{\Delta}$ is the ramp-inclusive total (§2); **(5)** the in-frame diagnostics are normalized by the measured contrast $\hat{F}$ (the draft's unnormalized checks would flag a frame failure at the design visibility), a $\hat{\Delta}$ bias ledger was added (§8), the $\Delta = \pi$ raw-frame value 0.951655 was re-attributed to the committed-geometry locals at τ_π (the committed-point locals forced to $\Delta = \pi$ give 0.948713), and title, figure riders, and citation strings were corrected. Seven derived shot-budget integers proposed by the steelman leg were re-certified by exact ceiling (23344 / 2612 / 2386 / 1478, entering the §6 witness column, and 2916 / 11663 / 467 for its (a″) budget triple, recorded in the finalize ledger, replacing its rounded 23343 / 2611 / 2385 / 1477 / 2915 / 11660 / 466). **No value of record moved; no licensed sentence text changed.** Verification record at the end of this note. Supersedes draft v0. **Posting is governed by program policy, not by this note: licensing is never posting; defer-until-big governs timing; every quotation event owes the standing literature re-sweep first.** Experiment-facing sibling of the E1 visibility note [E1] and an additive layer over the Witness Theorem [W]; licensed by the witness–magic decision document [WM] (v1.1, external quotation unblocked whole-with-riders). Self-contained: no reading of the program documents is assumed. Parameters checked against Bose et al. *A Spin Entanglement Witness for Quantum Gravity*, arXiv:1707.06050 / PRL 119, 240401 (2017) (`papers/06_witness_program/Bose_2017_BMV_spin_entanglement_witness.pdf`, PRL version alongside). Every number below is a certified value of record, an exact-arithmetic evaluation of one by the figure engine or the finalize certifier (new numbers labeled "derived"; the §8 bias ledger is a float-grade harness estimate, labeled as such), or a parameter quoted from Bose et al. / [E1] / [W] as labeled; Appendix A names the source and grade of each. Figures:

`analysis/e1m_fig_1_antialignment.png` (headline), `e1m_fig_2_thresholds.png`, `e1m_fig_3_precision.png`, `e1m_fig_4_extraction.png` (SVG twins alongside; data tables in `analysis/e1m_data.md`).

Companion documents: [W] `06_witness_theorem.md` (Theorem W, assumptions W1–W6, Definition 1, inherited verbatim and unamended here); [E1] `05a_E1_visibility_note.md`; [WM] `21_witness_magic.md`; [REC] `analysis/witness_magic_results.md` + `analysis/witness_magic_rows.jsonl` (certified values of record); [FIG] `analysis/e1m_data.md` + `analysis/e1m_figures.py`.

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1. The question, in one page

The BMV proposal entangles two ~10^{-14} kg masses, each held in a two-position superposition in adjacent Stern–Gerlach interferometers, through their mutual gravitational interaction alone; the signal is spin entanglement after recombination, read out through a two-setting witness. The Witness Theorem [W] settles what a certified positive result licenses, under six explicitly priced assumptions W1–W6 and a three-clause definition of "certified positive" (Definition 1): *no classical degree of freedom carried this entanglement, either a non-classical mediator did, or the interaction is a nonseparable direct quantum gate with no carrier.* That disjunction is the sentence a positive run should quote. It is a statement about the **channel's classicality**.

This note asks the next question, which is about the **channel's gate type**. Every two-qubit gate is either inside the Clifford group up to local unitaries or it is not; what Clifford gates and stabilizer states lack is called *magic* (nonstabilizerness), the resource that universal quantum computation needs beyond stabilizer operations. The gravitational hold implements a controlled-phase gate CZ(Δ) with $\Delta = -2K\Delta x^2/[d(d^2-\Delta x^2)]$, $K = Gm^2\tau/\hbar$, dressed by two local phase gates (§2). That a controlled-phase gate at a generic angle is non-Clifford, so that the output state carries magic, is elementary and folkloric, it is literally the opening example of Macêdo et al. arXiv:2503.18734, and it is never the claim here. The claim is a *certification* statement: which measured quantities, under which added assumptions, certify that the **gravitationally mediated gate** was non-Clifford; with what precision; at what visibility cost; and where the conclusion provably fails.

What the witness cannot do, at any value. The committed witness statistic is a sum of two commuting Pauli products (in the Schmidt-adapted calibrated frame $\langle Z\otimes Z\rangle + \langle X\otimes X\rangle$; the PRL readout pair $\{X\otimes Z, Y\otimes Y\}$ has the same commuting-stabilizer-pair structure in the PRL's own readout convention, its value 1 + C is attained in the Schmidt-adapted frame, reached by the local-phase readjustment Bose et al. themselves prescribe, the same readjustment M2 prices, not on CZ(Δ)$|{+}{+}\rangle$ read in the computational axes; only the commutation is used here). Commuting Paulis are jointly diagonalized by a stabilizer basis, so every value such a statistic can take, including its quantum maximum, is attained by a stabilizer state or a stabilizer mixture. Concretely, the Bell-diagonal mixture $\rho\bigstar = 0.578175\ |\Phi^+\rangle\langle\Phi^+| + 0.421825\ |\Phi^-\rangle\langle\Phi^-|$ reproduces both design correlators ($\langle ZZ\rangle = 1$, $\langle XX\rangle = C = 0.156350$) and the design witness value $W = 1 + C = 1.156350$ *exactly*; it is entangled (largest Bell weight > 1/2) and carries zero magic by every monotone. The entire correlator square $[-1, 1]^2$, including W = 2, is stabilizer-reachable; a standard CHSH test cannot witness magic at any violation either (arXiv:2503.18734), and the delivered state's CHSH value is only $2\sqrt{(1 + C^2)} = 2.024298$. **No function of the committed correlators certifies magic at any observed witness value** (Lemma L-M1 of [WM]). Added assumptions are therefore not an instrument artifact but a permanent price: the small-angle (T-type) magic species is not computationally self-testable in the setting of arXiv:2111.02700.

What the M-ledger adds. Five added assumptions M0–M4 (§3), stabilizer input, branch-diagonal channel, calibrated local frame, a measurement route, per-run statistics, lift the recorded data to a gate-typing certificate. The instrument is a re-scoring of data the campaign already takes: **the Z-conditioned fringes the campaign already records, A's X/Y fringe phase given B's branch outcome, from the (X_A Z_B), (Y_A Z_B) settings of the witness and of E1's V_c, give the gauge-invariant entangling angle directly: Δ = ϕ_c(B = R) − ϕ_c(B = L) = γ′ − a′ − b′, exactly invariant under local Z phases on either side, with the mirror (Z_A X_B), (Z_A Y_B) as an independent second estimate** and a shot-noise error bar with no model term (route (a″), §4.1; the Pauli-quadruple reading (a′) survives as a model-calibrated consistency check). If the extracted interval [Δ_lo, Δ_hi] excludes the Clifford exceptions {0, ±π}, Theorem M (§3, licensed sentence S-WM-2) certifies that the gravitational coupling implemented a **nonlocal non-Clifford gate, magic injected**, with monotone floors evaluated on that interval. At the committed BMV operating point (450 μm, 250 μm, τ = 2.5 s) the angle is Δ = –211/672 = –0.313988 rad, 12.3σ from Δ = 0 at 10^4 shots per setting and 70% visibility (angle resolution; the witness must itself certify at the run's visibility), and **at any visibility at which the witness certifies entanglement at 5σ with 10^4 shots per setting ($F \geq 0.926$ on the white-noise benchmark, derived), the committed angle is ≥ 14σ from Δ = 0, the extraction is never the bottleneck; the witness is**, with zero added hardware and zero added shots.

The structure worth remembering. Along the hold-time scan the entanglement witness grows *linearly* ($W \approx 1 + |\Delta|/2$) while the certified magic floor grows *quadratically* ($M_2 \approx 1.0820\,\Delta^2$), weak-field gravity is entanglement-first, magic-second, and the magic floor returns to exactly zero at Δ = π, where the witness is *maximal* (W = 2) on a stabilizer state (Fig. 1). The best entanglement point is a zero-magic point; non-classicality is angle-generic, magic is angle-conditional. The two certificates decouple within one protocol family, and the **state-level** magic witness costs strictly more visibility than the entanglement witness (thresholds 0.975555 vs 0.864790 on the white-noise benchmark, gap 0.1108); the recommended extraction route pays no visibility premium beyond F_ent, the gap is the width of the window in which the extraction + M-lift is the only magic instrument (rider (i) is operative exactly there).

Why run it. (i) It is free: the primary route re-scores the conditional fringes of settings the witness and E1 already run. (ii) A positive run then delivers a second, sharper typed conclusion, at gate grade, about the induced gate on the matter sector, stable across both readings of Theorem W's disjunction. (iii) The extracted angle $\hat{\Delta}$ with its error bar is a quantitative measurement of the gravitational second-difference phase that checks the ±5% phase model the witness itself relies on. (iv) The ledger states explicitly what a positive result does *not* certify, mediator ontology, computational hardness, or state magic without the frame assumption. A null outcome of any branch never feeds "gravity is classical" (§7).

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2. Setup, gate of record, and BMV-realistic numbers

Geometry and notation follow Bose et al. 2017 and [E1] §2. Two masses $m_1 = m_2 = m = 10^{-14}$ kg (micro-diamonds with embedded NV spin), released from adjacent traps separated by d, each split spin-dependently into $|L\rangle$ and $|R\rangle$ separated by Δx, held for τ, recombined; readout is two-spin correlations. Branch separations $s_{RL} = d - \Delta x$, $s_{LL} = s_{RR} = d$, $s_{LR} = d + \Delta x$; branch phases $\phi_b = K/s_b$ with $K = Gm^2\tau/\hbar$.

Gate of record. On the spin (branch-label) qubits the hold is the diagonal unitary U = diag(e^{iϕ_LL}, e^{iϕ_LR}, e^{iϕ_RL}, e^{iϕ_RR}), which factorizes exactly, up to a global phase, as

> **$U = (P_1(a) \otimes P_2(b)) \cdot CZ(\Delta)$, $a = \Delta\phi_{RL} = \phi_{RL} - \phi_{LL}$, $b = \Delta\phi_{LR} = \phi_{LR} - \phi_{LL}$, $\Delta = \phi_{LL} + \phi_{RR} - \phi_{LR} - \phi_{RL} = -(a + b) = -2K\Delta x^2/[d(d^2 - \Delta x^2)]$,**

with P(θ) = diag(1, e^{iθ}) the local phase gate and CZ(Δ) = diag(1, 1, 1, e^{iΔ}) the controlled-phase gate. Only Δ, the second difference of the branch phases, is invariant under local phase changes; a and b are gravitational in origin but locally counterfeitable, and they never enter the certificate. $\Delta < 0$ for the attractive interaction; figures show $|\Delta|$. The entangling phase of [E1] is $\Delta\phi = \Delta\phi_LR + \Delta\phi_RL = a + b = |\Delta|$; the concurrence of the delivered pure state is $C = |\sin(\Delta/2)|$ and the optimal two-setting witness value is $W = 1 + C$.

Committed operating point (Bose et al. design: d = 450 μm, Δx = 250 μm, closest approach 200 μm, τ = 2.5 s; $Gm^2/\hbar$ = 63.3 μm·rad/s at m = 10^{-14} kg). All values are exact rationals or rigorous-interval evaluations (Appendix A).

quantity	exact	value	note
K/d	211/600	0.351667	
a = Δφ_RL	211/480	0.439583 rad	Bose et al. print "Δφ_RL ~ 0.7" (arXiv v1 and PRL alike), loose; the PRL's own "W ~ 1.16" corroborates the corrected value ([E1] §2)
b = Δφ_LR	−211/1680	−0.125595 rad	printed "Δφ_LR ~ −0.2", same remark
Δ = −(a + b)	**−211/672**	**−0.313988 rad**	two independent routes agree exactly
dΔ/dτ at (450, 250)	211/1680 rad/s	0.125595 rad/s	Family I shape r = Δx/d = 5/9
dΔ/dτ at (800, 600)	5697/28000 rad/s	0.203464 rad/s	Family II A shape r = 3/4 ([E1] §5)
C = |sin(Δ/2)|	-	0.156350	
W = 1 + C	-	1.156350	PRL 119, 240401 prints "W ~ 1.16"
CHSH of the delivered state	$2\sqrt{(1 + C^2)}$	2.024298	> 2, far below $2\sqrt{2}$
margin of Δ to the exceptions	|Δ|, π − |Δ|	0.3140 / 2.8276 rad	interval-certified

Ramp honesty. The data return the **total** second difference including the split/recombine segments ([E1] §2: ~0.5–0.9 rad at the design point, ramp-dependent); the hold-only Δ = −211/672 is the reference of record; a larger ramp-inclusive $|\hat{\Delta}|$ moves every distance-from-0 up and the $0.05|\Delta|$ model term up proportionally; the ±5% trajectory-integrated model of M2 is precisely a model of that total.

Where the gate is certifiably magic-free (within the committed family model). The nonlocal part CZ(Δ) is Clifford-equivalent exactly when $\Delta \in \pi\mathbb{Z}$, and this set splits physically: $\Delta \in 2\pi\mathbb{Z}$ is the no-entanglement accident (witness-free *and* magic-free, the Δ = 0 endpoint of every scan); $\Delta \in \pi + 2\pi\mathbb{Z}$ is the Clifford CZ at the *maximal* witness value W = 2, the site of the separation theorem (§7). Requiring the local phase gates to be Clifford as well gives a discrete ladder in K/d on each rational-shape family: $K/d \in (28\pi/5)\mathbb{Z}$ for the Bose shape (generator 17.5929; the half-ladder $(14\pi/5)(2\mathbb{Z}+1)$ has Clifford locals with a CS-grade $\Delta \equiv \pi/2 \bmod \pi$ residue, a natural control family) and $K/d \in (7\pi/6)\mathbb{Z}$ for r = 3/4 (generator 3.66519); the first such point on the committed ray is τ = 125.07 s. The odd-π exception is a family theorem, never a laboratory point: **$\tau_\pi = 1680\pi/211 = 25.0136$ s at the committed geometry (15.4405 s for Family II A), equivalently $m_\pi = 3.1631\times10^{-14}$ kg at τ = 2.5 s; in quantized form, gravity delivers a Clifford nonlocal gate iff $m^2\tau = k\cdot(\pi\hbar/2G)\cdot d(d^2 - \Delta x^2)/\Delta x^2$, a quantum of 2.5014×10^{-27} kg²·s at this geometry.** No registered E1 contour ($|\Delta| = 0.10 \ldots 0.50$, [E1] §5) comes near either exception; within every registered family the gate is non-Clifford everywhere except the trivial $\tau \to 0$ endpoint.

Frame of record. All state-level quantities below refer to the delivered state in the *calibrated* frame, $\psi(\Delta) = CZ(\Delta)|{+}{+}\rangle$, i.e. after the local phases a, b are gauged away by the calibration that assumption M2

supplies. The raw-frame state (branch phases as they stand) carries *more* magic (§8), so the calibrated-frame numbers never overclaim.

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3. Theorem M and its ledger

3.1 Inherited without change: Theorem W, W1–W6, Definition 1

Theorem M is an additive layer: it assumes everything Theorem W assumes, adds M0–M4, and draws an additional conclusion. Nothing in W1–W6 or Definition 1 is altered; their full text is [W] §1.1–§1.2. For the reader's convenience the ledger is restated here with each assumption's laboratory certificate ([W] §1.2 certification map):

assumption	content (abridged)	operational certificate
W1 closed labs / A1′	all branch-dependent operations generated inside each lab's local algebra; every exterior branch record is dressed and erased at recombination	the E1 pair/single visibility ratio on fixed-$\Delta\phi$ contours [E1], the visibility the campaign measures anyway
W2 / W2b transfer exclusion, EM budget	no coherent matter transport at witness order; the Casimir–Polder/EM channel priced explicitly	binding spectroscopy (trivial for solids); single-mass control interferograms; the (d, τ) scan separating $1/r$ from $1/r^4$ scaling
W3 linear-order composition	sector-wise tomography exact at the order the protocol composes (abelian scope declared)	theorem-level only
W4 mediation semantics	"mediated" read per the mediation definition of [W]; cite Martín-Martínez & Perche arXiv:2208.09489 in this row	the retardation/interposition/autonomy signatures are infeasible at BMV, hence Theorem W's conclusion is a disjunction
W5 subsystem independence	commuting part algebras, independent preparation, free settings, separable initial state (sector-wise exact; strict factorization not assumed); cite Boulle & Franzmann arXiv:2512.17024 in this row	preparation/calibration runs
W6 no superdeterminism; QM/Born for matter	settings independent of the channel microstate; fair sampling; standard spin readout	single-mass fringe statistics (Born); (i) not certifiable, as in every Bell test
Definition 1 certified positive result	(1) a spin–spin statistic with a *sound* separable bound above the certified noise/systematics budget, the corrected two-setting witness (bound 1, design value $1 + C = 1.1563$; the PRL version of record prints the sound pair) or a physical Bell-grade statistic; (2) the excess over the EM channel budgeted explicitly (margin or scan discriminator); (3) a per-run i.i.d. correlator estimate, no joint quantum processing of runs	the witness itself; the scan; standard correlator estimation

Theorem W (condensed from [W] §1.3 and §6 item 1; the statement of record is [W] §1.3, inherited verbatim and unamended). Under W1–W6 and a certified positive result, the degree of freedom that coupled the two labs is non-classical: either (I) a mediating degree of freedom exists and is non-classical in every corpus sense (finite-GPT, classical measure-space/CQ, c-number field, locally-classical non-tomographic, or any other classical mediation), or (II) there is no mediator at all and the interaction is a nonseparable direct quantum gate on the joint matter system. Mass-side data cannot separate (I) from (II).

3.2 The gate at the operating point (licensed sentence S-WM-1, verbatim)

> "Within the committed BMV family, the gravitational entangling gate is $(P_1(a)\otimes P_2(b))\cdot CZ(\Delta)$ with $\Delta = -2K\Delta x^2/[d(d^2-\Delta x^2)]$; its nonlocal Clifford-exception set is exactly $\Delta \in \pi Z$, the no-entanglement points $2\pi Z$ and the maximal-witness Clifford points $\pi + 2\pi Z$, and the strict exception set is an explicit arithmetic K-ladder on each registered rational-shape family ($(28\pi/5)Z$ with a CS-grade half-ladder; $(7\pi/6)Z$, parity-automatic); at the committed operating point ($\Delta = -0.3140 \pm 5\%$ calibration) the gate lies outside $LU\cdot C_2\cdot LU$, with $M_2 \geq 0.104444$ and $e_p = (2/9)C^2 = 0.005432$ (monotone definitions and closed forms cited, evaluated not re-derived)."

Here $LU\cdot C_2\cdot LU$ is the two-qubit Clifford group C_2 (11520 elements, exactly four local-unitary classes) dressed by arbitrary local unitaries; M_2 is the stabilizer 2-Rényi entropy of the delivered state (Leone–Oliviero–Hamma arXiv:2106.12587; monotone status Leone–Bittel arXiv:2404.11652); e_p is the entangling power of the gate.

3.3 Theorem M (licensed sentence S-WM-2, verbatim)

> "Under W1–W6, Definition 1, and M0–M4, the stated input, channel-form, calibration, measurement-route, and statistics assumptions, each with its named operational certificate, a certified positive BMV result **whose extracted angle interval excludes πZ** additionally certifies that the gravitational coupling injected magic: it implemented a nonlocal non-Clifford gate, with $\Delta \in [\Delta_lo, \Delta_hi]$ and the cited monotone floors evaluated on that interval. Without the M-assumptions no magic conclusion follows from the committed correlators at any observed witness value (L-M1)."

The honesty sentence travels with it: *certifying magic is strictly harder than certifying non-classicality, and W1–W6 + M0–M4 are the added premises; without them no magic conclusion follows.*

3.4 The M-ledger

assumption	what it means in the lab	operational certificate	cost
M0, stabilizer input	the pre-interaction spin state is certified close to the	++⟩-class product (each spin in an X-eigenstate before the spin-dependent split)	preparation/calibration runs, the W5 row extended to fidelity grade. *Honesty row:* this is a protocol requirement, not a computed budget; the collaboration sets the fidelity target. Robustness anchor (derived): a π/2-pulse angle error ε leaves per-qubit stabilizer fidelity $\cos^2\varepsilon$, $1° \to 3\times10^{-4}$ infidelity. *Consumption:* the state-level floors consume M0 in full; the LU-class typing needs only support on all four branches ($V_s > 0$)
M1, channel form	over the interaction window the two-lab evolution is a branch-diagonal unitary (branch phases only; no spin flips or leakage) plus a residue inside a certified budget	E1's conditional-visibility ratio $V_c/V_s \approx 1$ certifies near-unitarity; single-mass controls certify branch-diagonality	zero, [E1] runs inside the campaign
M2, calibrated family / fixed local frame	the branch phases follow $\phi_b = K/s_b$ within the ±5% trajectory phase model that the witness already requires ([E1] §5, §10.3); the local readout frames are	++⟩ up to the certified tolerance; the (d, τ) scan certifies the 1/r law. *Consumption:* consumed by the	the ±5% phase model; the (d, τ) scan; the K-2 doping control (§4.2)

assumption	what it means in the lab	operational certificate	cost
	calibrated so that the delivered state is CZ(Δ)	state-level floors and by the witness frame; **not** by the LU-class typing, which is frame-free by construction, under route (a″) the phase model is a cross-check (Δ̂ vs Δ_model within 5%), not an input (§4.1)	
M3, measurement route	primary (a″): the Z-conditioned fringe difference, A's X/Y fringe phase given B's Z outcome, from the (X_A Z_B), (Y_A Z_B) settings of the witness and of E1's V_c, with the mirror (Z_A X_B), (Z_A Y_B) as an independent second estimate; consistency (a′): the local-Pauli quadruple {XX, XY, YX, YY} + X/Y marginals in the model-calibrated axes, a re-scoring of recorded two-qubit tomography (Cieśliński et al. arXiv:2512.25068 class), which at the symmetric BMV geometry does not determine a′ + b′ and whose Δ̂ is model-supplied (§4.1); secondary (b): tilted CHSH at the committed Schmidt angle (arXiv:2503.18734); (c): the triangle criterion at finite visibility (Liu et al. arXiv:2512.16777); (d): stabilizer-entropy quantification licensed by arXiv:2404.11652	trust-base honesty: certificates are relative to calibrated Pauli axes, the settings' own magic is part of the trust base; the K-2 doping control tests exactly this, and under (a″) it is a genuine null test (§4.2)	zero added settings and zero added shots on (a″); (a′) as dedicated settings: 4×10^4 shots ≈ 4.6 machine-days **per contour point** (≈ 23 over the five registered contours; full 9-setting tomography 10.4 machine-days per point)
M4, statistics	per-run i.i.d. shots (Definition 1, clause 3, inherited); σ_Δ propagated as in §6	standard correlator estimation	none new

3.5 Reading the theorem

- **"Nonlocal non-Clifford gate" is a local-unitary-class statement, certified by invariants.** The Makhlin invariants of CZ(Δ) are $G_1 = \cos^2(\Delta/2)$ and $G_2 = 2 + \cos\Delta$; the four Clifford classes carry $G_2 \in \{\pm1, \pm3\}$, while any $\Delta \notin \pi\mathbb{Z}$ gives G_2 strictly inside (1, 3), so an interval excluding πZ certifies the gate outside $\mathrm{LU}\cdot C_2\cdot\mathrm{LU}$ with no further computation (committed values $(G_1, G_2) = (0.9756, 2.9511)$). The entangling power corroborates it one-way: $e_p = (2/9)\sin^2(\Delta/2) = 0.005432$ lies strictly inside (0, 2/9), while Clifford gates attain only the anchors {0, 2/9} (2/9 is the maximum over all two-qubit gates).
- **The monotone floors** are evaluations of cited closed forms on the calibrated-frame state $\psi(\Delta) = \mathrm{CZ}(\Delta)|{+}{+}\rangle$: $M_2(\Delta) = 4 - \log_2[4 + 3(1 + \cos^2\Delta)^2]$ bits (zero iff $\Delta \in \pi\mathbb{Z}$; maximum 1.192645 at $\Delta = \pi/2$); the stabilizer fidelity $F_{\mathrm{STAB}}(\Delta) = (5 + 3|\cos\Delta|)/8$ (= 1 iff $\Delta \in \pi\mathbb{Z}$); e_p as above. Committed values: **$M_2 = 0.104445$ (certified ≥ 0.104444), $F_{\mathrm{STAB}} = 0.981666$, $e_p = 0.005432$.** On an extracted interval the floor is the weakest value each quantity takes over the interval, the smallest M_2 and e_p, the largest F_{STAB}; for $|\Delta| \le \pi/2$ all three sit at the endpoint nearest zero. Channel-level monotones (channel robustness and magic capacity for Z-rotations, Seddon & Campbell

arXiv:1901.03322; the unitary stabilizer entropy of arXiv:2503.20873, which also supplies the phrase "magic injection") are cited, not re-derived.

- **Reading-stable across Theorem W's disjunction.** Under reading (I) the mediated channel and under reading (II) the direct gate deliver the *same* diagonal gate on the masses (the polaron identity of [W]); the gate typing therefore does not split, unlike the mediator typing. The certificate is about the **induced gate on the matter sector**. "The graviton is magic" / "the mediator is magic" is not a licensed sentence at any grade; the mediator conclusion stays Theorem W's.
- **Not certified:** state magic at readout as *gravitational* without M2 (the labs' own pulses are non-Clifford-capable, §4.2); any mediator-sector magic; anything at the Clifford-exception couplings (§7); any hardness or speedup claim (nonstabilizerness ≠ hardness, classically simulable nonstabilizer states exist, arXiv:2509.25790).

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4. The protocol

4.1 Primary route (a″): the conditional-fringe difference, re-scoring data the campaign already takes

The E1 design records W, the conditional visibilities, and (recommended) full two-qubit spin tomography from the same shot data ([E1] §4: "should still be recorded"). In the readout frame as it stands the delivered state is, with lab A the first factor,

> ψ = ½ (|00⟩ + e^{ib′}|01⟩ + e^{ia′}|10⟩ + e^{iγ′}|11⟩), calibrated frame a′ = b′ = 0, γ′ = Δ; raw BMV frame a′ = a, b′ = b, γ′ = ϕ_RR − ϕ_LL ≡ 0 (since s_RR = s_LL).

The estimator. Condition A's equatorial fringe on B's Z outcome: ϕ_z = atan2(⟨Y_A|z⟩, ⟨X_A|z⟩) for z ∈ {L, R} (all shots used, no post-selection; correct ⟨X_A|z⟩, ⟨Y_A|z⟩ and the Z-conditioning through the measured readout confusion matrices before the atan2). Then

> **Δ̂_A = ϕ_R − ϕ_L (wrapped to (−π, π]) = (γ′ − b′) − a′ = γ′ − a′ − b′,** exactly invariant under local Z phases on either side, doping A shifts both conditional phases equally, doping B commutes with Z_B, **with no model input**; the mirror Δ̂_B from (Z_A X_B), (Z_A Y_B) is an independent second estimate. Report both and their mean.

σ_Δ = 2/(F√N) per side, √2/(F√N) with both mirrors averaged, identical to the (a′) statistical term, plus the readout-bias residual of §8; **no ±5% model term and no 20σ ceiling** (the phase model becomes a cross-check: Δ̂ vs Δ_model within 5%). The attractive sign Δ̂ < 0 (A's fringe phase larger when B sits in the near branch) is a free prediction, not an input. At the committed point the raw-frame conditional phases are a = 0.439583 and −b = 0.125595, difference −211/672 = −0.313988 (ring-exact, [FIG] D06; Fig. 4B).

Shot accounting (per setting, per contour point). N is per setting. A 5σ witness at the design point needs ~10^4 shots per witness setting at F ≥ 0.926 anyway (§6); route (a″) uses those settings, X_A Z_B is in the witness pair, Y_A Z_B in E1's V_c, **zero added shots**. Route (a′) needs three settings the witness and E1 do not run (XX, XY, YX): 4×10^4 shots ≈ 4.6 machine-days per contour point (≈ 23 over the five registered contours) unless nine-setting tomography at 10^4 per setting is already planned (10.4 machine-days per point).

Settings table (per contour point; N shots per setting).

(A, B) setting	PRL witness	E1 V_c	(a′) quadruple	(a″) conditional	yields
X Z	yes	yes	-	yes	⟨XZ⟩; A's X-fringe given z_B; ⟨XI⟩, ⟨IZ⟩
Y Z	-	yes	-	yes	A's Y-fringe given z_B; ⟨YI⟩
Z X, Z Y	(mirror)	(mirror)	-	yes (mirror)	B's fringes given z_A; Δ̂_B
Y Y	yes	-	yes	-	⟨YY⟩
X X, X Y, Y X	-	-	yes	-	γ′ and a′ − b′ only
Z Z	-	-	diagnostic	optional	equal branch weights (⟨ZZ⟩ = 0); a ninth setting

Consistency route (a′): the Pauli quadruple in the model-calibrated frame. The extraction identities hold exactly for any phase state of this form (Fig. 4):

> **cos γ′ = ⟨XX⟩ − ⟨YY⟩, sin γ′ = ⟨XY⟩ + ⟨YX⟩ ⇒ γ′ = atan2(⟨XY⟩ + ⟨YX⟩, ⟨XX⟩ − ⟨YY⟩)** (both quadratures, no reflection ambiguity; equals Δ̂ only in the calibrated frame, M2); **⟨XX⟩ + ⟨YY⟩ = cos(a′ − b′), ⟨YX⟩ − ⟨XY⟩ = sin(a′ − b′)** (frame diagnostics; the sign of the second is labeling-convention-dependent and carries no load); **⟨XI⟩ + i⟨YI⟩ = (e^{ia′} + e^{i(γ′−b′)})/2, |⟨XI⟩ + i⟨YI⟩| = |cos(Δ̃/2)|, Δ̃ ≡ γ′ − a′ − b′** (frame-free modulus: the marginal-contrast identity of [E1] §4; exactly invariant under local Z-doping, but quadratically insensitive near the committed angle: a cross-check, never a certificate).

Why (a′) is not the primary route (the steelman verification's finding). The quadruple returns γ′ and a′ − b′; the equatorial marginals return (γ′ ± (a′ − b′))/2 and the modulus: **no combination of equatorial-only data determines a′ + b′, hence none determines Δ̃ = γ′ − (a′ + b′) with linear sensitivity.** At the symmetric BMV geometry the raw-frame quadruple reads γ′ ≡ 0, the Clifford-exception value, for any coupling strength, and after the axes are calibrated by the ±5% model it reads Δ_model whatever gravity does (a true coupling at 0.5×/1×/1.5× the design value returns −0.313988 in all three cases; only the a′ − b′ diagnostic moves). Under (a′) alone, "12.3σ from Δ = 0" is the precision with which γ′_raw = 0 is confirmed, with the magnitude supplied by the model. The (a′) reading is therefore a **consistency check of the calibration model**, valuable, and quoted throughout §6 as the record σ-budget, but not by itself the measurement; the σ-ladder survives unchanged because the (a″) both-mirror statistical term √2/(F√N) is identical.

Step by step. (1) From the recorded settings compute the conditional fringe phases ϕ_L, ϕ_R on side A and the mirror pair on side B; Δ̂ = ϕ_R − ϕ_L wrapped to (−π, π], sign checked against Δ < 0 (attractive). (2) Where the quadruple is recorded, run the (a′) consistency reading in the calibrated axes and check γ′_cal against Δ̂. (3) *Protocol remark (not a value of record):* run the in-frame consistency checks with the visibility divided out, F̂ ≡ √((⟨XX⟩ − ⟨YY⟩)² + (⟨XY⟩ + ⟨YX⟩)²): (⟨XX⟩ + ⟨YY⟩)/F̂ should equal 1 within tolerance (a′ = b′: the local frames are calibrated), ⟨ZZ⟩ should vanish (equal-weight phase state, at any F; a ninth setting, optional under (a″)), and the marginal modulus |⟨XI⟩ + i⟨YI⟩|/F̂ (and its B-side twin) should reproduce |cos(Δ̂/2)|, the frame-free cross-check of the same angle. The division by F̂ is exact on the white-noise benchmark; under non-white channels the marginal and correlator contrasts need not share one F, and the marginal check then tests the noise model rather than the frame. (4) Propagate errors. Route (a″): σ_Δ = √2/(F√N) (both mirrors; 2/(F√N) one side) plus the readout-bias residual (§8). Route (a′): σ_Δ = √(2/N)/F ⊕ 0.05|Δ| (statistical term from the two quadrature combinations ⟨XX⟩ − ⟨YY⟩ and ⟨XY⟩ + ⟨YX⟩, four correlators from four settings, each at shot noise ≤ 1/√N, and visibility F; the ±5% calibration systematic of M2 in quadrature). Report [Δ_lo, Δ_hi] at the stated confidence. (5) **Decision rule** (every clause at one stated k; k = 5 recommended): (D-1) certified positive, W − 1 ≥ kσ_W in the readjusted frame, EM budget clause, per-run i.i.d. (Definition 1); (D-2) |Δ̂| ≥ kσ_Δ and π − |Δ̂| ≥ kσ_Δ (exclusion of πZ); (D-3) Δ̂_A and Δ̂_B agree within 2σ, and Δ̂ < 0; (D-4) readout confusion matrices measured on the same

runs and inverted, residual bias ≤ 0.2σ_Δ; (D-5) K-2 doping control: with deliberate Z(ϕ) pulses (ϕ non-Clifford, e.g. 0.4 rad) on either side before readout, $\hat{\Delta}$ moves by < 2σ with nothing subtracted (§4.2); (D-6) M-certificates: M1 from E1's R = V_c/V_s = 1 within budget on the same runs; M0 from V_s > 0 and the preparation-fidelity runs; M2 in its residual role, $\hat{\Delta}$ agrees with the trajectory-integrated model within 5% and the (d, τ) scan gives 1/r. Then Theorem M fires: report the k-σ interval, the LU-class certificate $G_2 = 2 + \cos\Delta$ on it, and the floors M_2, F_STAB, e_p evaluated at the interval endpoint nearest the exception, with S-WM-2 whole-with-riders.

Worked numbers at the committed point (ideal state, F = 1; Fig. 4). Route (a″), raw frame: ϕ_L = a = 0.439583, ϕ_R = −b = 0.125595, $\hat{\Delta} = \phi_R - \phi_L = -0.313988 = -211/672$. Route (a′), calibrated frame: ⟨XX⟩ = 0.975555, ⟨YY⟩ = 0.024445, ⟨XY⟩ = ⟨YX⟩ = −0.154427, ⟨XI⟩ = 0.975555, ⟨YI⟩ = −0.154427, ⟨ZZ⟩ = 0; hence $\cos\hat{\Delta} = 0.951109$, $\sin\hat{\Delta} = -0.308854$, $\hat{\Delta} = -0.313988$; $|\langle XI\rangle + i\langle YI\rangle| = 0.987702 = |\cos(\Delta/2)|$; ⟨XX⟩ + ⟨YY⟩ = 1. Two coincidences worth knowing: ⟨XX⟩ = (1 + cos Δ)/2 equals both the Makhlin invariant G_1 and the visibility threshold of the fidelity-type magic witness (§6). With finite visibility every correlator scales by F and the ratios in the identities are unchanged; only σ_Δ grows.

4.2 The frame requirement (M2) and the doping-invariance guarantee

Two facts about local operations shape the certificate. First, a local phase gate Z(ϕ) at a non-Clifford angle ϕ is itself magic-injecting, and nonstabilizerness that enables Bell-type violations can sit entirely in local, asymmetric corrections (arXiv:2504.03351): **state-level magic in the delivered spins is locally counterfeitable**, the labs' own pulses can produce it. That is why the certified quantity is only the second difference $\hat{\Delta}$, which no local phase can move, and why M2 (a calibrated, fixed local frame) is load-bearing **for the state-level floors and for the (a′) reading**: without it, the local-unitary ambiguity lets state magic be booked either in the local corrections a′, b′ or in the mediated angle, and only the calibrated frame ties the quadruple reading γ′ to Δ. The (a″) conditional difference is $\hat{\Delta} = \gamma' - a' - b'$ by construction and needs no frame, which is exactly why it is the primary route.

Second, the guarantee runs in the other direction as well. Under two-sided local Z-doping $Z(\phi_1) \otimes Z(\phi_2)$ the bare quadruple moves ($\gamma' \to \Delta + \phi_1 + \phi_2$, $a' - b' \to \phi_1 - \phi_2$) while $e^{i\hat{\Delta}}$ and the marginal modulus $|\cos(\hat{\Delta}/2)|$ are *exactly* invariant, a ring identity, verified on the committed point and in a deliberate doping control (Fig. 4B: with illustrative $\phi_1 = 0.4$, $\phi_2 = -0.7$ rad the quadruple reads $\gamma' = -0.613988 \neq \Delta$ and ⟨XX⟩ + ⟨YY⟩ = 0.453596, while $|\langle XI\rangle + i\langle YI\rangle|$ stays 0.987702 and $\hat{\Delta} = \gamma' - a' - b' = -0.313988$). **The K-2 control is therefore a laboratory test, not a thought experiment:** apply deliberate local Z(ϕ) pulses at non-Clifford ϕ before readout and verify that $\hat{\Delta}$ is unchanged, the instrument never books local magic as gravitational. The two frame diagnostics (⟨XX⟩ + ⟨YY⟩ and ⟨YX⟩ − ⟨XY⟩) report the doping directly. **Under route (a″) the K-2 run is a genuine null test, nothing is subtracted; the doping must simply not move Δ̂. Under route (a′) it tests the instrument's linear response to *known* phases only, since they must be subtracted through the calibration model.**

4.3 Fallback routes and their honest fragility

All alternatives are *state-level* witnesses and are visibility-starved at the committed angle; they are listed for completeness and as cross-checks, not as the teeth of the protocol.

- **(b) Tilted CHSH at the committed Schmidt angle** (arXiv:2503.18734): the delivered state is LU-equivalent to $\cos\theta|00\rangle + \sin\theta|11\rangle$ with $\sin 2\theta = C$, $\theta = 0.078497$ rad (4.4975°). The committed point *is* inside the ideal magic-witnessing window, by a margin of 2.95×10^{-4} in the inequality value, and the white-noise visibility needed to keep it there is F_b = 0.999925, noise-fragile at any realistic visibility.

- **Fidelity-hyperplane witness:** a fidelity with ψ(Δ) exceeding the maximal stabilizer overlap F_STAB(Δ) certifies the delivered state outside the stabilizer polytope (stabilizer-polytope witnesses: arXiv:2409.18570; mixed-state witnessing: Haug & Tarabunga arXiv:2504.18098). On the white-noise benchmark $\rho_F = F|\psi\rangle\langle\psi| + (1 - F)I/4$ this needs $F > F_{magic} = (1 + \cos\Delta)/2 = 0.975555$ at the committed point (§6).
- **(c) Triangle criterion** (arXiv:2512.16777): the finite-visibility, mixed-state form of the state-level certificate, the right tool if a decohered output is to be typed directly.

Under M0–M2 any state-level certificate lifts to the channel (stabilizer input + branch-diagonal unitary + calibrated frame ⇒ the gate mapped a stabilizer state outside the stabilizer polytope ⇒ the channel is not completely stabilizer-preserving), but at the committed angle none of these routes carries the load. **Rider (i), verbatim: "at realistic noise the teeth are the (a′) extraction route + M0–M2 gate-grade lift, not the state-witness route."** (Reading note, outside the licensed text: the rider's "(a′) extraction route" names M3's extraction lane as opposed to the state-witness routes (b)/(c); within that lane the primary estimator of this note is route (a″), the conditional-fringe difference, with the quadruple (a′) as the model-calibrated consistency reading. The rider travels verbatim and unamended.)

4.4 If the tomography is not recorded

Run the four correlators (plus marginals) as dedicated settings, interleaved with the witness settings per [E1] §7: 4–6 settings × 10^4 shots at ~10 s/shot ≈ 5–7 machine-days **per contour point** (≈ 23 machine-days over the five registered contours for the four-setting quadruple), the same shot class as the witness. The primary instrument remains the (a″) re-scoring of settings already run, which needs no fallback; this is the (a′) fallback cost.

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5. The anti-alignment exhibit (headline figure)

Fig. 1 (`analysis/e1m_fig_1_antialignment.png`) plots, against |Δ| from 0 to π with the equivalent hold time at the committed geometry (0 to 25.0 s) on the upper axis: (A) the witness $W = 1 + |\sin(\Delta/2)|$; (B) the magic floor $M_2(\Delta)$; (C) the stabilizer fidelity F_STAB(Δ); (D) the entangling power $e_p(\Delta)$; with the committed point, the weak-field laws, the registered E1 contours |Δ| = 0.10 … 0.50, and the Clifford exception at Δ = π. Exact closed forms, evaluated in rigorous interval arithmetic:

|Δ|	τ [s] at (450, 250)	W = 1 + C	M_2 [bits]	F_STAB	e_p	F_ent	F_magic
0	0	1	0	1	0	1	1
0.10	0.796	1.049979	0.010798	0.998127	0.000555	0.952400	0.997502
0.20	1.592	1.099833	0.042918	0.992525	0.002215	0.909229	0.990033
0.30	2.389	1.149438	0.095526	0.983251	0.004963	0.869990	0.977668
211/672 (committed)	**2.500**	**1.156350**	**0.104445**	**0.981666**	**0.005432**	**0.864790**	**0.975555**
0.40	3.185	1.198669	0.167188	0.970398	0.008771	0.834258	0.960530
0.50	3.981	1.247404	0.255806	0.954093	0.013602	0.801665	0.938791
π/2	12.507	1.707107	1.192645	0.625000	0.111111	0.585786	0.500000
π	25.014	2	0	1	2/9	0.500000	1

Physical reading. In the weak field the witness grows linearly, $W \approx 1 + |\Delta|/2$ (1.156994 vs the exact 1.156350 at the committed point), while the certified magic grows quadratically, $M_2 \approx (3/(4 \ln 2))\,\Delta^2 = 1.0820\,\Delta^2$ (0.106675 vs 0.104445): **weak-field gravity is entanglement-first, magic-second**, $M_2/C \to 0$ as $\Delta \to 0$. This is the quantitative reason the magic certificate needs a precision instrument (§6) while the

witness is O(1), and why the near-term monotone floors are small (0.104 bits) although the witness is comfortably above its bound. Continuing along the family, the magic floor peaks at Δ = π/2 (the controlled-S point; $M_2 = 4 - \log_2 7 = 1.192645$ bits, which is the global two-qubit maximum ln(16/7) nats identified in arXiv:2502.17550) and then **returns to exactly zero at Δ = π**, where the witness is *maximal*: CZ(π)|++⟩ is a stabilizer state (stabilizer group {II, XZ, YY, ZX}) on which W = 2, F_STAB = 1, $M_2 = 0$, and e_p takes its Clifford value 2/9. The two certificates are not co-monotone; more witness is not more magic; the best entanglement point is a zero-magic point.

Reach-honesty row (travels with every quotation of τ_π or m_π). The Δ = π separation site is a **family theorem, never a lab point**: τ_π ≈ 25.0 s at the committed geometry / 15.4 s for Family II A; $m_\pi = 3.16\times10^{-14}$ kg at τ = 2.5 s. Every registered E1 contour sits at |Δ| ≤ 0.5, on the rising quadratic flank. **Both curves are functions of the single measured angle Δ̂: the witness is measured, the floor is $M_2(\hat{\Delta})$ evaluated under the M-ledger; the scan measures Δ̂ at five contours and tests nothing else about the figure.** The π-point is quoted as the family's limiting structure.

The ambient plane for this figure, the exact two-qubit magic-versus-entanglement Pareto frontiers, is arXiv:2603.24902; the stabilizer minimum frontier is the ambient fact behind the impossibility statement of §1 (stabilizer states reach every witness value).

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6. Precision and cost

The precision instrument (Fig. 3, `analysis/e1m_fig_3_precision.png`). The record σ-budget is quoted in the (a′) form, each correlator at shot noise ≤ 1/√N, visibility F, and the ±5% calibration model of M2 entering as a systematic in quadrature:

> **$\sigma_\Delta = \sqrt{(2/N)}/F \oplus 0.05|\Delta|$.**

The (a″) primary route shares the identical both-mirror statistical term √2/(F√N) and drops the model term (§4.1), so the distances below are conservative for (a″) up to its readout-bias residual (§8). What Theorem M's premise consumes is the distance of Δ̂ from the nearest exception in units of σ_Δ. At the committed point and $N = 10^4$ shots per setting:

visibility F	witness W_F = F(1 + C) on this benchmark	σ_Δ	distance from Δ = 0	distance from Δ = ±π	N_min for 5σ from Δ = 0
1.0	1.156350	0.02113	14.9σ	134σ	541 (derived)
0.926	1.070780, 5σ witness at $N = 10^4$ (derived)	0.02190	14.3σ	129σ	631 (derived)
0.7	0.809, **no certified positive; angle resolution only**	0.02559	12.3σ	111σ	**1105 (exact; value of record)**
0.5	0.578, **no certified positive; angle resolution only**	0.03235	9.7σ	87σ	2164 (derived)

Consistency caveat (derived, not a value of record). F in σ_Δ is the correlator contrast; on the white-noise benchmark of the threshold table the witness itself certifies only for F > F_ent = 0.864790, so the F = 0.7 and F = 0.5 rows are angle-resolution statements, on that benchmark Theorem M's premise (a certified positive, Definition 1) is not met there, and the benchmark state is in fact separable outright (partial-transpose minimum eigenvalue +0.020 / +0.086; separability boundary F_sep = 1/(1 + 2C) = 0.7618). At F = F_ent the (a′) route sits at 13.9σ from Δ = 0 ($\sigma_\Delta = 0.02267$ at $N = 10^4$; derived). For the dephasing-type channels of [E1] §8 the witness threshold is not 1/(1 + C) and the two uses of F do not

coincide; the campaign's measured visibility, not the benchmark, decides whether both certificates are live.

The 5σ condition is the exact predicate $(|\Delta|/5)^2 \geq 2/(NF^2) + (|\Delta|/20)^2$, i.e. $N \geq 160/(3F^2\Delta^2)$; at F = 0.7 the real-valued bound is 1104.018, so **N_min = 1105 exactly**, 1104 fails the predicate by 6.14×10^{-8} in σ^2 units (exact deficit 17/276971520). Because the calibration term does not shrink with N, the (a′) significance saturates at $|\Delta|/(0.05|\Delta|)$ = **20σ** at any angle: beyond ~10^5 shots that reading is calibration-limited, not shot-limited (Fig. 3A). Route (a″) carries no model term and no 20σ ceiling; its systematic floor is the readout-bias residual (§8). The two Clifford rungs are asymmetric by construction, the committed point is 0.3140 rad from Δ = 0 and 2.8276 rad from Δ = ±π, so the operative exclusion is always the no-entanglement rung.

Across the registered E1 contours (N = 10^4, F = 0.7; Fig. 3B):

|Δ| contour	σ_Δ	distance from Δ = 0	N_min(5σ)	F needed for 5σ at N = 10^4	witness N for 5σ (F = 0.926; derived)	status
0.10	0.02081	**4.80σ**	10885	0.730	23344	**marginal, for the witness first** (the 4.80σ is a value of record): the witness needs N ≥ 20017 even at F = 1 (derived); the extraction's 4.8σ is not the binding constraint
0.20	0.02254	8.87σ	2722	0.365	5851	≥ 5σ (derived)
0.30	0.02516	11.92σ	1210	0.243	2612	≥ 5σ (derived)
211/672 (committed)	0.02559	12.27σ	1105	0.233	2386	value of record (12.3σ)
0.40	0.02843	14.07σ	681	0.183	1478	≥ 5σ (derived)
0.50	0.03214	15.56σ	436	0.146	953	≥ 5σ (derived)

The witness-N column is the dephasing-contrast reading (margin FC, so $N \geq 50/(C^2F^2)$; exact ceilings, [FIG] D07); its ratio to the extraction N is $(15/16)(\Delta/C)^2 \approx 3.75$–$3.83$ on every registered contour: **wherever the campaign can certify entanglement at 5σ, it types the gate at ≥ 5σ with a ~3.8× shot margin to spare, the witness, not the extraction, is the binding cost.** For route (a″) with no model term, N_min(5σ from 0, F = 0.7) = 2071 one-sided, 1036 with both mirrors (derived, exact ceilings).

What it costs in shots. [E1] §7 budgets 10^4 shots per (geometry, τ) point; the numbers above are per **setting**. A 5σ witness at the design point needs ~10^4 shots per witness setting at F ≥ 0.926 on the benchmark anyway, and route (a″) rides those same settings at zero added shots (§4.1). At N = 10^4 per setting the committed angle is resolved at 12.3σ even at 70% visibility (angle resolution; on the benchmark the witness itself certifies only above F_ent) and every registered contour except |Δ| = 0.10 is ≥ 5σ from the exceptions. The |Δ| = 0.10 contour is E1's long τ-flatness contour and is marginal **for the witness first** (N ≥ 20017 at F = 1); for the extraction the choices there are N ≥ 10885 (F = 0.7), F ≥ 0.730 at N = 10^4, or 3σ-grade typing on that contour. If the quadruple is wanted as dedicated settings, it costs 4–6 × 10^4 shots × 10 s ≈ 4.6–6.9 days, i.e. **5–7 machine-days per contour point** (≈ 23 machine-days over the five registered contours for the four-setting version), against E1's own 21–25 machine-day baseline.

What it costs in visibility (Fig. 2, `analysis/e1m_fig_2_thresholds.png`). On the white-noise benchmark $\rho_F = F|\psi(\Delta)\rangle\langle\psi(\Delta)| + (1 - F)I/4$:

certificate	threshold (closed form)	committed value	note
entanglement, two-setting witness W_F = F(1 + C) > 1	F_ent = 1/(1 + C)	**0.864790**	
magic, fidelity-hyperplane witness (1 + 3F)/4 > F_STAB(Δ)	F_magic = (1 + cos Δ)/2 (	Δ	≤ π/2)
gap F_magic – F_ent	-	**0.110765 ≈ 0.1108**	the experimental price of the **state-level** upgrade route, equivalently the width of the visibility window (0.8648, 0.9756) in which the extraction + M-lift is the only magic instrument
magic, tilted CHSH (route (b))	stabilizer/quantum bound ratio at sin 2θ = C	0.999925	noise-fragile

The gap is the O(Δ²)-versus-O(Δ) resource price made visible: the magic threshold stays strictly above the entanglement threshold across the whole weak-field regime and the two cross only where sin(Δ/2) = (√5 − 1)/2, |Δ| = 1.3325 rad (τ ≈ 10.6 s at the committed geometry), beyond every registered contour (family-level, derived). Two cautions. "F" here is the white-noise fraction, a *benchmark*, not a model of the apparatus; and the state-level routes are the ones this table starves, the extraction instrument keeps 9.7σ **angle resolution** at F = 0.5, where on this benchmark no certificate of any kind is live (below F_ent there is no certified positive; below F_sep = 1/(1 + 2C) = 0.7618 the benchmark state is separable outright, derived). **Rider (i), verbatim: "at realistic noise the teeth are the (a′) extraction route + M0–M2 gate-grade lift, not the state-witness route."**

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7. What a null means

Theorem M has a chain of premises, a certified positive result (Definition 1), an extracted interval excluding πZ, and M0–M4, and "null" can occur at each link. **None of them licenses a classicality claim.** Mandatory rider (ii): **a null never feeds "gravity is classical".**

1. **No certified positive (witness null at sufficient sensitivity).** This is Theorem W′′s territory ([W] §5): at sufficient sensitivity a null certifies unconditionally that the gravitational channel imparts no second-difference phase at the probed regime; its interpretation is conditional on linear phase additivity, and even then the null refutes only the conjunction {quantized gravity ∧ linearized coupling ∧ QM for matter at these scales}. *A null never certifies* "*gravity is classical*", every classical model in the corpus predicts a null, but so do modified-coupling quantum models; affirming the consequent is unavailable. **Theorem M has no premise here. The extraction is not a non-classicality certificate: a classical channel that reads one mass's branch and phases the other imprints the same second difference on the conditional fringes while destroying the entanglement (the E1 H1b class). Two sub-branches: witness null with $\bar{\Delta} = 0$ at sufficient sensitivity is Theorem W′′s case; witness null with $\bar{\Delta} \neq 0$, together with a visibility deficit tracking Δ², is the classical-channel fingerprint: a diagnostic, never a certificate, and never "gravity is classical".**
2. **Certified positive, but the extracted interval contains 0 (precision null).** Theorem M correctly declines, its premise is false, and nothing about the gate's type follows. The cure is statistical (§6), not interpretive. *Protocol remark (not a value of record):* since the witness certifies C > 0, i.e. Δ ∉

2πZ, an interval containing 0 at comparable confidence would signal a frame or visibility inconsistency between the two readings and should be treated as a diagnostic.

3. **Angle resolved, but no certified positive** (visibility below the witness threshold at the run's noise). Theorem M declines for lack of its Definition-1 premise; the extracted $\hat{\Delta}$ is then a phase measurement checking the ±5% model, not a gate-typing certificate; nothing about magic, or classicality, follows.
4. **The extraction certificate fires while every state-level magic witness is silent.** Expected at the committed angle: the state-level routes are visibility-starved ($F_{magic} = 0.975555$, $F_b = 0.999925$). Silence of a state witness below its threshold is not evidence against magic injection; rider (i) applies.
5. **The Clifford exception, the separation theorem (licensed sentence S-WM-3, verbatim):**

> "At the Clifford-exception couplings $\Delta \in \pi + 2\pi\mathbb{Z}$ the same protocol certifies non-classicality at the maximal witness value $W = 2$ while provably certifying no nonlocal magic, the delivered state is a stabilizer state, the gate is Clifford, and every sound magic witness is silent by construction; the two certified conclusions separate within one protocol family. (Family theorem: the exception coupling lies outside every registered platform, $\tau_\pi \approx 25.0$ s at the committed geometry.)"

Both directions hold: at $\Delta = \pi$ the delivered state $CZ(\pi)|{+}{+}\rangle$ is the witness maximizer (stabilizer group {II, XZ, YY, ZX}; $W = 2$), Theorem W fires at full strength with every W-assumption untouched, while every monotone vanishes identically and every sound magic witness is silent, not for lack of settings but because there is nothing to witness; at $\Delta \in 2\pi\mathbb{Z}$ neither theorem fires. Reach-honesty row: $\tau_\pi \approx 25.0$ s / 15.4 s; $m_\pi = 3.16\times10^{-14}$ kg, a family theorem, not a lab point. 6. **Classical-channel worlds** (E1's hypothesis class H1b: gravity as a classical information channel, forced to decohere at the rate it would have entangled) produce no witness and no magic certificate, but they **do** produce the second-difference phase in the conditional fringes (item 1); the co-suppression is of the certificates, not of the extracted angle. E1's visibility scan, not E1-M, is the laboratory handle on that class. 7. **Failure of an M-assumption** (calibration drift beyond the ±5% model, preparation infidelity beyond the M0 target, a non-diagonal residue beyond the M1 budget) voids the magic certificate, not the witness, and not Theorem W.

The asymmetry of stakes is inherited from [E1] §9: a positive delivers a gate-grade typed conclusion for free; a null in any branch delivers a bound, a diagnostic, or a family theorem, never a verdict on classicality.

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8. Confounds, and where the degeneracies honestly lie

Everything that reduces visibility reduces σ-significance; that is priced in §6. The confounds specific to a *magic* certificate are about bookkeeping: what could be booked as gravitational magic without being it.

channel / degeneracy	effect on the certificate	killed or bounded by
local non-Clifford phases (the labs' own pulses; the LU ambiguity of arXiv:2504.03351)	state-level magic is locally counterfeitable; the quadruple reading γ' shifts under local doping	the certified quantity is $\hat{\Delta} = \gamma' - a' - b'$ only, exactly doping-invariant; M2 fixes the frame; the K-2 doping run tests it in the lab (§4.2)
local-axis miscalibration (measurement frame off the calibrated X/Y axes)	the identities presuppose calibrated Pauli axes, the settings' own magic is part of the trust base	normalized frame diagnostics $(\langle XX\rangle + \langle YY\rangle)/\hat{F} = 1$, $\langle ZZ\rangle = 0$, marginal-modulus cross-check |

channel / degeneracy	effect on the certificate	killed or bounded by
		$\langle XI\rangle + i\langle YI\rangle/\hat{F} = \lvert\cos(\hat{\Delta}/2)\rvert$ (§4.1 step 3); residual inside the 0.05
visibility loss (all channels of [E1] §8: gas, blackbody, spin dephasing, recombination, partner jitter, CoM spread)	scales every correlator by F; the ratio identities are unchanged; σ_Δ grows	the extraction σ-budget (9.7σ **angle resolution** at F = 0.5, below F_ent no certificate is live on the benchmark, §6); state-level routes starve at 0.975555 / 0.999925 and are not the teeth
classical-channel phase imprint (E1's H1b: B's branch read, A phased classically)	$\hat{\Delta} \neq 0$ with witness null and full conditional contrast, the extraction alone excludes nothing	Definition 1 (the certified positive) is the premise that excludes it; witness-null + $\hat{\Delta} \neq 0$ + deficit $\propto \Delta^2$ is the classical-channel fingerprint, a diagnostic only (§7 item 1)
readout errors (confusion, asymmetry; B's Z-readout errors enter the conditioning)	asymmetric readout $\varepsilon_0/\varepsilon_1$ = 1%/5% biases the (a′) quadruple by +0.093 ≈ 30% of Δ; the (a″) conditional route by ~2ε sin Δ (+0.029), calibratable	measure the confusion matrices on the same runs and invert; residual ≤ $0.2\sigma_\Delta$ enters the systematic budget (decision rule D-4)
EM / Casimir–Polder second-difference phase	$\hat{\Delta}$ is the *total* pair-channel second difference; the gravitational attribution is Theorem W's Step 1, inherited (W2b + Definition 1's EM budget: of order 0.05–0.1 of the gravitational phase at the 200 μm geometries, order-of-magnitude only; Bose et al. quote ~0.1, [W] §1.2 quotes ≈ 0.05, and 0.7% / 0.08% / 0.01% at Family I's C/D/E; Family II's A2/A3 not separately quoted in [E1] §5)	the (d, τ) scan (1/r vs $1/r^4$) and the deterministic CP correction inside the M2 phase model; E1-M adds nothing here and claims nothing beyond the inherited budget
proximity to a Clifford exception	premise fails if the interval touches πZ	margins 0.3140 / 2.8276 rad; every registered contour
preparation infidelity (M0)	the state → gate lift needs a certified stabilizer input	protocol row (fidelity target set by the collaboration), not a computed budget, typed soft spot
channel residue (M1)	non-diagonal or non-unitary residue breaks the phase-state form	E1's $V_c/V_s \approx 1$; single-mass controls; residue budget
joint processing of runs	would void the per-run structure of Definition 1	clause 3 (inherited): standard i.i.d. correlator estimation
bookkeeping frame	raw-frame state (branch phases as they stand) has F_STAB = 0.954825, M_2 = 0.249310 vs calibrated 0.981666 / 0.104445; at Δ = π on the committed geometry (τ = τ_π; locals a = 1.4π, b = –0.4π) the raw state is not a stabilizer state (F_STAB = 0.951655)	the extra raw-frame magic is *local* content, exactly what the certificate excludes; direction conservative (calibrated numbers never overclaim)
"how much magic did the gate inject" as a scalar	the maximum over stabilizer inputs of 1 – F_gate is C^2 = 0.024445, attained by the control-	1⟩ *product* input where the gate acts as a local $R_z(\Delta)$ (the

Bias ledger for $\hat{\Delta}$ (steelman-harness estimates at the committed point; float-grade, not values of record). $\hat{\Delta}$ is unbiased under every branch-diagonal (dephasing-type) channel, however correlated; non-diagonal residues (spin flips, amplitude damping) and readout errors bias it toward Δ = 0 at first order, conservative for the certificate; readout confusion matrices are measured on the same runs and inverted, the residual enters the systematic budget.

channel	(a′) quadruple bias	(a″) conditional bias	direction
branch-diagonal dephasing, uniform, asymmetric (0.6/0.95), or coherence-selective (00↔11 only)	0	0	none: every Z-diagonal channel preserves the phase of each coherence, exactly M1's class
white noise F	0	0	none
amplitude damping 5%/5%	0	+0.015 (4.7%)	toward 0 (conservative), Z-conditioning is mixed
bit flips 2%/2% (an M1 violation)	+0.012	+0.023	toward 0
readout, symmetric 2%	0	+0.012	toward 0
readout, **asymmetric** $\varepsilon_0 = 1\%$, $\varepsilon_1 = 5\%$	**+0.093 (30% of Δ)**	+0.029	toward 0, the quadruple is hostage to readout asymmetry (marginal-proportional offsets on near-unit marginals)
B's Z-readout, 5% symmetric	0	+0.031 ≈ 2ε·sin Δ	toward 0

Few-percent single-shot readout asymmetry is the norm on NV platforms, so the quadruple's 30%-of-Δ sensitivity is disqualifying for it as a primary route on its own; the conditional route's ~2ε sin Δ is calibratable through the measured confusion matrices (decision rule D-4).

The honest degeneracy. The one thing no measurement on the spins can do is distinguish, without a fixed frame, magic injected by the gravitational coupling from magic injected by the labs' own local operations: state magic is local-unitary-ambiguous. The protocol does not pretend otherwise. It certifies the one number local operations cannot move ($\hat{\Delta}$), makes the frame an explicit, lab-tested assumption (M2 + K-2), and states that without M2 no magic conclusion follows from any of the data. **Second honest caveat:** the visibility thresholds of §6 are white-noise benchmark numbers; the apparatus noise is whatever [E1]'s channel table says it is, and it enters E1-M only through F in σ_Δ. **Third:** the (a′) consistency reading is calibration-limited at 20σ, a claim of "n-sigma typing" beyond that on route (a′) is a claim about the phase model, not about the data; route (a″) carries no model term, and its systematic floor is the readout-bias residual above.

Matchgate remark (a second restriction class, for completeness). Against the free-fermion (matchgate) class the gate's leak is $4\sin^2(\Delta/2) = 4C^2$, zero iff $\Delta \in 2\pi\mathbb{Z}$, so gravity's gate is beyond the matchgate class at *every* entangling coupling, including the Clifford points (leak = 4 at $\Delta = \pi$; committed leak 0.0977812). The two restriction classes cut the same physical family on different exception sets ($\pi\mathbb{Z}$ vs $2\pi\mathbb{Z}$). Licensed ceiling wording: **"beyond both registered restriction classes"**; the absolute "beyond every known classical-simulable sector" is banned, and neither certificate is a hardness or simulability claim.

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9. Relation to the literature

The precedent (cite with delta, never a pivot). Howl, Vedral, Naik, Christodoulou, Rovelli & Iyer, *Non-Gaussianity as a signature of a quantum theory of gravity*, PRX Quantum 2, 010325 (2021), arXiv:2004.01189, is the precedent for "gravity certified by injecting a resource beyond entanglement": only quantum gravity creates non-Gaussianity in matter, on a single Bose–Einstein condensate in a single location, read out through higher-order statistics, with their own entanglement witness inapplicable by construction. The delta of this note is exactly: discrete magic (nonstabilizerness) on the committed two-party *mediated-gate* protocol; an explicit added-assumption ledger over the entanglement witness; the

exception-set arithmetic with the converse separation; and a zero-added-hardware re-scoring of recorded tomography. The line is active: Chatterjee, Sen & Gangopadhyay, *Post-Newtonian analysis of the quantum signatures of gravity*, arXiv:2606.09119 (2026) extends the BEC non-Gaussianity program (no discrete-magic content). The contextuality-currency sibling of the same move is P. D. Andriolo, *Emaranhamento, não localidade e contextualidade em sistemas quânticos gravitacionalmente correlacionados* (USP master's dissertation, advisor B. L. Amaral, 2025; DOI 10.11606/D.43.2025.tde-13032025-144401): Kochen–Specker contextuality proposed as the nonclassicality signature for gravitationally coupled masses (Bell locality being unclosable at micron distances), with a numerical comparison of the robustness of the contextuality signature against that of entanglement under Lindblad decoherence, contextuality viable but *less* robust than entanglement. That robustness ordering is the genre parallel of the F_magic > F_ent threshold gap of §6 (rider (iv)): in both currencies the beyond-entanglement signature costs more visibility than the entanglement signature. The two results are disjoint by construction, this note makes no Bell/KS/contextuality theorem claim. On posting, the announced pipeline paper *Contextuality as a signature of gravity's nonclassicality* (Andriolo, Cius, Comar, Amaral) becomes a mandatory cite-and-distinguish. Resource-injection genre row: arXiv:2602.09900, *Gravitationally-induced conversion of local coherence to entanglement* (a coherence-to-entanglement resource reading of the BMV channel).

Building blocks (cited, never re-derived; citation strings as currently listed). Macêdo, Andriolo, Zamora, Poderini & Chaves, *Witnessing nonstabilizerness with Bell inequalities*, arXiv:2503.18734 (arXiv v1 title *Witnessing Magic with Bell inequalities*; retitled *Witnessing nonstabilizerness with Bell inequalities* in the INSPIRE/journal record, which supplies the citation string of record, [WM] §7.6.5): tailored Bell inequalities (tilted CHSH, CGLMP) whose stabilizer-state maxima lie below the quantum maxima witness magic, and standard CHSH provably cannot, the premise of §1's impossibility statement; its opening example is the folkloric core credited above. Cieśliński, Knips, Weinfurter & Laskowski, *No-cost Bell nonlocality certification from quantum tomography*, arXiv:2512.25068 (the arXiv title, v1–v2 alike, carries the full subtitle *…and its applications in quantum-magic-resource witnessing*; the short form is the citation string of record, [WM] §7.6.5): Pauli-basis tomographic data, including archival datasets, re-evaluated against tailored inequalities with computed stabilizer maxima, the re-scoring class of route (a′). Liu, Haug, Ye, Liu & Roth, *Triangle Criterion: a mixed-state magic criterion with applications in distillation and detection*, arXiv:2512.16777 (route (c)). Leone, Oliviero & Hamma, arXiv:2106.12587 (stabilizer Rényi entropy: definition, measurability); Leone & Bittel, *Stabilizer entropies are monotones for magic-state resource theory*, arXiv:2404.11652 (monotone status, $\alpha \geq 2$); Bravyi, Browne, Calpin, Campbell, Gosset & Howard, arXiv:1808.00128 (exact stabilizer extent of rotation states); Seddon & Campbell, arXiv:1901.03322 (channel robustness and magic capacity of Z-rotations, the channel-level pair for this gate); arXiv:2503.20873 (unitary stabilizer entropy; the "magic injection" terminology precedent); Warmuz, Dokudowiec, Radhakrishnan & Byrnes, arXiv:2409.18570 (stabilizer-polytope witness/monotone); Haug & Tarabunga, arXiv:2504.18098 (efficient mixed-state magic witnessing and testing); arXiv:2504.03351, *Non-stabilizerness and violations of CHSH inequalities* (the CHSH-enabling nonstabilizer resource is local and asymmetric, the LU-ambiguity row of §8); Mizutani, Takeuchi, Hiromasa, Aikawa & Tani, *Computational self-testing for entangled magic states*, arXiv:2111.02700 (CCZ-type magic computationally self-testable, T-type not, the M-assumptions are not removable); arXiv:2412.11918 (Pauli-only precedent, cited at the separation theorem); optional toolbox: arXiv:2506.02226 (semi-device-independent nonstabilizerness certification). Theorem M's *shape*, data + W1–W6 + M0–M4 ⇒ the mediated gate injected magic, with the Clifford-exception converse, is, per the sweeps of record, unoccupied; the nearest inference-shape relative is arXiv:2606.17287 (induced resource theories), whose inference runs in the opposite direction (free interaction ⇒ environment-state typing, versus certified stabilizer input ⇒ gate typing).

Tomography and assumption rows inside the GIE literature. Chevalier, Paige & Kim, *Witnessing the non-classical nature of gravity in the presence of unknown interactions*, arXiv:2005.13922, close an unknown-interaction loophole via state tomography, the precedent that the campaign's recorded tomography carries certification load; route (a′) is a re-scoring, not new hardware. W-row citations: Martín-Martínez & Perche, *What gravity mediated entanglement can really tell us about quantum gravity*, arXiv:2208.09489 (W4: mediator locality); Boulle & Franzmann, *Subsystems (in)dependence in GIE proposals*, arXiv:2512.17024 (W5: strict factorization under dressing and gauge constraints).

Adjacent occupied ground (cite-and-distinguish on every export). *Amplitudes:* arXiv:2508.14967, *Spin versus magic: lessons from gluon and graviton scattering*, and arXiv:2603.04148, *Non-local nonstabiliserness in gluon and graviton scattering*, magic generically produced in graviton scattering on S-matrix helicity qubits; no gate, no witness, no exception structure; arXiv:2605.19117 (a closed-form stabilizer entropy over a physical CP phase with Clifford-phase zeros and "magic-inspired witnesses", the closest phrase-level neighbor of the dichotomy $M_2 = 0$ iff $\Delta \in \pi\mathbb{Z}$); arXiv:2606.31501 (a 60-stabilizer-input census as a gate-grade magic functional on a physical family, the same census instrument); arXiv:2603.24902 (two-qubit magic–entanglement Pareto frontiers, the ambient plane of Fig. 1); arXiv:2502.17550, *Maximal magic for two-qubit states* (the global maximum ln(16/7), attained by this family at $\Delta = \pi/2$). *Mediator-produced magic in particle physics:* arXiv:2503.03098, *Quantum Magic in Quantum Electrodynamics*; arXiv:2607.00229, *Production of magic states via Z bosons and dark photons*, the genre precedent that a fundamental interaction's 2→2 processes produce magic; the gravity slot was unoccupied. *Relativistic quantum information / harvesting:* arXiv:2409.11473, *Harvesting magic from the vacuum*; arXiv:2412.11918, *Harvesting stabilizer entropy and non-locality from a quantum field* (its "non-Clifford interaction" is the detector–field coupling; BMV is cited there as motivation only); arXiv:2508.16466; arXiv:2607.16403, *Universality of Magic in Local Quantum Field Theory*; contextuality harvesting arXiv:2508.15001, *Contextuality from the vacuum*, and arXiv:2508.11773 (the no-contextuality-theorem rider guards this boundary); arXiv:2507.21194, *Vacuum-induced quantum gate* (Clifford-only; vocabulary neighbor). *Holography:* arXiv:2403.07056, *Gravitational back-reaction is magical* (AdS/CFT; non-local magic ⟺ back-reaction; a different arena, and the title collision is one reason the slogan of §10 is banned), with arXiv:2607.01930, 2602.12339, 2603.13475, 2607.10663; Mathur, arXiv:2606.20334, *The magic of the gravitational vacuum*, is a homonym (figurative), not nonstabilizerness. *Wording hazard:* arXiv:2509.25790, "Nonstabilizerness without magic" (short form; the arXiv title continues *…: Classically Simulatable Quantum States That Are Indistinguishable by Classically Simulatable Quantum Circuits*), classically simulable nonstabilizer states exist; hence the nonstabilizerness ≠ hardness rider on every magic sentence.

Matchgate remark citations. Valiant; Terhal & DiVincenzo; Jozsa & Miyake (matchgate simulability, standard); Hebenstreit et al. arXiv:1905.08584 (every pure fermionic non-Gaussian state is matchgate-magic, with their locality/SWAP caveats); Sierant, Turkeshi & Tarabunga, arXiv:2603.12392 (k-replica matchgate commutant).

Inherited must-cite block (carried on every export of this line by program policy; quoted whole from the companion magic-typing document, rider R8-am). "Every export carries the G20-2 must-cite block in cite-and-differentiate: (i) **2608.12512 + 2607.16403v2**, universality-of-magic at algebra grade (type III von Neumann); differentiate: continuum structural necessity vs carrier-located exact finite certificates with a two-sided instrument (B-1/B-2 in-class); (ii) **2510.18967 + 2608.12472**, Wigner negativity of gravitational states via the gravitational path integral; differentiate: state-complexity monotone vs protocol-carrier adjudication with impossibility+location halves; no epistricted class, no pre-registration; (iii) **2512.10126 + 2601.22219**, law-level negativity adjudication in cosmology (CV/Hudson lane, verdicts in both directions; also parity precedent, an in-class/deflation verdict published as a discovery); mandatory with any instrument-novelty sentence (PC-D's PARTIAL carriers);

(iv) **2606.19180v2**, stage-located SRE accumulation inside a gravity protocol (the SRE-corroboration-only rider does the work; necessity-one-sided already in-sentence); (v) **2607.12047** with 2306.14996/2403.07056/2607.10663 by name, Cao-line experimental arrival (magic-enriched HaPPY code on trapped ions); (vi) **2606.09971** alongside 2601.15842, cited in both verdict directions (the constraint-null's positive complement); (vii) **2604.06087 (+2604.06149)**, Gauss-law sectors are stabilizer codes (QRF-side null corroboration); (viii) with any W-B export: **2607.08508 + 2607.16968**, the magic-located-in-protocol genre (PC-B pressure set)." This note quotes no in-class (odd-prime) sentence, so the in-class rider stack R1-am..R8-am is not invoked here beyond this must-cite block.

Credit line. The naive core, a phase gate at a generic angle is non-Clifford, so the BMV output state carries magic, is elementary and folkloric and is credited in full; every closed form above is an evaluation of a cited definition, presented as computation. The contribution of this note is the instantiation on the committed BMV protocol, the forced and priced assumption ledger, the converse separation at the Clifford exceptions, the exception arithmetic, and the costing.

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10. Assumptions and caveats ledger; riders; never-claimables

1. **Frame of record.** All state-level monotones and thresholds refer to the calibrated-frame state $\psi(\Delta) = CZ(\Delta)|{+}{+}\rangle$ (the local phases gauged away by M2). The raw-frame state carries more magic ($F_{STAB} = 0.954825$, $M_2 = 0.249310$ at the committed point) and at $\Delta = \pi$ on the committed geometry ($\tau = \tau_\pi$; locals $a = 1.4\pi$, $b = -0.4\pi$) is not a stabilizer state ($F_{STAB} = 0.951655$); that surplus is local content, excluded from the certificate by construction. The calibrated numbers never overclaim.
2. **Benchmarks, not noise models.** The visibility thresholds (0.864790 / 0.975555 / 0.999925) assume white noise ρ_F; they benchmark the *state-level* routes. The operative instrument at realistic noise is the extraction σ-budget ((a″) primary; the record σ-ladder quoted in the (a′) form), in which F enters only through σ_Δ, and the witness must itself certify at the run's visibility (§6).
3. **M0 is a protocol row.** The stabilizer-input fidelity is a requirement the collaboration certifies with preparation runs; this note supplies no computed fidelity budget (typed soft spot).
4. **The ±5% phase model's role is now residual.** It is the calibration the witness already needs ([E1] §5, §10.3); here it fixes the local frame for the state-level floors and for the (a′) consistency reading (M2), where it caps that reading's significance at 20σ. Under the primary route (a″) it is a cross-check ($\hat{\Delta}$ vs Δ_{model} within 5%), not an input; the attractive sign $\hat{\Delta} < 0$ comes out of the data as a free prediction and resolves the modulus reflection $\hat{\Delta} \leftrightarrow -\hat{\Delta}$.
5. **Per-run statistics.** Definition 1's third clause is inherited: standard i.i.d. correlator estimation; joint quantum processing of runs would need separate analysis.
6. **Gravitational attribution is inherited, not new.** $\hat{\Delta}$ is the total pair-channel second difference; that it is gravitational rests on W2/W2b and Definition 1's EM budget with the (d, τ) scan, exactly as for the witness.
7. **The $|\Delta| = 0.10$ contour is marginal, for the witness first** (witness $N \geq 20017$ at $F = 1$, derived; the extraction's 4.8σ at $N = 10^4$, $F = 0.7$ is a value of record but not the binding constraint); design choices in §6.
8. **Reach.** $\tau_\pi \approx 25.0$ s / 15.4 s and $m_\pi = 3.16 \times 10^{-14}$ kg: family theorem, never a lab point; the exhibit is exercised through the scan's shape.
9. **Exact arithmetic and print windows (rider (iii)).** The threshold decimals are 6dp record values whose row strings in the certified ledger are print windows (rounded to nearest at final print, clipping at most half a print-ulp); the underlying values are certified at the exact-Fraction level. In

particular the exact F_magic = (1 + cos Δ)/2 lies below 0.975555 by less than 5×10^{-7}, and its 7dp value is **0.9755547**. **Erratum values (fused rider):** 0.9755547 (7dp) and 6.14×10^{-8} (the deficit by which N = 1104 fails the 5σ predicate) supersede the slipped display literals 0.9755548 and 6.5×10^{-8} wherever those appear in the record's display strings; neither literal occurs in any licensed sentence.

10. **Grade of the theorem.** Theorem M is a corollary-grade assembly: the lemma chain is cited (§9); the contribution is the instantiation, the ledger, and the converse. Time budget and platform constraints are inherited unchanged from [E1] §10.7.
11. **Scope of "certifiably magic-free".** The phrase travels only as "certifiably magic-free *within the committed family model*", the exception set is a codimension-one resonance condition of that model, not a regime.

Mandatory riders (quoted verbatim from the licensing ruling, [WM] §7.6):

> "Mandatory in-quotation riders: (i) the noise-honesty clause verbatim ("at realistic noise the teeth are the (a′) extraction route + M0–M2 gate-grade lift, not the state-witness route"); (ii) a null never feeds "gravity is classical"; (iii) AD-1 caveat, the threshold decimals are 6dp record values whose row STRINGS are print windows, certified at the Fraction level; (iv) when the note is assembled it carries the full §7.6 cite-and-distinguish stack (the Andriolo robustness-gap parallel bears directly on the threshold-gap exhibit) and the citation-string refreshes; (v) defer-until-big."

Base rider set of record ([WM] §7.6): the §7.3 never-claimable stack in full, the reach-honesty row, the R8-am must-cite block, the S1–S5 fold-ins (only S3 fired, the corollary-grade annotation of item 10 above), and the standing re-sweep (G21-5). Rider (i)'s "(a′) extraction route" names M3's extraction lane, see the reading note of §4.3; the rider travels verbatim and unamended.

Never-claimables (binding on every sentence of this note and on every quotation from it):

- The slogan **"gravity is a magic factory" is banned** everywhere (abstracts, theorem statements, verdicts, exported sentences); at most an explicitly marked informal gloss in discussion prose, never here.
- "BMV witnesses magic" only in the ledgered form of S-WM-2, whole-with-riders. **"The graviton / the mediator is magic" is banned**, the certificate is gate-grade, about the induced gate on the matter sector; the mediator conclusion stays Theorem W's.
- The naive core ("phase gates are non-Clifford, so the state has magic") is folkloric and never claimed bare; only the deltas are claimed.
- **Nonstabilizerness ≠ hardness** (arXiv:2509.25790): no speedup, simulability, or "not simulable" claim at any grade; magic-witness statements are not Bell/Kochen–Specker/contextuality theorems (same currency, different theorem).
- Semantics inherited from the companion magic-typing program: negativity-required wording only (never "no nonnegative model exists"; never "not simulable"); necessity-one-sided; stabilizer-entropy statements are corroboration-only, never a witness where geometric-response claims might arise; the qubit-side stabilizer/Clifford currency and the odd-prime Wigner currency never exchange numerics.
- **"Beyond every known classical-simulable sector" is banned as an absolute**; the licensed ceiling is "beyond both registered restriction classes".
- "Certifiably magic-free" travels only as "…within the committed family model"; the Δ = π separation site always travels with the reach-honesty row.
- **A null or deflation outcome anywhere, including any future E1-M null, never feeds "gravity is classical" or "quantum mechanics unnecessary".**

- Licensing is never posting; defer-until-big governs timing; every export carries the cite-and-distinguish stack of §9 and owes the standing literature re-sweep first.

One-line summary for the collaboration: *from the fringes you already record, condition A's X/Y fringe phase on B's branch outcome and take the difference, $\hat{\Delta} = \phi_c(B = R) - \phi_c(B = L)$, with $\sigma_\Delta = \sqrt{2}/(F\sqrt{N})$ (both mirrors, no model term; the record (a′) budget is $\sqrt{(2/N)/F \oplus 0.05|\Delta|}$); where the quadruple is recorded, check the frame with $(\langle XX\rangle + \langle YY\rangle)/\sqrt{((\langle XX\rangle - \langle YY\rangle)^2 + (\langle XY\rangle + \langle YX\rangle)^2)} = 1$ and the normalized marginal modulus $|\cos(\hat{\Delta}/2)|$; run one control with deliberate local Z-pulses to show $\hat{\Delta}$ does not move (nothing subtracted). If a certified positive run's interval excludes 0 and ±π, and the witness must itself certify at your visibility: $F \geq 0.926$ on the white-noise benchmark for 5σ at 10^4 shots per setting, the record 12.3σ at 70% visibility being angle resolution, with at least 1105 shots per setting for 5σ on the (a′) budget, then, under the witness assumptions plus the five ledgered ones, gravity implemented a nonlocal non-Clifford gate with stabilizer-entropy floor 0.104 bits; the witness alone can never say this, at the maximal-witness coupling nothing could, a classical channel can fake the angle but never the certified positive, and a null of any branch says nothing about gravity being classical.*

Appendix A, Numbers provenance

Sources of record: **[ROWS]** `analysis/witness_magic_rows.jsonl` (8 certified rows, sha256 `5167e96fa3813abd…`), **[REC]** `analysis/witness_magic_results.md` (sha256 `ac48023d33d2203d…`), engine `analysis/witness_magic_lattice.py` (sha256 `50a53ea9144812a5…`), the pins of [WM] v1.1; **[FIG]** `analysis/e1m_figures.py` (v1.0 regeneration, sha256 `d13cd064593f005c…`; exact-Fraction / rigorous-interval evaluation of the record's closed forms; 40/40 record checks, 8/8 self-tests, 7/7 derived-consistency checks incl. the finalize-leg D06 (a″ ring identity) and D07 (witness gating); tables in `analysis/e1m_data.md`, check ids Rnn/Tnn/Dnn as cited below); **[FC]** `analysis/e1m_finalize_checks.py` (finalizer-leg certifier of the remaining v1.0 derived protocol numbers: exact-Fraction ceilings, 50-dps trig anchored on the record C = 0.156350); **[SH]** `analysis/e1m_experimentalist_steelman_checks.py` + `.json` (steelman harness; float-grade, NOT of record, source only for the §8 bias-ledger estimates); **[WM]** `21_witness_magic.md` v1.1; **[MECH]** `analysis/witness_magic_mechanism.md`; **[E1]** `05a_E1_visibility_note.md`; **[W]** `06_witness_theorem.md`; **[Bose]** arXiv:1707.06050 / PRL 119, 240401. Grades: **record** = certified value of record (interval or exact Fraction; decimals are both-endpoint roundings); **exact** = exact rational identity; **derived** = new number evaluated by [FIG] from the same exact predicates, *not* a value of record; **source** = quoted from the cited text.

numbers	used in	source of record	grade
d = 450 μm, Δx = 250 μm, d − Δx = 200 μm, τ = 2.5 s, m = 10^{-14} kg; "W ~ 1.16"	§2	[Bose] text (PRL version for the witness value); [FIG] T08	source
$Gm^2/\hbar$ = 63.3 μm·rad/s	§2	[E1] §2 pin; CODATA check 63.2892 within 0.1 % ([FIG] T07)	pin
K/d = 211/600 = 0.351667; a = 211/480 = 0.439583; b = −211/1680 = −0.125595; Δ = −211/672 = −0.313988	§1, §2, §4	[ROWS] MA-ladders `x_committed`; [REC] §1 (K-4); [WM] §1.2, §3.2 (two routes); [FIG] R05–R07	exact
dΔ/dτ = 211/1680 = 0.125595 rad/s; 5697/28000 = 0.203464 rad/s	§2	[REC] K-4; [WM] §1.6(iv); [FIG] R09	exact
C = 0.156350; W = 1.156350	§1, §2, §5	[REC] §1; [WM] §1.2; [FIG] R08	record (6dp)

numbers	used in	source of record	grade
ρ★ weights 0.578175 / 0.421825	§1	[REC] §3 (L-M1); [WM] §1.3; [FIG] R10	record
CHSH $2\sqrt{(1 + C^2)}$ = 2.024298	§1, §2	[REC] §3; [FIG] R11	record
margins 0.3140 / 2.8276 rad	§2, §6, §8	[REC] §1; [WM] §1.2; [FIG] R15	record (4dp)
ladder generators 28π/5 = 17.5929, 7π/6 = 3.66519; half-ladder (14π/5) (2Z+1); first strict point 8400π/211 = 125.07 s	§2	[ROWS] MA-ladders; [REC] §1; [FIG] R16	record
τ_π = 1680π/211 = 25.0136 s; Family II A 15.4405 s; $m_\pi = 3.1631\times10^{-14}$ kg; $m^2\tau$ quantum 2.5014×10^{-27} $kg^2\cdot s$	§2, §5, §7, §10	[ROWS] MA-ladders `tau_pi` [25.013628, 25.013629]; [REC] §1; [WM] §1.2; [FIG] R12–R14	record
F_STAB = 0.981666 (0.98166602); M_2 = 0.104445 (0.10444461; floor 0.104444); e_p = 0.005432 (0.00543229)	§3, §5	[ROWS] MA-monotones-committed windows; [REC] §1; [WM] §1.2, §3.2, §7.6; [FIG] R17–R19	record
gate-grade 1 − F_gate = C^2 = 0.024445; (3/4)C^2 = 0.018334	§8	[ROWS] MA-monotones-committed `one_minus_F_gate`; [REC] §1; [FIG] R20	record
M_2 maximum 4 − $\log_2 7$ = 1.192645 at Δ = π/2 ($\Sigma\langle P\rangle^4$ = 7/4); F_STAB(π/2) = 5/8; ring points Δ = π → (W, M_2, F_STAB, e_p, leak) = (2, 0, 1, 2/9, 4), Δ = 0 → (1, 0, 1, 0, 0)	§3, §5	[REC] §1, §3, K-2; [FIG] R21–R23	exact
weak-field coefficients 3/(4 ln 2) = 1.0820, 1/ln 2 = 1.4427	§1, §5	[REC] §1; [WM] §1.2; [FIG] R24	record
weak-field comparisons 1.156994 (vs 1.156350), 0.106675 (vs 0.104445)	§5	[FIG] D05	derived
Makhlin (G_1, G_2) = (0.9756, 2.9511); 11520 Cliffords, 4 LU classes; Clifford $G_2 \in \{\pm1, \pm3\}$ vs (1, 3) off πZ; e_p anchors {0, 2/9}	§3	[ROWS] MB-census-separation; [REC] §3; [WM] §1.2, §3.2; [MECH] AM-1; [FIG] R25	record / exact
matchgate leak 4 $\sin^2(\Delta/2)$: committed 0.0977812; 4 at Δ = π	§8	[ROWS] MC-matchgate-rider; [REC] §5; [FIG] R26	record
σ_Δ = 0.02113 / 0.02559 / 0.03235; 14.9 / 12.3 / 9.7σ; 134 / 111 / 87σ (N = 10^4, F = 1 / 0.7 / 0.5)	§1, §6	[REC] §3; [WM] §1.3; [FIG] R27–R29	record
N_min(5σ, F = 0.7) = 1105; real-valued bound 1104.018; deficit of N = 1104: 6.14×10^{-8} (6.1378×10^{-8}; exact 17/276971520)	§6, §10	[ROWS] MB-design `Nmin_5sigma`; [REC] §3; [WM] §1.3, §5 (AD-3), §7.6; [FIG] R30–R31	record / exact
N_min(5σ) at F = 1 / 0.5 = 541 / 2164; systematic ceiling 20σ	§6	[FIG] D04, D03	derived
|Δ| = 0.10 contour: σ_Δ = 0.02081, 4.80σ	§6, §10	[REC] §3; [WM] §1.3; [FIG] R32	record
|Δ| = 0.10 contour: N ≥ 10885 or F ≥ 0.730 for 5σ	§6	[FIG] D02	derived
other contours: σ_Δ = 0.02254 / 0.02516 / 0.02843 / 0.03214; 8.87 / 11.92 / 14.07 / 15.56σ; N_min 2722 /	§6	[FIG] §4 table (same exact predicate)	derived

numbers	used in	source of record	grade
1210 / 681 / 436; F needed 0.365 / 0.243 / 0.233 / 0.183 / 0.146			
witness gating: $F \geq 0.926$ for a 5σ witness at $N = 10^4$/setting on the white-noise benchmark (3σ: 0.9015); $W_F = F(1 + C) = 1.070780$ / 0.809 / 0.578 at $F = 0.926$ / 0.7 / 0.5; $\sigma_\Delta(F = 0.926) = 0.02190 \rightarrow 14.3\sigma$ / 129σ, N_{min} 631; $\sigma_\Delta(F = F_{ent}) = 0.02267 \rightarrow 13.9\sigma$	§1, §6, §10	[FIG] D07; [FC]	derived
witness N per setting for 5σ at $F = 0.926$ (contrast reading, $N \geq 50/(C^2F^2)$, exact ceilings): 23344 / 5851 / 2612 / 2386 / 1478 / 953; at $F = 1$ on the 0.10 contour: 20017; ratio to the (a′) extraction $N = (15/16)(\Delta/C)^2 \approx 3.75$–3.83	§6, §10	[FIG] D07; [FC]	derived
(a″) conditional route: $\phi_L = a = 211/480 = 0.439583$, $\phi_R = -b = 211/1680 = 0.125595$, $\hat{\Delta} = \phi_R - \phi_L = -211/672$ (ring-exact; doping-invariant); $\sigma_\Delta = 2/(F\sqrt{N})$ one side, $\sqrt{2}/(F\sqrt{N})$ both mirrors; N_{min}(5σ from 0, $F = 0.7$) = 2071 one-sided / 1036 both (exact ceilings); no model term	§1, §4, §6, §10	[FIG] D06 (exact); [FC]	exact identity / derived
benchmark separability: PPT minimum eigenvalue $(1 - F)/4 - FC/2 = +0.020$ / +0.086 at $F = 0.7$ / 0.5 (−0.034 at F_{ent}); separability boundary $F_{sep} = 1/(1 + 2C) = 0.7618$	§1 (header), §6	[FC] (standard partial-transpose identity for a pure state of concurrence C under white noise); [SH] cross-check	derived
machine-day arithmetic: 4 settings × 10^4 shots × 10 s = 4.63 d per contour point, ≈ 23.15 d over five contours, 9-setting tomography 10.42 d per point; M0 anchor $\sin^2(1°) = 3.05\times10^{-4}$	§3, §4, §6	[FC]	derived
§8 bias-ledger estimates (+0.015 / +0.012 / +0.023 / +0.012 / +0.093 / +0.029 / +0.031; classical-channel imprint $\hat{\Delta} = -0.313988$ with witness 0)	§7, §8	[SH] (float-grade harness; NOT of record)	harness estimate
committed-point locals forced to $\Delta = \pi$: $F_{STAB} = 0.948713$ (vs 0.951655 on the committed-geometry locals at τ_π)	§8, §10	rigor-verifier leg, definition-level float route (`analysis/e1m_rigor_verifier.md` §2 block D)	derived (float-grade)
$F_{ent} = 0.864790$ (0.86479012); $F_{magic} = 0.975555$ (0.97555470; 7dp 0.9755547; exact < 0.975555 by $< 5\times10^{-7}$); gap 0.110765 → 0.1108 (0.11076458)	§1, §5, §6, §10	[ROWS] MB-design `F_ent` [0.864790, 0.864791], `F_fid_witness` [0.975555, 0.975556] (print window); [REC] §3 + display amendment; [WM] §1.3, §5 (AD-1, AD-2), §7.6; [FIG] R33–R35	record
route (b): $F_b = 0.999925$ (0.99992529); $\theta = 0.078497$ rad = 4.4975°; window margin 2.95×10^{-4} (record print 2.9×10^{-4})	§4, §6	[ROWS] MB-design `Fb_tilted`; [REC] §3; [MECH] §1.4 Step 4; [FIG] R36	record

numbers	used in	source of record	grade
threshold crossover $\sin(\Delta/2) = (\sqrt{5} - 1)/2$, $\lvert\Delta_{\times}\rvert = 1.3325$ rad, $\tau_{\times} = 10.6$ s	§6	[FIG] D01	derived
fallback cost 4–6 settings × 10^4 shots × 10 s = 4.6–6.9 ≈ 5–7 machine-days (re-scoped **per contour point** in v1.0, D4); E1 baseline 10^4 shots/point, 21–25 machine-days	§3, §4, §6	[WM] §1.5; [REC] §3; [FIG] R37; [E1] §7	record / [E1]
extraction values $\langle XX\rangle = 0.975555$, $\langle YY\rangle = 0.024445$, $\langle XY\rangle = \langle YX\rangle = \langle YI\rangle = -0.154427$, $\langle XI\rangle = 0.975555$, $\langle ZZ\rangle = 0$; $\cos\Delta = 0.951109$; $\sin\Delta = -0.308854$; $\lvert\langle XI\rangle + i\langle YI\rangle\rvert = 0.987702$; doped-frame illustration $\gamma' = -0.613988$, $\langle XX\rangle + \langle YY\rangle = 0.453596$	§4	[FIG] §5, R38 (identities exact on rational circle points), R38b (doping invariance exact), R39 (committed-point enclosures); [REC] §3 (L-M2)	exact identities; engine evaluations
raw-frame values $F_{STAB} = 0.954825$, $M_2 = 0.249310$; $\Delta = \pi$ raw $F_{STAB} = 0.951655$ (committed-geometry locals at τ_{π}: a = 1.4π, b = –0.4π, attribution corrected in v1.0, E1M-V7)	§2, §8, §10	[REC] §1 frame note; [WM] §5 (RV-D3), §7.6	record
stabilizer group {II, XZ, YY, ZX} of $CZ(\pi)\lvert{+}{+}\rangle$; W = 2 there	§5, §7	[REC] §3 (L-M3); [MECH] §1.1, §1.5	exact
Casimir–Polder fractions 10.0 % / 0.7 % / 0.08 % / 0.01 %; registered contours $\lvert\Delta\rvert = 0.10$–$0.50$; Family II A = (800, 600) μm	§2, §5, §6, §8	[E1] §5	[E1]
Theorem W, W1–W6, Definition 1, certification map	§1, §3, §7	[W] §1.1–§1.3, §5, §6	inherited verbatim (abridged in the table of §3.1)

Licensed sentences quoted verbatim from [WM] §1.5: S-WM-1 (§3.2), S-WM-2 (§3.3), S-WM-3 (§7 item 5). Riders (i)–(v) quoted verbatim from [WM] §7.6 (§10). The inherited must-cite block is quoted verbatim from the companion magic-typing document's rider R8-am (§9).

Appendix B, Figures

All four figures are generated by `analysis/e1m_figures.py` from the record (floats only in the drawing code); every panel carries the frame-of-record label (calibrated frame, $\psi(\Delta) = CZ(\Delta)\lvert{+}{+}\rangle$; $\Delta < 0$, $\lvert\Delta\rvert$ shown). PNG and SVG twins share a basename; PNG twins are byte-reproducible run to run, while the SVG twins differ only in matplotlib's per-run element ids and date stamp.

figure	file	content	riders printed on the figure
Fig. 1, the anti-alignment exhibit (headline)	`analysis/e1m_fig_1_antialignment.png` / `.svg`	four stacked panels vs $\lvert\Delta\rvert \in [0, \pi]$ with the hold-time axis at the committed geometry: (A) $W = 1 + \lvert\sin(\Delta/2)\rvert$ with the weak-field law $1 + \lvert\Delta\rvert/2$; (B) $M_2(\Delta)$ with the law $1.0820\,\Delta^2$, maximum at π/2, zero at π; (C) $F_{STAB}(\Delta)$; (D) $e_p(\Delta)$	full reach-honesty row ($\tau_{\pi} \approx 25.0$ s / 15.4 s Family II A; $m_{\pi} = 3.16\times10^{-14}$ kg at τ = 2.5 s, family theorem, not

figure	file	content	riders printed on the figure
		with the 2/9 ceiling; committed point, registered E1 contours, Clifford exception	a lab point); frame label
Fig. 2, visibility thresholds	`analysis/e1m_fig_2_thresholds.png` / `.svg`	F_ent, F_magic, F_b across the family; committed values 0.864790 / 0.975555 / 0.999925, gap 0.1108; crossover at 1.3325 rad marked as family-level (derived)	rider (i) verbatim; rider (ii); rider (iii) with the 7dp value 0.9755547; frame label
Fig. 3, the precision instrument	`analysis/e1m_fig_3_precision.png` / `.svg`	(A) σ-distance from Δ = 0 vs shots for F = 1 / 0.7 / 0.5 at the committed angle, the 5σ threshold, N_min = 1105, the 20σ ceiling; (B) 5σ contours in the (F, N) plane for |Δ| = 0.10 … 0.50 and the committed angle, the design point (F = 0.7, N = 10^4); reading line carries the angle-resolution qualifier and the (a″) statistical-term note	rider (i) verbatim; rider (ii); record-vs-derived labeling; frame label
Fig. 4, extraction identities worked at the committed point	`analysis/e1m_fig_4_extraction.png` / `.svg`	(A) the six calibrated-frame expectation values and the hand-checkable identities (cos Δ, sin Δ, atan2, marginal modulus, frame diagnostics); (B) the K-2 doping test, the quadruple moves, Â and the marginal modulus do not; the (a″) conditional phases 0.439583 / 0.125595 → −211/672 printed ring-exact (D06)	frame label; the S-WM-2 exclusion clause and the L-M1 impossibility sentence in the caption; the (a″)-primary reading note

Verification record

Two independent adversarial legs verified draft v0 (sha256 `d01b6bfdaa38bb7f…`) on 2026-08-24; this v1.0 folds every repair from both. Full repair ledger: `analysis/e1m_finalize_record.md`.

Leg 1, certificate-rigor verifier (`analysis/e1m_rigor_verifier.md`; script `analysis/e1m_rigor_verifier.py`, python3.12, own Decimal-70 trig cross-checked against mpmath, exact Fractions, definition-level numpy routes). **96/102 checks PASS; the six FAILs were intended defect probes.** Every number of record re-derived independently (Δ = −211/672 by two routes; M_2, F_STAB, e_p, F_ent, F_magic to 8–12 digits; N_min = 1105 with the 1104 deficit exactly 17/276971520; the AD-1..AD-3 errata re-confirmed: 7dp F_magic = 0.9755547, deficit 6.14×10^{-8}); the figure engine's exactness claims and self-tests verified with a byte-identical isolated rerun; S-WM-1/2/3, riders (i)–(v), and the R8-am block byte-verbatim; canon pins untouched. Verdict PASS-WITH-REPAIRS. **Defects resolved here: E1M-V1, E1M-V2 (MED); E1M-V3–V8 (LOW); E1M-V9–V16 (INFO, all folded).**

Leg 2, experimentalist steelman (`analysis/e1m_experimentalist_steelman.md`; harness `analysis/e1m_experimentalist_steelman_checks.py` + `.json`, float-grade). Confirmed everything licensed survives untouched (Theorem M, S-WM-1..3 verbatim, riders, never-claimables, all record values, the Bose-parameter checks), then found the four blocking protocol defects. Verdict PASS-WITH-

REPAIRS, D1–D4 blocking for v1.0. **Defects resolved here: D1–D4 (MED, blocking); D5–D6 (LOW-MED); D7–D9 (LOW); D10–D11 (INFO; the 1808.00128 / 1901.03322 author lists of D11 were verified on the PDF first pages by leg 1).**

Repair mechanics of this finalization. Route (a″) installed as primary with (a′) demoted to a model-calibrated consistency reading (D1; §1, §3.4, §4.1, §4.2, §10); witness-gating caveats and columns added with the F = 0.7/0.5 rows relabeled angle-resolution (E1M-V2 + D2; §1, §6, §7 item 3, §10); the classical-channel branch corrected and added to §7/§8 (D3); shot accounting re-scoped per setting and per contour point with the ramp-honesty paragraph (D4; §2, §3.4, §4.1, §4.4, §6); normalized frame diagnostics (E1M-V1; §4.1, §8, §10); bias ledger added (D5; §8); Fig. 1 reach row completed and Fig. 3 rider paraphrase replaced by the verbatim rider, with Fig. 4 now printing the (a″) conditional phases, one `--force` regeneration, engine checks 40/40 · 8/8 · 7/7 including the new D06 ((a″) ring identity, exact) and D07 (witness gating) (E1M-V3/V4 + D1; Appendix A shas refreshed); title re-formed to the ledgered gate-grade shape (E1M-V5 + D7); Theorem W label, Δ = π attribution, citation strings, EM-ratio phrasing, and the remaining INFO wording folded (E1M-V6–V16, D8–D10). New derived numbers entered only after engine certification: [FIG] D06/D07 (exact/interval) and [FC] `analysis/e1m_finalize_checks.py` (exact ceilings; seven of the steelman's proposed rounded integers corrected, 23344, 2612, 2386, 1478 in the §6 witness column; 2916, 11663, 467 in its (a″) budget triple, recorded in the finalize ledger). **No value of record changed; no licensed sentence text changed; no canon file touched.** The record-side dated-amendment candidate for [WM] §1.3 M3(a′) / §1.5 E1-M "Settings" and mechanism §5 (the (a′)→(a″) finding, verdict-neutral) is drafted in `analysis/e1m_finalize_record.md` for the program owner's decision, this note does not amend the record.

End of E1-M note, Final v1.0.