When gravitationally induced entanglement certifies magic: Clifford exceptions and a gate-typing upgrade of the witness theorem

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The companion paper [What gravitationally induced entanglement certifies] establishes what a certified positive result of the Bose et al.–Marletto–Vedral (BMV) protocol certifies: under six priced assumptions, no classical degree of freedom carried the observed entanglement. We ask the next question, which is about the channel's *gate type* rather than its classicality. Within the committed family the gravitational hold implements $U = (P_1(a) \otimes P_2(b))\,\mathrm{CZ}(\Delta)$ with $\Delta = -2K\Delta x^2/[d(d^2 - \Delta x^2)]$, $K = Gm^2\tau/\hbar$. That a controlled-phase gate at a generic angle is non-Clifford, so that the output state carries magic (nonstabilizerness), is elementary and folkloric; it is never the claim here. The claims are certification statements. (i) The committed witness statistic is magic-blind at every value, including its maximum $W = 2$: an explicit stabilizer mixture reproduces the design correlators and $W = 1.156350$ exactly, the entire correlator square is stabilizer-reachable, and standard CHSH cannot witness magic at any violation, so added assumptions are forced, not optional. (ii) The family's nonlocal Clifford-exception set is exactly $\Delta \in \pi\mathbb{Z}$, splitting into the no-entanglement points $2\pi\mathbb{Z}$ and the maximal-witness Clifford points $\pi + 2\pi\mathbb{Z}$; requiring Clifford local phases as well yields explicit arithmetic ladders in K/d on each rational-shape family; at the committed operating point ($\Delta = -211/672 = -0.313988$, $\pm 5\%$ calibration) the gate is certified outside $\mathrm{LU} \cdot \mathcal{C}_2 \cdot \mathrm{LU}$ by an exact census of all 11520 two-qubit Cliffords, with stabilizer 2-Rényi entropy $M_2 \geq 0.104444$ and entangling power 0.005432. (iii) Theorem M: under the companion theorem's assumptions plus five added, operationally certified assumptions M0–M4, a certified positive result whose extracted phase interval excludes $\pi\mathbb{Z}$ additionally certifies that the gravitational coupling implemented a nonlocal non-Clifford gate, magic injected at gate grade, with cited monotone floors evaluated on the interval, stable across both readings of the companion theorem's mediation disjunction. (iv) The converse: at $\Delta = \pi$ the same protocol attains its maximal witness value $W = 2$ on a stabilizer state delivered by a Clifford gate, so the family's best entanglement point is provably a zero-magic point; non-classicality is angle-generic, magic is angle-conditional, and along the hold-time scan the witness grows linearly while the certified magic floor grows quadratically (anti-alignment). The experimental price is measured: the primary instrument is a re-scoring of conditional fringes the witness protocol already records, the Z-conditioned fringe difference $\hat{\Delta} = \varphi_c(B{=}R) - \varphi_c(B{=}L)$, exactly invariant under local phase doping, with zero added settings and shots, resolving the committed angle at 12.3σ with 10^4 shots per setting at 70% visibility, and needing 1105 shots per setting for 5σ; the witness itself, not the extraction, is the binding cost ($F \geq 0.926$ for a 5σ witness at 10^4 shots on the white-noise benchmark), while a state-level magic witness would need visibility 0.9756 against the entanglement witness's 0.8648, a gap of 0.1108 that prices the upgrade on that route. A null of any branch never certifies that gravity is classical; in particular a classical measure-and-act channel imprints the full second-difference phase in the conditional fringes, so the extraction alone is never a non-classicality certificate. Every certified value is verified in deterministic exact-arithmetic ancillary scripts.

I. INTRODUCTION

Two adjacent matter-wave interferometers, each holding a mass $m \approx 10^{-14}\,\mathrm{kg}$ in a spin-dependent spatial superposition, are predicted to become spin-entangled through their mutual gravitational interaction [1, 2]. The companion paper [3] supplies the theorem the experiment needs: under six explicitly priced assumptions W1–W6 and a three-clause definition of a certified positive result, an observed, certified entanglement statistic implies that the degree of freedom that coupled the two labs is non-classical, either a non-classical mediator carried the entanglement, or the interaction is a nonseparable direct quantum gate with no carrier, the two readings separated in principle only by retardation. That is a statement about the channel's *classicality*.

This paper asks the next question, which is about the channel's *gate type*. Every two-qubit unitary either lies in the Clifford group up to local unitaries or it does not; what Clifford gates and stabilizer states lack is called *magic* (nonstabilizerness), the resource that universal quantum computation needs beyond stabilizer operations [6, 7]. The

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gravitational hold of the BMV protocol implements a controlled-phase gate at the second-difference angle Δ, dressed by two local phase gates (Sec. II). That a controlled-phase gate at a generic angle is non-Clifford, so that the output state carries magic, is elementary and folkloric, it is literally the opening example of Macêdo et al. [8], and it is never the claim of this paper. The claims are *certification* statements: which measured quantities, under which added assumptions, certify that the *gravitationally implemented* gate was non-Clifford; with what precision; at what visibility cost; and where the conclusion provably fails.

The certification question has a sharp obstruction, which we prove first (Sec. IV): the committed witness statistic, the two-setting spin witness the protocol was designed around, is *magic-blind at every value it can take*, including its quantum maximum $W = 2$. An explicit stabilizer mixture reproduces the design correlators and the design witness value exactly; the entire correlator square, $W = 2$ included, is reachable inside the stabilizer polytope; and a standard CHSH statistic cannot witness magic at any violation [8]. No function of the committed correlators certifies magic at any observed witness value. Any magic conclusion therefore requires added premises; the content of this paper is to make the added premises explicit, operationally certified, and priced, and to prove the converse, which shows the premises' exclusion clause is not decorative: the coupling at which the witness is *maximal* is one at which the delivered gate is provably Clifford and every sound magic witness is silent.

A. Contributions

(1) The exception arithmetic of the committed family (Sec. III). Within the committed BMV family the entangling gate is $U = (P_1(a)\otimes P_2(b))\,\mathrm{CZ}(\Delta)$ with $\Delta = -2K\Delta x^2/[d(d^2-\Delta x^2)]$, $K = Gm^2\tau/\hbar$. Its nonlocal Clifford-exception set is exactly $\Delta \in \pi\mathbb{Z}$, a codimension-one resonance condition of the family model, not a regime, and it splits physically: $\Delta \in 2\pi\mathbb{Z}$ is the no-entanglement accident, $\Delta \in \pi + 2\pi\mathbb{Z}$ is a Clifford CZ at the *maximal* witness value. Requiring the local phases to be Clifford as well yields explicit strict-exception ladders, arithmetically different on the two registered geometry shapes: $K/d \in (28\pi/5)\mathbb{Z}$ for the Bose et al. shape (with a half-ladder of Clifford-local, CS-grade points) and $K/d \in (7\pi/6)\mathbb{Z}$ for the 3:4 shape; irrational shapes have no nontrivial strict exception at all. At the committed operating point the gate is certified outside $\mathrm{LU}\cdot\mathcal{C}_2\cdot\mathrm{LU}$ by an exact enumeration of all 11520 two-qubit Cliffords (four local-unitary classes) and the Makhlin invariants of the family, on the whole $\pm5\%$ calibration interval; the monotone values are evaluated from cited closed forms.

(2) The impossibility lemma (Sec. IV). The committed witness statistic is a sum of two commuting Pauli products; every value it can take is attained by a stabilizer state or mixture. The lemma forces the assumption ledger and fixes the logical shape of any honest upgrade claim.

(3) Theorem M with its assumption ledger (Sec. V). Under W1–W6 and the certified-positive definition of the companion paper, plus five added assumptions M0–M4, stabilizer input, branch-diagonal unitary channel, calibrated local frame, a stated measurement route, per-run statistics, each with a named laboratory certificate, a certified positive result whose extracted angle interval excludes $\pi\mathbb{Z}$ additionally certifies that the gravitational coupling implemented a nonlocal non-Clifford gate, with monotone floors evaluated on that interval. The conclusion is stable across both readings of the companion theorem's mediation disjunction: the mediated channel and the direct gate deliver the identical mass-side diagonal gate, so the gate typing, unlike the mediator typing, does not split. The certificate is about the induced gate on the matter sector; no conclusion about a mediator's own magic is drawn at any grade.

(4) The converse separation, and the anti-alignment law (Sec. VI). At $\Delta \in \pi + 2\pi\mathbb{Z}$ the delivered state is the witness maximizer ($W = 2$ on a stabilizer state) while every magic monotone vanishes exactly and every sound magic witness is silent by construction: the two certified conclusions, non-classicality and magic injection, separate within one protocol family. Along the hold-time scan the witness grows linearly, $W \approx 1 + |\Delta|/2$, while the certified magic floor grows quadratically, $M_2 \approx 1.0820\,\Delta^2$, peaks at the controlled-S point $\Delta = \pi/2$ (where the family attains the global two-qubit maximum of M_2 [28]), and returns to exactly zero at $\Delta = \pi$: weak-field gravity is entanglement-first, magic-second (Fig. 1).

(5) The experiment-facing protocol, priced (Sec. VII). The primary instrument is a re-scoring of data the witness campaign already takes: the Z-conditioned fringe difference $\hat{\Delta} = \varphi_c(B{=}R) - \varphi_c(B{=}L)$ on the witness's own settings, exactly invariant under local Z phases on either side, with an independent mirror estimate, a shot-noise error bar with no model term, and a laboratory null test (deliberate local phase doping must not move $\hat{\Delta}$). A Pauli-quadruple reading is demoted to a model-calibrated consistency check, for a reason with teeth: at the symmetric BMV geometry equatorial data alone cannot determine the second difference. We give the shot and visibility accounting: 12.3σ angle resolution at the committed point with 10^4 shots per setting at 70% visibility; $N = 1105$ shots for 5σ (exact); witness-first gating ($F \geq 0.926$ for a 5σ witness at 10^4 shots on the white-noise benchmark); and the visibility thresholds $F_{\mathrm{magic}} = 0.9756$ versus $F_{\mathrm{ent}} = 0.8648$, whose gap 0.1108 prices the state-level upgrade route and delimits the window in which the extraction route is the only live magic instrument.

(6) Honest failure semantics (Sec. VIII). A null of any branch never certifies that gravity is classical. In particular,

a classical channel that reads one mass's branch and phases the other imprints the *full* second-difference phase in the conditional fringes while destroying the entanglement: the extraction alone is never a non-classicality certificate; the certified-positive premise carries that load.

Every certified value in this paper traces to a deterministic, exact-arithmetic ancillary script (Table IV); displayed decimals are roundings of certified rational or interval-arithmetic values. The two readout-bias figures of Sec. VII B, labeled float-grade in place, are the only numbers quoted below certificate grade.

B. Relation to prior and adjacent work

The precedent. Howl, Vedral, Naik, Christodoulou, Rovelli, and Iyer [19] established the genre this paper works in: certifying gravity's quantum character by the injection of a *resource beyond entanglement*, in their case continuous-variable non-Gaussianity, created in a single Bose–Einstein condensate at a single location, read out through higher-order statistics, with their own entanglement witness inapplicable by construction. The present work differs in every load-bearing ingredient while sharing the move's shape: the resource here is discrete magic (nonstabilizerness); the platform is the committed two-party *mediated-gate* protocol itself; the certificate rides on an explicit added-assumption ledger over the entanglement witness; the exception set and its converse are part of the theorem; and the primary instrument costs no added hardware, settings, or shots. The non-Gaussianity line is active [20]. A contextuality-currency sibling of the same move is Andriolo's dissertation [21]: Kochen–Specker contextuality as the non-classicality signature for gravitationally coupled masses, with the numerical finding that the contextuality signature is *less robust* to decoherence than the entanglement signature, the genre parallel of our threshold gap $F_{\mathrm{magic}} > F_{\mathrm{ent}}$ (Sec. VII); this paper makes no Bell/Kochen–Specker/contextuality claim, and the two results are disjoint by construction. A further resource-injection reading of the BMV channel is the coherence-to-entanglement conversion analysis of [22].

Building blocks, cited and never re-derived. The magic-witnessing toolbox this paper consumes is recent and sharp: tailored Bell inequalities whose stabilizer maxima lie below the quantum maxima witness magic, and standard CHSH provably cannot [8]; Pauli-basis tomography, including archival data, re-scored against such inequalities at no experimental cost [9]; a mixed-state triangle criterion [10]; stabilizer Rényi entropies and their monotone status [6, 7]; exact stabilizer extents of rotation states [11]; channel-level robustness and capacity for Z-rotations [12]; unitary stabilizer entropy, whose authors also supply the phrase "magic injection" [13]; stabilizer-polytope and mixed-state witnesses [14, 15]; the locality analysis showing that CHSH-enabling nonstabilizer resources can sit entirely in local, asymmetric corrections [16]; and the self-testing boundary: small-angle (T-type) magic is not computationally self-testable [17], which is why an assumption ledger is permanent rather than provisional. The theorem *shape* assembled here, witness data plus the W- and M-ledgers implying that the mediated gate injected magic, with the Clifford-exception converse, appears nowhere in this literature; the nearest inference-shape relative is the induced-resource-theory analysis of [29], whose inference runs in the opposite direction (a free interaction types environment states, versus a certified stabilizer input typing the gate).

Adjacent ground, distinguished. Magic production by fundamental interactions is an active genre: graviton and gluon scattering on S-matrix helicity qubits [23, 24], closed-form stabilizer entropies over physical phases with Clifford-phase zeros [25], stabilizer-input censuses as gate-grade magic functionals on physical families [26], QED and electroweak $2 \to 2$ processes [30, 31]. None of these treats a mediated two-party protocol, a witness-based certificate, or an exception-set converse. In relativistic quantum information, magic and stabilizer entropy are harvested from quantum fields by local detectors [32–34, 36], with universality results at algebra grade [35] alongside [34], or accumulate stage-by-stage inside gravity protocols [37]; the nearest vocabulary neighbor is a Clifford-only vacuum-induced gate [38]; a contextuality-harvesting branch [39, 40] sits on the far side of the no-contextuality-claim boundary drawn above; and the sole BMV-citing paper in the harvesting line [33] types a detector–field coupling, not a mediated gate. In holography, non-local magic is tied to gravitational back-reaction in AdS/CFT [41], with a growing cluster [42], a different arena, whose title we note and do not adopt as a slogan for the present, assumption-ledgered setting; the "magic" of the gravitational vacuum in [43] is a homonym, not nonstabilizerness. The two-qubit magic–entanglement Pareto frontiers of [27] are the ambient plane of our Fig. 1, and the global two-qubit maximum of M_2 identified in [28] is attained by this family at $\Delta = \pi/2$. Within the gravitationally-induced-entanglement literature itself, the precedent that recorded tomography carries certification load is the unknown-interaction loophole closure of Chevalier, Paige, and Kim [44]; the assumption rows this paper inherits from the companion theorem engage the mediator-locality and subsystem-independence analyses of [45, 46].

II. SETTING: THE COMMITTED FAMILY AND ITS OPERATING POINT

A. Protocol and gate of record

Geometry and notation follow Bose et al. [1] and the companion paper [3]. Two masses $m_1 = m_2 = m = 10^{-14}\,\mathrm{kg}$ (microdiamonds with embedded NV spins), released from adjacent traps separated by d, are each split spin-dependently into branches $|L\rangle$, $|R\rangle$ separated by Δx, held for a time τ, and recombined; readout is two-spin correlations. The branch separations are $s_{RL} = d - \Delta x$, $s_{LL} = s_{RR} = d$, $s_{LR} = d + \Delta x$, and the branch phases are $\varphi_b = K/s_b$ with $K = Gm^2\tau/\hbar$. On the spin (branch-label) qubits the hold is the diagonal unitary $U = \mathrm{diag}(e^{i\varphi_{LL}}, e^{i\varphi_{LR}}, e^{i\varphi_{RL}}, e^{i\varphi_{RR}})$, which factorizes exactly, up to a global phase, as

$$U = \big(P_1(a) \otimes P_2(b)\big) \cdot \mathrm{CZ}(\Delta), \qquad a = \varphi_{RL} - \varphi_{LL}, \quad b = \varphi_{LR} - \varphi_{LL}, \quad \Delta = \varphi_{LL} + \varphi_{RR} - \varphi_{LR} - \varphi_{RL}, \tag{1}$$

with $P(\theta) = \mathrm{diag}(1, e^{i\theta})$ the local phase gate and $\mathrm{CZ}(\Delta) = \mathrm{diag}(1, 1, 1, e^{i\Delta})$ the controlled-phase gate. For the symmetric geometry ($s_{LL} = s_{RR}$),

$$\Delta = -(a + b) = -\frac{2K\,\Delta x^2}{d\,(d^2 - \Delta x^2)}. \tag{2}$$

Only Δ, the second difference of the branch phases, is invariant under local phase changes; a and b are gravitational in origin but locally counterfeitable, and they never enter any certificate below. $\Delta < 0$ for the attractive interaction; figures show $|\Delta|$. The concurrence of the delivered pure state is $C = |\sin(\Delta/2)|$ and the optimal two-setting witness value is $W = 1 + C$ [1, 3].

B. Committed operating point

At the Bose et al. design geometry ($d = 450\,\mu\mathrm{m}$, $\Delta x = 250\,\mu\mathrm{m}$, closest approach $200\,\mu\mathrm{m}$, $\tau = 2.5\,\mathrm{s}$; $Gm^2/\hbar = 63.3\,\mu\mathrm{m\,rad/s}$ at $m = 10^{-14}\,\mathrm{kg}$) all committed values are exact rationals or rigorous-interval evaluations:

$$\frac{K}{d} = \frac{211}{600}, \qquad a = \frac{211}{480} = 0.439583, \qquad b = -\frac{211}{1680} = -0.125595, \qquad \Delta = -\frac{211}{672} = -0.313988, \tag{3}$$

giving $C = 0.156350$, $W = 1 + C = 1.156350$ (the published "$W \sim 1.16$" [1]), CHSH value of the delivered state $2\sqrt{1 + C^2} = 2.024298$, and margins of Δ to the Clifford exceptions $|\Delta| = 0.3140$ and $\pi - |\Delta| = 2.8276$ (interval-certified). The scan rate is $|d\Delta/d\tau| = 211/1680 = 0.125595$ rad/s at this geometry and $5697/28000 = 0.203464$ rad/s at the second registered geometry $(d, \Delta x) = (800, 600)\,\mu\mathrm{m}$.

Ramp honesty. The recorded data return the *total* second difference including the split and recombine segments (of order 0.5–0.9 rad at the design point, ramp-dependent [4]); the hold-only $\Delta = -211/672$ is the reference value, and the ±5% trajectory-integrated phase model that the witness protocol already carries (assumption M2 below) is precisely a model of that total.

Frame of record. All state-level quantities below refer to the delivered state in the *calibrated* frame, $\psi(\Delta) = \mathrm{CZ}(\Delta)\,|{+}{+}\rangle$, i.e. after the local phases a, b are gauged away by the calibration that assumption M2 supplies. The raw-frame state (branch phases as they stand) carries strictly *more* magic ($F_{\mathrm{STAB}} = 0.954825$, $M_2 = 0.249310$ at the committed point), so the calibrated-frame numbers never overclaim; the surplus is local content, which the certificate excludes by construction (Sec. VII).

C. Certified positive result, inherited

The definition of a certified positive result is the companion paper's [3], inherited verbatim and unamended, in three clauses: (1) a spin–spin statistic with a *sound* separable bound above the certified noise/systematics budget, the corrected two-setting witness (separable bound 1, design value $1 + C = 1.1563$; the version-of-record operator pair) or a physical Bell-grade statistic; (2) the excess over the electromagnetic channel budgeted explicitly (a stated margin, or the (d, τ)-scan discriminator separating the gravitational $1/r$ phase scaling from the Casimir $1/r^4$ scaling); (3) a per-run i.i.d. correlator estimate, no joint quantum processing of runs. Assumptions W1–W6 (closed labs, transfer exclusion and EM budget, linear-order composition, mediation semantics, subsystem independence, no superdeterminism plus Born-rule readout) are likewise inherited with their operational certificates; the reader needs only their names here, and their full statements and certification map are in [3].

III. THE CLIFFORD-EXCEPTION ARITHMETIC OF THE FAMILY

A. The nonlocal exception set

Proposition 1 (nonlocal Clifford exceptions)**.** *Within the family* (1)*, the nonlocal part* $\mathrm{CZ}(\Delta)$ *is Clifford-equivalent under local unitaries exactly when* $\Delta \in \pi\mathbb{Z}$*. The set splits physically:* $\Delta \in 2\pi\mathbb{Z}$ *is the no-entanglement accident (*$C = 0$*: witness-free and magic-free, the* $\tau \to 0$ *endpoint of every scan);* $\Delta \in \pi + 2\pi\mathbb{Z}$ *is a Clifford* CZ *at the maximal witness value* $W = 2$*, the site of the separation theorem (Theorem 3).*

The certificate behind Proposition 1 is invariant-theoretic and exact. The Makhlin invariants of $\mathrm{CZ}(\Delta)$ are

$$G_1 = \cos^2(\Delta/2), \qquad G_2 = 2 + \cos\Delta, \tag{4}$$

and an exact enumeration of the two-qubit Clifford group $\mathcal{C}_2$ (all 11520 elements, in the ring $\mathbb{Z}[\zeta_8]$) yields exactly four local-unitary classes, with $G_2 \in \{\pm 1, \pm 3\}$ on the Clifford classes while any $\Delta \notin \pi\mathbb{Z}$ gives G_2 strictly inside the open interval $(1, 3)$. An extracted angle interval excluding $\pi\mathbb{Z}$ therefore certifies the gate outside $\mathrm{LU} \cdot \mathcal{C}_2 \cdot \mathrm{LU}$ with no further computation. At the committed point $(G_1, G_2) = (0.9756, 2.9511)$, and the separation from all four Clifford classes is certified on the entire $\pm 5\%$ calibration interval around the committed Δ as well as on a uniform $(\pi/60)$-grid of the family. The entangling power corroborates the typing one-way: $e_p = \frac{2}{9}\sin^2(\Delta/2) = \frac{2}{9}C^2 = 0.005432$ lies strictly inside $(0, 2/9)$, whereas Clifford gates attain only the anchor values $\{0, 2/9\}$ ($2/9$ being the maximum over *all* two-qubit gates). Throughout this paper, "certifiably magic-free" is scoped: it means certifiably magic-free *within the committed family model*, the exception set is a codimension-one resonance condition of that model, not a regime of couplings.

B. Strict exceptions: the arithmetic ladders

Theorem 1 (strict Clifford ladders)**.** *Call the family point strictly Clifford when both local phase gates and the nonlocal part are Clifford, i.e.* $a, b \in (\pi/2)\mathbb{Z}$ *and* $\Delta \in \pi\mathbb{Z}$ *(the characterization is necessary and sufficient). On a rational-shape family with* $r = \Delta x/d$ *fixed, this is an explicit arithmetic ladder in* K/d*:*

(i) Bose et al. shape $r = 5/9$*:* $K/d \in (28\pi/5)\mathbb{Z}$ *(generator* 17.5929*), with a half-ladder* $(14\pi/5)(2\mathbb{Z} + 1)$ *of points whose local gates are Clifford while the nonlocal residue is* $\Delta \equiv \pi/2 \pmod{\pi}$*, controlled-S-grade points, a natural control family;*

(ii) shape $r = 3/4$*:* $K/d \in (7\pi/6)\mathbb{Z}$ *(generator* 3.66519*), parity-automatic (no half-ladder);*

(iii) irrational shapes: the strict exception set is $\{K = 0\}$ *alone.*

The committed operating point is off-ladder with margins 0.3517 *(to the origin) and* > 17 *(to the first rung) in* K/d*; the first strict point on the committed ray is* $\tau = 125.07\,s$*.*

The two registered shapes carry arithmetically *different* exception ladders, the exception structure is a property of the geometry's rationality class, not of gravity's strength. In quantized form: within the family model, gravity delivers a Clifford nonlocal gate iff

$$m^2\tau \;=\; k \cdot \frac{\pi\hbar}{2G} \cdot \frac{d\,(d^2 - \Delta x^2)}{\Delta x^2}, \qquad k \in \mathbb{Z}, \tag{5}$$

a quantum of $2.5014 \times 10^{-27}\,\mathrm{kg}^2\,\mathrm{s}$ at the committed geometry. The odd rung is reached at $\tau_\pi = 1680\pi/211 = 25.0136\,\mathrm{s}$ at the committed geometry ($15.4405\,\mathrm{s}$ at the $(800, 600)\,\mu\mathrm{m}$ geometry), equivalently $m_\pi = 3.1631 \times 10^{-14}\,\mathrm{kg}$ at $\tau = 2.5\,\mathrm{s}$. These are *family-level statements, never accessible laboratory points*: every registered scan contour of the companion visibility protocol [4] sits at $|\Delta| \le 0.5$, far from every nonzero exception; within every registered family the gate is non-Clifford everywhere except the trivial $\tau \to 0$ endpoint. The exception structure is exercised through the scan's shape, not by visiting the exception couplings.

C. Monotone closed forms and the weak-field laws

On the calibrated-frame state $\psi(\Delta) = \mathrm{CZ}(\Delta)|{+}{+}\rangle$ the relevant magic quantifiers have exact closed forms, obtained as evaluations of cited definitions (never re-derivations): the stabilizer fidelity

$$F_{\mathrm{STAB}}(\Delta) = \frac{5 + 3|\cos\Delta|}{8} \qquad (= 1 \text{ iff } \Delta \in \pi\mathbb{Z}), \tag{6}$$

the stabilizer 2-Rényi entropy [6, 7]

$$\sum_P \langle P\rangle^4 = 1 + \tfrac{3}{4}\,(1+\cos^2\Delta)^2 \quad\Longrightarrow\quad M_2(\Delta) = 4 - \log_2\!\big[4 + 3(1+\cos^2\Delta)^2\big] \quad \text{bits} \qquad (= 0 \text{ iff } \Delta \in \pi\mathbb{Z}), \tag{7}$$

with maximum $M_2 = 4 - \log_2 7 = 1.192645$ at $\Delta = \pi/2$ (the global two-qubit maximum $\ln(16/7)$ nats [28]); and the entangling power $e_p(\Delta) = \frac{2}{9}\sin^2(\Delta/2)$. Committed values:

$$M_2 = 0.104445 \;\; (\text{certified} \geq 0.104444), \qquad F_{\text{STAB}} = 0.981666, \qquad e_p = 0.005432. \tag{8}$$

Channel-level monotones (channel robustness and magic capacity for Z-rotations [12]; unitary stabilizer entropy [13]; stabilizer extents [11]) are cited, not re-derived. In the weak field,

$$C \approx \frac{|\Delta|}{2} \quad \text{(linear)}, \qquad\qquad M_2 \approx \frac{3}{4\ln 2}\,\Delta^2 = 1.0820\,\Delta^2 \quad \text{(quadratic)}, \tag{9}$$

so $M_2/C \to 0$ as $\Delta \to 0$: this is the quantitative reason the magic certificate needs a precision instrument (Sec. VII) while the witness is $O(1)$, and the origin of the anti-alignment law of Sec. VI. One bookkeeping fact deserves early notice: the maximum over stabilizer inputs of the gate's output "magic content" $1 - F_{\text{gate}}$ is $C^2 = 0.024445$, attained by a control-$|1\rangle$ *product* input on which the gate acts as a *local* $R_z(\Delta)$ (the $|{+}{+}\rangle$ route gives only $\frac{3}{4}C^2$): locally realized magic dominates that scalar bookkeeping [16]. The nonlocal typing of this paper is therefore carried by the local-unitary-class certificate (4) and the exception arithmetic, never by a magic magnitude.

IV. THE WITNESS IS MAGIC-BLIND AT EVERY VALUE

Lemma 1 (magic-blindness of the committed witness)**.** *The committed witness statistic is a sum of two commuting two-qubit Pauli products (in the Schmidt-adapted calibrated frame $W = \langle Z\otimes Z\rangle + \langle X\otimes X\rangle$; the published readout pair has the same commuting-stabilizer-pair structure in its own frame). Consequently:*

(a) every value the statistic can take, its quantum maximum $W = 2$ included, is attained by a stabilizer state or stabilizer mixture; the reachable correlator square is the full $[-1,1]^2$;

(b) the Bell-diagonal stabilizer mixture $\rho^\star = 0.578175\,|\Phi^+\rangle\langle\Phi^+| + 0.421825\,|\Phi^-\rangle\langle\Phi^-|$ reproduces both design correlators ($\langle ZZ\rangle = 1$, $\langle XX\rangle = C$) and the design witness value $W = 1 + C = 1.156350$ exactly; it is entangled and carries zero magic by every monotone;

(c) no function of the committed correlators certifies magic at any observed witness value.

Proof. Commuting Pauli products are jointly diagonalized by a common stabilizer basis; the four joint eigenprojectors are stabilizer states, so every point of the correlator square is a convex combination of stabilizer-state values, which proves (a) and, by the explicit mixture, (b) (weights $(1 \pm C)/2$; verified exactly in the ancillary suite, including entanglement of $\rho^\star$ and the census matching of the four eigenprojectors). Claim (c) follows: any function of the correlators is constant on the fiber containing a zero-magic state that attains the observed value. □

Two published results close the flanking escape routes. A standard CHSH statistic cannot witness magic at any violation [8], and the delivered BMV state's CHSH value is in any case only $2\sqrt{1+C^2} = 2.024298$, far below $2\sqrt{2}$. And the added assumptions of Sec. V are not an instrument artifact that better theory could remove: the small-angle (T-type) magic species relevant here is not computationally self-testable [17], and CHSH-enabling nonstabilizer resources can sit entirely in local, asymmetric corrections [16], so state-level magic in the delivered spins is *locally counterfeitable*, the labs' own pulses can produce it, and only a frame assumption can attribute it. Lemma 1 thus forces the shape of any honest upgrade: added premises, stated and certified, or no magic conclusion at all.

V. THEOREM M: THE LEDGERED UPGRADE

Theorem M is an additive layer over the companion witness theorem: it assumes everything Theorem W of [3] assumes, W1–W6 and the three-clause certified positive result, adds five assumptions M0–M4, and draws one additional conclusion. Nothing in the W-ledger is altered. Table I states the M-ledger with each assumption's laboratory meaning, operational certificate, and cost.

TABLE I. The M-ledger: the five assumptions added to the companion theorem's W1–W6, each with its operational certificate. V_s and V_c are the single-mass and conditional interference visibilities of the companion visibility protocol [4], recorded in the same campaign.

Assumption	Laboratory content	Operational certificate	Cost
M0 stabilizer input	the pre-interaction spin state is certified close to the $\|{+}{+}\rangle$-class product (each spin an X eigenstate before the split)	preparation/calibration runs at fidelity grade; robustness anchor: a $\pi/2$-pulse angle error ϵ leaves per-qubit stabilizer fidelity $\cos^2\epsilon$ ($1^\circ \to 3\times10^{-4}$ infidelity). The state-level floors consume M0 in full; the LU-class typing needs only support on all four branches ($V_s > 0$)	standard calibration; no new hardware
M1 channel form	over the interaction window the two-lab evolution is a branch-diagonal unitary (branch phases only; no spin flips or leakage) plus a residue inside a certified budget	the conditional-visibility ratio $V_c/V_s \approx 1$ certifies near-unitarity; single-mass controls certify branch-diagonality	zero, runs inside the campaign
M2 calibrated family / fixed local frame	branch phases follow $\varphi_b = K/s_b$ within the $\pm5\%$ trajectory-integrated phase model *the witness already requires*; local readout frames calibrated so the delivered state is $\mathrm{CZ}(\Delta)\|{+}{+}\rangle$ up to certified tolerance	the $\pm5\%$ phase model; the (d,τ) scan certifying the $1/r$ law; the phase-doping null test of Sec. VII. Consumed by the state-level floors and the witness frame; *not* by the LU-class typing under the primary route, where the model is a cross-check, not an input	a precision upgrade of an assumption already made
M3 measurement route	primary: the Z-conditioned fringe difference on the witness's own settings (Sec. VII); consistency: the local-Pauli quadruple $\{XX, XY, YX, YY\}$ + marginals in model-calibrated axes, a rescoring of recorded tomography [9]; secondary: tilted CHSH at the committed Schmidt angle [8]; mixed-state route: the triangle criterion [10]	trust-base honesty: certificates are relative to calibrated Pauli axes, the settings' own magic is part of the trust base; the doping control tests exactly this	zero added settings and shots on the primary route
M4 statistics	per-run i.i.d. shots (clause 3 of the certified positive result, inherited); σ_Δ propagated as in Sec. VII	standard correlator estimation	none new

Theorem 2 (Theorem M: gate-grade magic certification)**.** *Assume W1–W6 and Definition 1 of the companion theorem [3], and assumptions M0–M4 of Table I, the stated input, channel-form, calibration, measurement-route, and statistics assumptions, each with its named operational certificate. Then a certified positive BMV result whose extracted angle interval $[\Delta_{\mathrm{lo}}, \Delta_{\mathrm{hi}}]$ excludes $\pi\mathbb{Z}$ additionally certifies that the gravitational coupling injected magic: it implemented a nonlocal non-Clifford gate, with $\Delta \in [\Delta_{\mathrm{lo}}, \Delta_{\mathrm{hi}}]$ and the cited monotone floors (6)–(8) evaluated on that interval (the weakest value each quantity takes over the interval). Without the M-assumptions, no magic conclusion follows from the committed correlators at any observed witness value (Lemma 1).*

Proof (assembly). By the certified positive result and W1–W6, Theorem W of [3] applies and the entangling channel is the gravitational coupling. By M0–M2 the recorded data are lifted from state grade to gate grade: a certified stabilizer input, a branch-diagonal unitary channel, and a calibrated frame identify the induced two-qubit gate with a member of the family (1) up to certified tolerance, with Δ the gauge-invariant family parameter. By M3 and M4 the extraction of Sec. VII returns an interval $[\Delta_{\mathrm{lo}}, \Delta_{\mathrm{hi}}]$ with stated confidence. If the interval excludes $\pi\mathbb{Z}$, the invariant certificate (4) places the gate outside $\mathrm{LU}\cdot\mathcal{C}_2\cdot\mathrm{LU}$ (Proposition 1); the monotone floors are then evaluations of the cited closed forms on the interval. The premise-exclusion clause is not removable: at $\Delta \in \pi + 2\pi\mathbb{Z}$ the conclusion is false while the witness is maximal (Theorem 3), and without M0–M4 Lemma 1 exhibits zero-magic states reproducing all committed data. □

The honesty sentence travels with the theorem: *certifying magic is strictly harder than certifying non-classicality, and W1–W6 plus M0–M4 are the added premises; without them no magic conclusion follows.*

Remark 1 (reading stability across the mediation disjunction). Theorem W's conclusion is a disjunction, (I) a non-classical mediator, or (II) a nonseparable direct quantum gate with no carrier, which mass-side data cannot separate. The magic typing does not split across it: under either reading the channel delivers the *same* diagonal gate on the masses (the polaron identity of the companion paper [3], re-verified at ring-exact grade for the slaved presentation in the ancillary suite). The certificate of Theorem 2 is therefore a statement about the *induced gate on the matter sector*, stable across the disjunction. For the same reason, no conclusion of the form "the mediator (graviton) is itself magic" is drawn, at any grade, anywhere in this paper: the mediator-sector conclusion remains Theorem W's, and mediator-sector resource claims would require mediator-sector instruments the protocol does not possess.

Remark 2 (what is not certified). Not certified by a positive Theorem-M outcome: (i) state magic at readout as *gravitational* without M2, the labs' own pulses are non-Clifford-capable, and the local-unitary ambiguity is physical [16]; (ii) any mediator-sector magic (Remark 1); (iii) anything at the Clifford-exception couplings (Sec. VI); (iv) any computational-hardness or speedup statement: nonstabilizerness is not hardness, classically simulable nonstabilizer states exist [18], and neither certificate here is a simulability claim.

VI. THE CONVERSE: SEPARATION AT THE CLIFFORD POINT, AND THE ANTI-ALIGNMENT LAW

Theorem 3 (separation at the Clifford exceptions)**.** *At the Clifford-exception couplings $\Delta \in \pi + 2\pi\mathbb{Z}$, the same protocol certifies non-classicality at the maximal witness value $W = 2$ while provably certifying no nonlocal magic: the delivered state $\mathrm{CZ}(\pi)|{+}{+}\rangle$ is a stabilizer state (stabilizer group $\{II, XZ, YY, ZX\}$) on which the witness attains 2; the gate is Clifford ($G_2 = 1$, $e_p = 2/9$ at its Clifford anchor); every magic monotone vanishes identically; and every sound magic witness is silent by definition of soundness, not for lack of settings, but because there is nothing to witness. Theorem 2's premise is correctly false there. At $\Delta \in 2\pi\mathbb{Z}$ neither theorem fires ($C = 0$). The two certified conclusions, non-classicality and magic injection, separate within one protocol family.*

Both directions are machine-verified exactly (stabilizer group membership, witness maximization, vanishing of M_2, $F_{\mathrm{STAB}} - 1$, and the e_p anchor). The physical reading is the useful one: *non-classicality is angle-generic; magic is angle-conditional; the family's best entanglement point is a zero-magic point.* The reach statement accompanies every appearance of the exception coupling: $\tau_\pi = 25.0136\,\mathrm{s}$ at the committed geometry (15.4405 s at the second registered geometry), $m_\pi = 3.1631 \times 10^{-14}\,\mathrm{kg}$ at $\tau = 2.5\,\mathrm{s}$, a family theorem, never a laboratory point (Sec. III).

Figure 1 displays the law that makes the separation quantitative along the accessible part of the family. Along the hold-time scan the witness grows linearly ($W \approx 1 + |\Delta|/2$; 1.156994 versus the exact 1.156350 at the committed point) while the certified magic floor grows quadratically ($M_2 \approx 1.0820\,\Delta^2$; 0.106675 versus 0.104445), peaks at the controlled-S point ($M_2 = 4 - \log_2 7$ bits, the global two-qubit maximum [28]), and returns to exactly zero at $\Delta = \pi$ where the witness is maximal. The two certificates are not co-monotone; more witness is not more magic. The ambient plane for this figure, the exact two-qubit magic-versus-entanglement Pareto frontiers, is [27]; the stabilizer-minimum frontier is the ambient fact behind Lemma 1 (stabilizer states reach every witness value).

VII. THE EXPERIMENTAL PROTOCOL: EXTRACTION, GATING, AND COSTS

This section is the experiment-facing content: what to compute from data the campaign already records, the decision rules under which Theorem 2 fires, and the complete shot and visibility accounting. Nothing in the primary route requires hardware, settings, or shots beyond the committed witness-plus-visibility campaign [1, 4]; the one addition is the doping null test of rule D-5, a control run of standard calibration grade.

A. Primary route: the conditional-fringe difference

In the readout frame as it stands, the delivered state is the phase state

$$\psi = \tfrac{1}{2}\big(|00\rangle + e^{ib'}|01\rangle + e^{ia'}|10\rangle + e^{i\gamma'}|11\rangle\big), \tag{10}$$

with (calibrated frame) $a' = b' = 0$, $\gamma' = \Delta$, or (raw BMV frame) $a' = a$, $b' = b$, $\gamma' = \varphi_{RR} - \varphi_{LL} \equiv 0$ at the symmetric geometry. Condition lab A's equatorial fringe on lab B's Z outcome: $\varphi_z = \mathrm{atan2}(\langle Y_A|z\rangle, \langle X_A|z\rangle)$ for $z \in \{L, R\}$ (all shots used, no postselection; readout confusion matrices inverted first). Then

$$\hat{\Delta}_A = \varphi_R - \varphi_L \ \ (\text{wrapped to } (-\pi, \pi]) \ = \ \gamma' - a' - b', \tag{11}$$

exactly invariant under local Z phases on either side (doping A shifts both conditional phases equally; doping B commutes with Z_B), with *no model input*; the mirror $\hat{\Delta}_B$ from the (Z_AX_B), (Z_AY_B) settings is an independent second estimate. The statistical error is $\sigma_\Delta = 2/(F\sqrt{N})$ per side and $\sqrt{2}/(F\sqrt{N})$ with both mirrors averaged, F the correlator contrast and N the shots per setting. The attractive sign $\hat{\Delta} < 0$ (A's fringe phase larger when B sits in the near branch) is a free prediction of the data, not an input; it also resolves the reflection ambiguity of any modulus-based cross-check. At the committed point the raw-frame conditional phases are $\varphi_L = a = 0.439583$ and $\varphi_R = -b = 0.125595$, difference $-211/672 = -0.313988$, ring-exact.

The settings involved, (X_AZ_B), (Y_AZ_B) and their mirrors, are the witness's own settings and those of the companion visibility protocol's conditional-visibility measurement [4]: the primary route costs *zero added settings and zero added shots*. The ±5% phase model of M2 is demoted, on this route, from input to cross-check ($\hat{\Delta}$ versus the model within 5%).

The doping null test. A local phase gate $Z(\phi)$ at non-Clifford ϕ is itself magic-injecting, and CHSH-enabling nonstabilizerness can sit entirely in such local corrections [16]; that is why the certified quantity is the one number no local phase can move. The guarantee is testable in the laboratory, not only on paper: apply deliberate local $Z(\phi)$ pulses (e.g. $\phi = 0.4\,\mathrm{rad}$) on either side before readout and verify that $\hat{\Delta}$ does not move, *with nothing subtracted*. Under two-sided doping $Z(\phi_1) \otimes Z(\phi_2)$ the bare quadruple observables move ($\gamma' \to \Delta + \phi_1 + \phi_2$) while $e^{i\hat{\Delta}}$ and the marginal modulus $|\cos(\hat{\Delta}/2)|$ are exactly invariant, a ring identity, verified on the committed point (illustratively: $\phi_1 = 0.4$, $\phi_2 = -0.7$ give $\gamma' = -0.613988 \neq \Delta$ while $\hat{\Delta} = -0.313988$ unmoved; Fig. 4). On the primary route this control is a genuine null test; on the quadruple route it tests only linear response to *known* phases, which must be subtracted through the calibration model.

B. Why the Pauli quadruple is not the primary route

The extraction identities of the quadruple route hold exactly for any phase state (10): $\langle XX\rangle - \langle YY\rangle = \cos\gamma'$, $\langle XY\rangle + \langle YX\rangle = \sin\gamma'$, $\langle XX\rangle + \langle YY\rangle = \cos(a' - b')$, $\langle XI\rangle + i\langle YI\rangle = (e^{ia'} + e^{i(\gamma' - b')})/2$ with modulus $|\cos(\hat{\Delta}/2)|$. But the quadruple returns γ' and $a' - b'$, and the equatorial marginals return $(\gamma' \pm (a' - b'))/2$: *no combination of equatorial-only data determines $a' + b'$, hence none determines $\hat{\Delta} = \gamma' - (a' + b')$ with linear sensitivity.* At the symmetric BMV geometry the raw-frame quadruple reads $\gamma' \equiv 0$, the Clifford-exception value, at *any* coupling strength, and after the axes are calibrated by the ±5% model it returns the model angle whatever gravity does (a true coupling at 0.5× / 1× / 1.5× the design value returns -0.313988 in all three cases). The quadruple reading is therefore a *consistency check of the calibration model*, valuable, and it supplies the conservative error budget quoted below, but not, by itself, the measurement. A second, independent reason is systematic: with realistic asymmetric single-shot readout ($\epsilon_0/\epsilon_1 = 1\%/5\%$, the norm on NV platforms) the quadruple estimate is biased by $+0.093 \approx 30\%$ of Δ, while the conditional-fringe estimate moves by $\sim 2\epsilon \sin\Delta$ ($+0.029$), calibratable through confusion matrices measured on the same runs; all identified biases push $|\hat{\Delta}|$ *toward* zero, i.e. are conservative for the certificate, and every branch-diagonal (dephasing-type) channel, however correlated, leaves $\hat{\Delta}$ exactly unbiased. (These bias figures are float-grade protocol estimates, not certified values; the certified content of this section is the exact identities and the error-propagation formulas.)

Where the quadruple *is* recorded, three normalized frame diagnostics should be run, with the measured contrast $\hat{F} \equiv [(\langle XX\rangle - \langle YY\rangle)^2 + (\langle XY\rangle + \langle YX\rangle)^2]^{1/2}$ divided out: $(\langle XX\rangle + \langle YY\rangle)/\hat{F} = 1$ (calibrated frames), $\langle ZZ\rangle = 0$ (equal branch weights, at any F), and the marginal modulus $|\langle XI\rangle + i\langle YI\rangle|/\hat{F} = |\cos(\hat{\Delta}/2)|$ (a frame-free cross-check of the same angle, quadratically insensitive near the committed angle). The division by $\hat{F}$ is exact on the white-noise benchmark; under non-white channels the marginal and correlator contrasts need not share one F, and the marginal check then tests the noise model rather than the frame.

C. Decision rules

Every clause at one stated significance k ($k = 5$ recommended):

D-1. *Certified positive:* $W - 1 \geq k\sigma_W$ in the readjusted frame; the EM budget clause; per-run i.i.d. statistics (the companion Definition 1, all three clauses).

D-2. *Exception exclusion:* $|\hat{\Delta}| \geq k\sigma_\Delta$ and $\pi - |\hat{\Delta}| \geq k\sigma_\Delta$.

D-3. *Internal consistency:* $\hat{\Delta}_A$ and $\hat{\Delta}_B$ agree within 2σ; $\hat{\Delta} < 0$ (attractive sign).

TABLE II. Committed operating point, $N = 10^4$ shots per setting. $W_F = F(1 + C)$ is the witness value on the white-noise benchmark $\rho_F = F|\psi\rangle\langle\psi| + (1 - F)\mathbb{1}/4$; rows with $W_F \leq 1$ are *angle-resolution* statements only (no certified positive on that benchmark; the benchmark state is separable outright below $F_{\text{sep}} = 1/(1 + 2C) = 0.7618$). $N_{\min}$ is the smallest N with 5σ exclusion of $\Delta = 0$ under (12).

F	W_F	σ_Δ	from $\Delta = 0$	from $\Delta = \pm\pi$	$N_{\min}(5\sigma)$
1.0	1.156	0.02113	14.9σ	134σ	541
0.926	1.071 (5σ witness at 10^4)	0.02190	14.3σ	129σ	631
0.7	0.809 (angle resolution only)	0.02559	12.3σ	111σ	1105 (exact)
0.5	0.578 (angle resolution only)	0.03235	9.7σ	87σ	2164

D-4. *Readout:* confusion matrices measured on the same runs and inverted; residual bias $\leq 0.2\,\sigma_\Delta$.

D-5. *Doping null test:* with deliberate local $Z(\phi)$ pulses at non-Clifford ϕ on either side, $\hat{\Delta}$ moves by $< 2\sigma$, nothing subtracted.

D-6. *M-certificates:* M1 from the conditional-visibility ratio $V_c/V_s = 1$ within budget on the same runs; M0 from $V_s > 0$ and the preparation-fidelity runs; M2 in its residual role, $\hat{\Delta}$ agrees with the trajectory-integrated model within 5% and the (d, τ) scan returns the $1/r$ law.

If D-1 through D-6 hold, Theorem 2 fires: report the $k\sigma$ interval, the invariant certificate $G_2 = 2 + \cos\Delta$ on it, and the floors M_2, F_{STAB}, e_p evaluated at the interval endpoint nearest the exception.

D. Precision, gating, and shots

The conservative (quadruple-form) error budget, which the primary route only improves on (identical statistical term, no model term), is

$$\sigma_\Delta = \frac{\sqrt{2/N}}{F} \oplus 0.05\,|\Delta| \qquad (\oplus = \text{addition in quadrature}), \tag{12}$$

whose calibration term caps that reading's significance at 20σ at any angle; the primary route carries no model term and no such ceiling, its systematic floor being the readout-bias residual of rule D-4. What Theorem 2 consumes is the distance of $\hat{\Delta}$ from the nearest exception in units of σ_Δ. Table II gives the committed-point ladder; Table III spans the registered scan contours; and Fig. 2 displays the instrument across the (F, N) plane.

Witness-first gating. The extraction significance must not be read as a certificate on its own: Theorem 2's premise includes the certified positive, and on the white-noise benchmark the witness itself certifies only above $F_{\text{ent}} = 1/(1 + C) = 0.864790$; a 5σ witness at 10^4 shots per setting needs $F \geq 0.926$. The correct summary is: *at any visibility at which the witness certifies entanglement at 5σ with 10^4 shots per setting, the committed angle is $\geq 14\sigma$ from $\Delta = 0$, the extraction is never the bottleneck; the witness is.* The exact 5σ predicate at the committed angle and $F = 0.7$ gives $N \geq 160/(3F^2\Delta^2) = 1104.018$, hence $N_{\min} = 1105$ exactly; $N = 1104$ fails the predicate by 6.14×10^{-8} in squared-σ units (exact deficit $17/276971520$). For the primary route with both mirrors and no model term, the corresponding minima are 2071 (one-sided) and 1036 (mirrors averaged) at $F = 0.7$.

Shot cost. The primary route rides the witness and conditional-visibility settings at zero added shots. If the Pauli quadruple is wanted as dedicated settings (it is a consistency check, not the certificate), it costs 4×10^4 shots ≈ 4.6 machine-days *per contour point* at 10 s per shot (≈ 23 machine-days over the five registered contours; full nine-setting tomography 10.4 machine-days per point), against the visibility campaign's own 21–25 machine-day baseline [4].

E. The visibility price of the state-level route

On the white-noise benchmark $\rho_F = F|\psi(\Delta)\rangle\langle\psi(\Delta)| + (1-F)\mathbb{1}/4$ the two certificates have sharply different visibility thresholds (Fig. 3):

$$F_{\text{ent}} = \frac{1}{1 + C} = 0.864790, \qquad F_{\text{magic}} = \frac{1 + \cos\Delta}{2} = 0.975555 \;\; (7\text{dp } 0.9755547), \qquad F_{\text{magic}} - F_{\text{ent}} = 0.110765 \approx 0.1108, \tag{13}$$

TABLE III. The registered scan contours [4] at $N = 10^4$, $F = 0.7$ (angle-resolution column), with the witness cost at the gating visibility. The witness column is the dephasing-contrast reading $N \geq 50/(C^2F^2)$ (exact ceilings) at $F = 0.926$; its ratio to the extraction $N_{\min}$ is $(15/16)(\Delta/C)^2 \approx 3.75$–$3.83$ on every contour: wherever the campaign can certify entanglement at 5σ, it types the gate at $\geq 5\sigma$ with a $\sim 3.8\times$ shot margin to spare.

$\lvert\Delta\rvert$	σ_Δ	from $\Delta = 0$	$N_{\min}(5\sigma)$	witness N $(5\sigma,\ F = 0.926)$	status
0.10	0.02081	4.80σ	10885	23344	marginal, for the witness first ($N \geq 20017$ even at $F = 1$)
0.20	0.02254	8.87σ	2722	5851	$\geq 5\sigma$
0.30	0.02516	11.92σ	1210	2612	$\geq 5\sigma$
211/672	0.02559	12.27σ	1105	2386	committed point
0.40	0.02843	14.07σ	681	1478	$\geq 5\sigma$
0.50	0.03214	15.56σ	436	953	$\geq 5\sigma$

where F_{magic} is the threshold of the fidelity-hyperplane magic witness ($(1 + 3F)/4 > F_{\text{STAB}}(\Delta)$; stabilizer-polytope and mixed-state witnessing per [14, 15]), and the tilted-CHSH route [8] is sharper still: the committed Schmidt angle $\theta = 0.078497\,\text{rad}$ lies inside the ideal magic-witnessing window by a margin of only 2.95×10^{-4} in the inequality value, surviving only to $F_b = 0.999925$, noise-fragile at any realistic visibility. The gap in (13) is the $O(\Delta^2)$-versus-$O(\Delta)$ resource price of Sec. VI made visible, and it is *correctly scoped*: it prices the *state-level* upgrade route only, and equivalently measures the width of the visibility window $(0.8648, 0.9756)$ in which the extraction route plus the M0–M2 gate-grade lift is the *only* live magic instrument. The thresholds cross only at $\sin(\Delta/2) = (\sqrt{5}-1)/2$, i.e. $|\Delta| = 1.3325$ ($\tau \approx 10.6\,\text{s}$ at the committed geometry), beyond every registered contour, a family-level fact. Two cautions: F here is a white-noise *benchmark* fraction, not a model of the apparatus (the campaign's measured visibility decides which certificates are live); and at realistic noise the teeth of the protocol are the extraction route together with the M0–M2 gate-grade lift, not the state-witness route, which is visibility-starved at the committed angle. Under M0–M2 any state-level certificate lifts to the channel (a certified stabilizer input mapped outside the stabilizer polytope implies the channel is not stabilizer-preserving), so the state-level routes remain valuable cross-checks at high visibility and the right primary tool only for directly typing a decohered output [10].

F. Systematics and degeneracies

The confounds specific to a magic certificate are bookkeeping degeneracies, what could be booked as gravitational magic without being it, and each is closed or priced: local non-Clifford pulses (closed by the doping-invariant $\hat{\Delta}$ and rule D-5); local-axis miscalibration (the settings' own magic is part of the trust base; closed to the stated tolerance by the normalized frame diagnostics and priced inside (12)); visibility loss of any origin (scales all correlators; priced in σ_Δ); asymmetric readout (rule D-4; the quadruple's 30%-of-Δ sensitivity is one of the two reasons it is not primary); electromagnetic second-difference phase (the gravitational attribution of $\hat{\Delta}$ is the companion theorem's Step 1, inherited unchanged, the EM budget and the (d, τ) scan; this paper adds no new EM claim); proximity to an exception (margins 0.3140 / 2.8276 at the committed point; every registered contour is at $|\Delta| \leq 0.5$); preparation infidelity and channel residue (assumptions M0/M1, certified as in Table I; the M0 fidelity target is a protocol requirement set by the collaboration, not a computed budget, a declared soft spot); and joint processing of runs (excluded by clause 3 of the certified positive result, inherited). The one degeneracy no spin-side measurement can break is stated plainly: without a fixed frame, magic injected by the gravitational coupling cannot be distinguished from magic injected by the labs' own local operations, state magic is local-unitary-ambiguous. The protocol does not pretend otherwise: it certifies the one number local operations cannot move ($\hat{\Delta}$), makes the frame an explicit, laboratory-tested assumption (M2 with the doping null test), and states that without M2 no *state-level* magic conclusion follows from any of the data.

VIII. WHAT A NULL MEANS

Theorem 2 has a chain of premises, a certified positive result, an extracted interval excluding $\pi\mathbb{Z}$, and M0–M4, and a "null" can occur at each link. *None of them licenses a classicality claim; a null of any branch never certifies that gravity is classical.*

1. *No certified positive (witness null at sufficient sensitivity).* This is the territory of the companion paper's null-

result converse [3]: at sufficient sensitivity it certifies that the gravitational channel imparts no second-difference phase at the probed regime, with the interpretation priced there. Theorem 2 has no premise here. Two sub-branches matter for this paper. A witness null with $\hat{\Delta} = 0$ at sufficient sensitivity belongs entirely to the companion analysis. A witness null with $\hat{\Delta} \neq 0$, together with a visibility deficit tracking Δ^2, is the fingerprint of a *classical measure-and-act channel*: a channel that reads one mass's branch and phases the other imprints the *full* Newtonian second difference in the conditional fringes while destroying the entanglement. The extraction alone is therefore never a non-classicality certificate; the certified-positive premise (Definition 1) carries that exclusion, and the fingerprint is a diagnostic, never a verdict.

2. *Certified positive, but the extracted interval contains* 0 *(precision null).* The theorem correctly declines, its premise is false, and nothing about the gate's type follows. The cure is statistical (Sec. VII D), not interpretive. Since the witness certifies $C > 0$, an interval containing 0 at comparable confidence signals a frame or visibility inconsistency between the two readings and should be treated as a diagnostic.

3. *Angle resolved, but no certified positive* (visibility below the witness threshold at the run's noise). The theorem declines for lack of its Definition-1 premise; the extracted $\hat{\Delta}$ is then a phase measurement checking the $\pm 5\%$ model, not a gate-typing certificate.

4. *Extraction certificate fires while every state-level magic witness is silent.* Expected at the committed angle: the state-level routes are visibility-starved ((13); $F_b = 0.999925$). Silence of a state witness below its threshold is not evidence against magic injection.

5. *The Clifford exception.* At $\Delta \in \pi + 2\pi\mathbb{Z}$ the protocol certifies non-classicality at $W = 2$ and provably no nonlocal magic (Theorem 3), the premise is correctly false, and the outcome is the converse theorem, not a failure. Reach honesty as in Sec. III.

6. *Failure of an M-assumption* (calibration drift beyond the $\pm 5\%$ model, preparation infidelity beyond the M0 target, a non-diagonal residue beyond the M1 budget) voids the magic certificate, not the witness, and not the companion theorem.

The asymmetry of stakes is inherited from the companion visibility protocol [4]: a positive run delivers a gate-grade typed conclusion at no added cost; a null in any branch delivers a bound, a diagnostic, or a family theorem, never a verdict on classicality.

IX. DISCUSSION

A. What the certificate adds, and to whom

For the experimental campaigns, Theorem 2 converts data already planned into a second, sharper typed conclusion: beyond "no classical degree of freedom carried this entanglement" [3], a certified positive run with the extraction interval of Sec. VII additionally reports "the gravitational coupling implemented a nonlocal non-Clifford gate," with quantified floors and with every added premise named, certified, and cheap. The extracted $\hat{\Delta}$ with its error bar is simultaneously a quantitative measurement of the gravitational second-difference phase that checks the $\pm 5\%$ phase model the witness itself relies on. For the theory of the experiments, the exception arithmetic and the converse theorem supply the structure: the certificate's boundary is not a sensitivity floor but a resonance set of the family, with the striking feature that the family's best entanglement point is a provably zero-magic point, more witness is not more magic, and the two certificates decouple within one protocol family.

B. Position among the precedents

The inference pattern, certify a quantum character of gravity by an injected resource *beyond* entanglement, is Howl et al.'s [19], and this paper is best read as the discrete, two-party, assumption-ledgered instance of that pattern, with four deltas stated in Sec. I B: discrete magic instead of CV non-Gaussianity; the committed mediated-gate protocol instead of a single-system platform; a forced and priced assumption ledger over the entanglement witness itself (Lemma 1 shows no ledger-free version exists); and the exception-set arithmetic with its converse. Andriolo's contextuality-currency proposal [21] independently exhibits the same economic structure we quantify in (13): in both currencies the beyond-entanglement signature costs strictly more visibility than the entanglement signature. The

magic-from-fundamental-interactions genre [23–26, 30, 31] and the relativistic-quantum-information harvesting line [32–34] type states or S-matrix outputs of fundamental processes; none addresses a witness-based, assumption-ledgered certificate on a mediated two-party protocol, which is exactly the gap this paper fills. The induced-resource-theory framework [29] is the nearest inference-shape relative, with the direction reversed.

C. Scope, assumptions, and limitations

The certificate's strength is exactly its ledger's, and the following caveats are part of the result.

1. *The folkloric core is credited, not claimed.* That a generic-angle controlled-phase gate is non-Clifford and its output carries magic is elementary [8]; the contribution here is the certification structure: the impossibility lemma, the ledger, the exception arithmetic with the converse, the extraction instrument, and the costing. Every monotone closed form is an evaluation of a cited definition.

2. *Gate grade, matter sector.* The certificate types the induced gate on the matter sector and is stable across the companion theorem's mediation disjunction; it licenses no statement about a mediator's own resources (Remark 1).

3. *Magic is not hardness.* Nonstabilizerness certifies a resource, not a speedup: classically simulable nonstabilizer states exist [18], and no simulability claim is made at any grade. The certified floors at the committed point are moreover small ($M_2 = 0.104$ bits), the honest weak-field statement is the quadratic law (9), not a large resource count.

4. *Frame dependence is physical and priced.* State-level magic is local-unitary-ambiguous [16]; the fixed calibrated frame (M2) is a genuine assumption, tested by the doping null test, and without it only the frame-free gate-grade content survives. All state-level numbers are calibrated-frame values and never overclaim relative to the raw frame (Sec. II).

5. *Benchmarks are not noise models.* The visibility thresholds (13) are white-noise benchmark numbers scoping the state-level routes; the apparatus noise enters the operative instrument only through F in σ_Δ, and the campaign's measured visibility, not the benchmark, decides which certificates are live.

6. *M0 is a protocol row.* The stabilizer-input fidelity is a requirement the collaboration certifies with preparation runs; this paper supplies the robustness anchor, not a computed fidelity budget.

7. *The measured angle is ramp-inclusive.* $\hat{\Delta}$ is the total pair-channel second difference; its gravitational attribution rests on the companion theorem's transfer-exclusion and EM-budget assumptions with the (d, τ) scan, exactly as for the witness.

8. *Exception-set scope.* "Certifiably magic-free" means within the committed family model; the strict ladders of Theorem 1 are statements about the family's arithmetic, and the exception couplings τ_π, m_π are family-level values outside every registered platform, the exception structure is exercised through the scan's shape, never as a laboratory point.

9. *Further certified structures are deliberately out of scope here*: the species identification of the located magic carriers, their in-class interconvertibility analysis, and the free-fermion (matchgate) restriction class, which cuts the same physical family on a different exception set, are treated in a companion work [5].

X. CONCLUSION

The witness experiments were designed to certify that gravity's channel is non-classical; the companion paper supplies that theorem. This paper establishes what more the same data can certify, and at what price. The committed witness statistic itself can never say anything about magic, at any value, including its maximum, so any gate-typing conclusion rides on added premises; we exhibit the forced ledger of five, each with a laboratory certificate, and the theorem it buys: a certified positive run whose extracted second-difference interval excludes $\pi\mathbb{Z}$ certifies that the gravitational coupling implemented a nonlocal non-Clifford gate, magic injected at gate grade, stable across the mediation disjunction, with quantified floors. The certificate's boundary is an arithmetic, not a sensitivity floor: the family's Clifford exceptions form explicit ladders, and at the odd rungs the protocol's *best* entanglement point is a provably zero-magic point, so non-classicality is angle-generic while magic is angle-conditional, the witness growing

linearly while the magic floor grows quadratically. The instrument is free, a re-scoring of conditional fringes the campaign already records, exactly immune to local phase doping, with the witness rather than the extraction as the binding cost, and its failure semantics are stated: a classical channel can fake the angle but never the certified positive, and a null of any branch never says that gravity is classical.

Acknowledgments., Numerical certificates, verification suites, and drafting made use of AI-assisted tools; all results were adversarially verified as described, and the author takes full responsibility for the content.

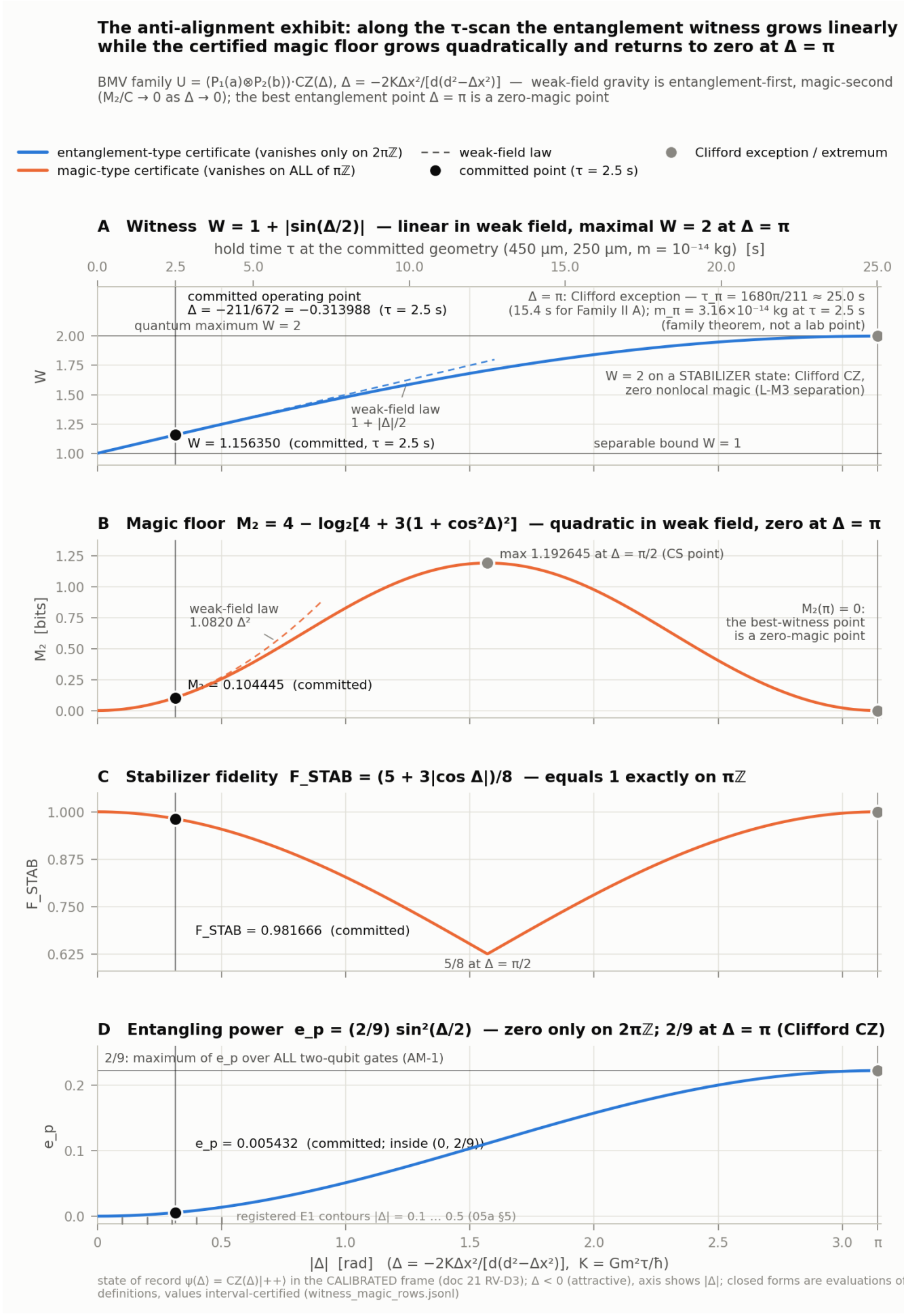
Data and code availability., The ancillary scripts of Table IV reproduce every quantitative claim deterministically and will be posted with the preprint, together with the certified-value ledger they generate and the figure data tables.

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TABLE IV. Ancillary verification suite (deterministic, `python3.12`; exact `Fraction`/interval/cyclotomic arithmetic in every certified quantity, floats display-only; byte-identical reruns; exit 0). The suites of the companion paper [3] are consumed as cited there.

Script	Self-tests
`witness_magic_lattice.py` (exception sets, Clifford census, monotones, blindness/extraction/separation lemmas, precision predicates)	77/77
`witness_magic_rigor_verifier.py` (independent-route verification)	53/53
`witness_magic_closing_audit.py` (third-band re-execution, independent arithmetic)	49/49
`e1m_figures.py` (figures + tables; record/self/derived checks)	40/40, 8/8, 7/7
`e1m_finalize_checks.py` (exact-ceiling shot budgets)	all exact
`e1m_rigor_verifier.py` (protocol-note verification; six intended defect probes)	96/102

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The anti-alignment exhibit: along the τ-scan the entanglement witness grows linearly while the certified magic floor grows quadratically and returns to zero at Δ = π
BMV family U = (P₁(a)⊗P₂(b))·CZ(Δ), Δ = −2KΔx²/[d(d²−Δx²)] — weak-field gravity is entanglement-first, magic-second (M₂/C → 0 as Δ → 0); the best entanglement point Δ = π is a zero-magic point
entanglement-type certificate (vanishes only on 2πℤ)
magic-type certificate (vanishes on ALL of πℤ)
weak-field law
committed point (τ = 2.5 s)
Clifford exception / extremum
A Witness W = 1 + |sin(Δ/2)| — linear in weak field, maximal W = 2 at Δ = π
hold time τ at the committed geometry (450 μm, 250 μm, m = 10⁻¹⁴ kg) [s]
committed operating point
Δ = −211/672 = −0.313988 (τ = 2.5 s)
quantum maximum W = 2
Δ = π: Clifford exception — τ_π = 1680π/211 ≈ 25.0 s
(15.4 s for Family II A); m_π = 3.16×10⁻¹⁴ kg at τ = 2.5 s
(family theorem, not a lab point)
W = 2 on a STABILIZER state: Clifford CZ,
zero nonlocal magic (L-M3 separation)
weak-field law
1 + |Δ|/2
W = 1.156350 (committed, τ = 2.5 s)
separable bound W = 1
W
B Magic floor M₂ = 4 − log₂[4 + 3(1 + cos²Δ)²] — quadratic in weak field, zero at Δ = π
max 1.192645 at Δ = π/2 (CS point)
weak-field law
1.0820 Δ²
M₂(π) = 0:
the best-witness point
is a zero-magic point
M₂ = 0.104445 (committed)
M₂ [bits]
C Stabilizer fidelity F_STAB = (5 + 3|cos Δ|)/8 — equals 1 exactly on πℤ
F_STAB = 0.981666 (committed)
5/8 at Δ = π/2
F_STAB
D Entangling power e_p = (2/9) sin²(Δ/2) — zero only on 2πℤ; 2/9 at Δ = π (Clifford CZ)
2/9: maximum of e_p over ALL two-qubit gates (AM-1)
e_p = 0.005432 (committed; inside (0, 2/9))
registered E1 contours |Δ| = 0.1 … 0.5 (05a §5)
e_p
|Δ| [rad] (Δ = −2KΔx²/[d(d²−Δx²)], K = Gm²τ/ħ)
state of record ψ(Δ) = CZ(Δ)|++⟩ in the CALIBRATED frame (doc 21 RV-D3); Δ < 0 (attractive), axis shows |Δ|; closed forms are evaluations of definitions, values interval-certified (witness_magic_rows.jsonl)

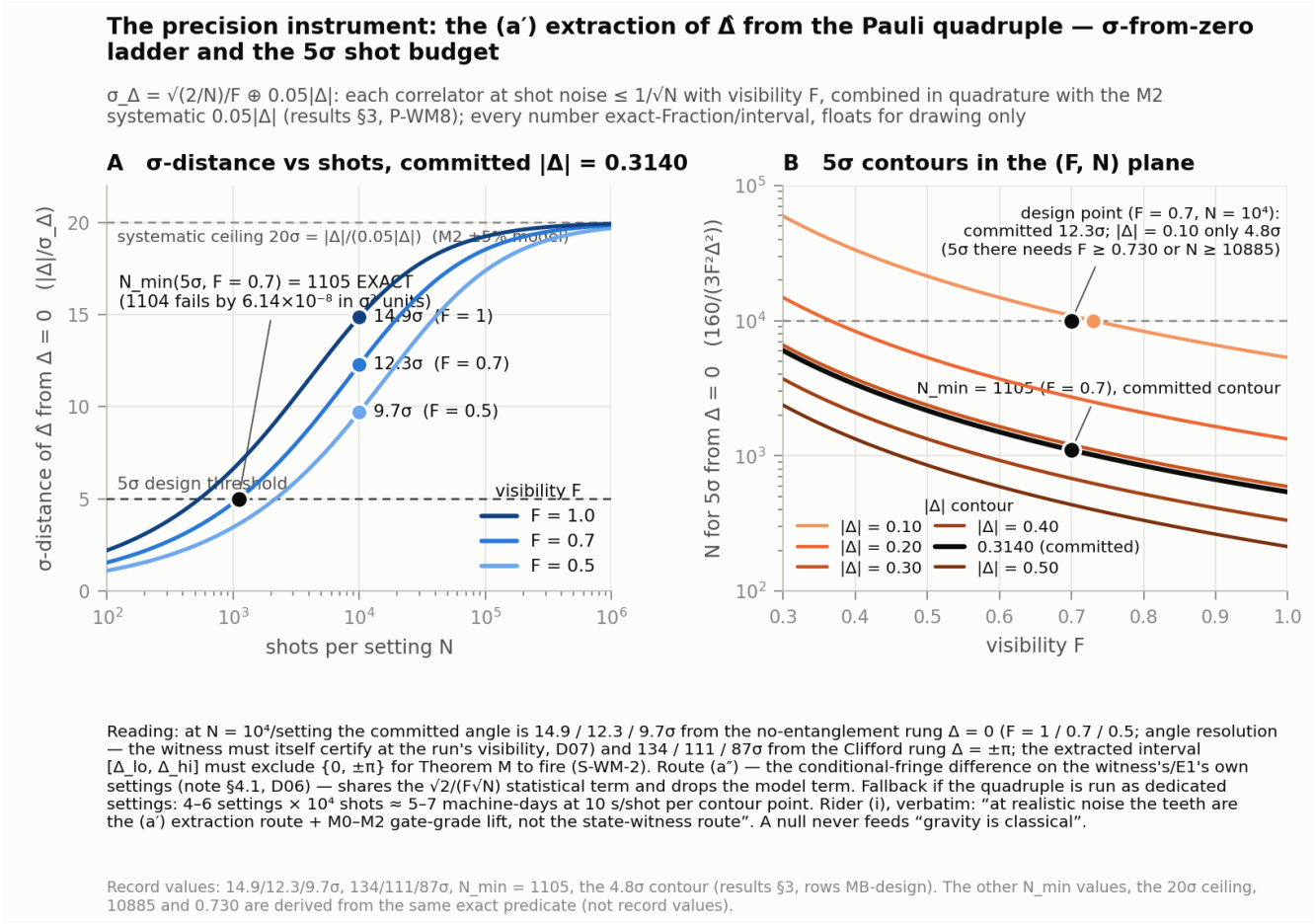


FIG. 2. The precision instrument. (A) σ-distance of the committed angle from $\Delta = 0$ versus shots per setting at $F = 1/0.7/0.5$, with the 5σ threshold, $N_{\min} = 1105$, and the 20σ calibration ceiling of the conservative budget (12); (B) 5σ contours in the (F, N) plane for $|\Delta| = 0.10$–0.50 and the committed angle, with the design point ($F = 0.7$, $N = 10^4$). Distances at sub-threshold visibility are angle-resolution statements; the witness must itself certify at the run's visibility (Table II).

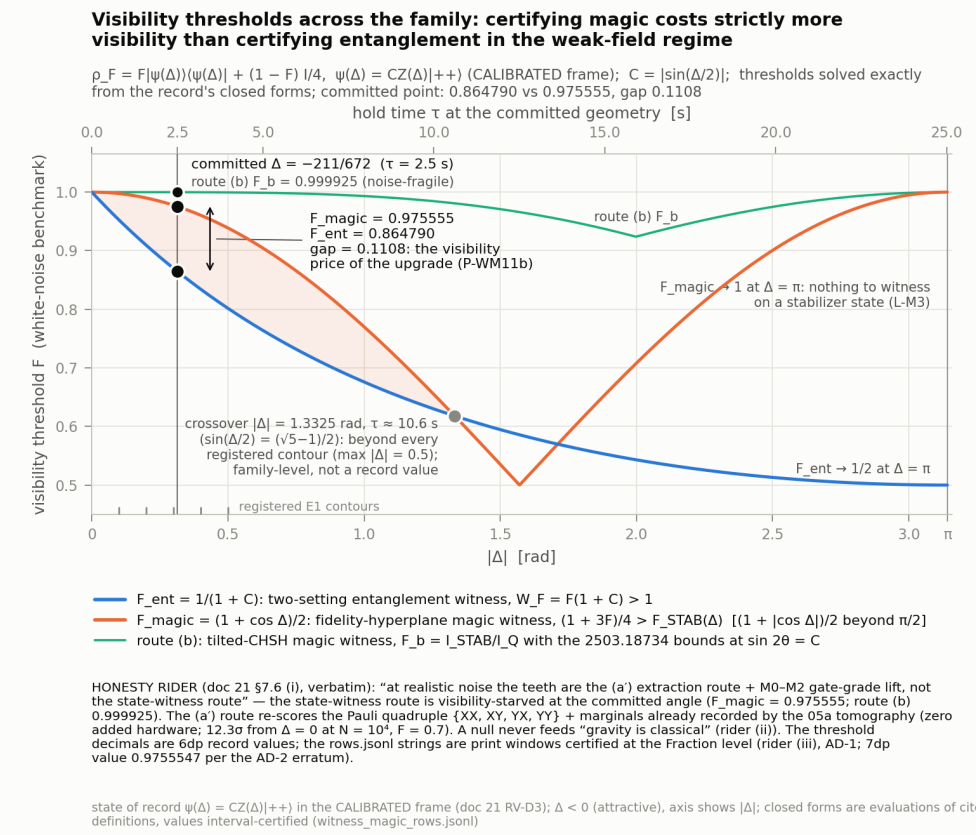


FIG. 3. Visibility thresholds on the white-noise benchmark across the family: F_{ent} (entanglement, two-setting witness), F_{magic} (state-level fidelity-hyperplane magic witness), and the tilted-CHSH threshold F_b; committed values 0.864790 / 0.975555 / 0.999925, gap 0.1108. The crossover at $|\Delta| = 1.3325$ is family-level, beyond every registered contour. The gap prices the state-level route only; the extraction route pays no visibility premium beyond F_{ent} and is the operative instrument inside the window.

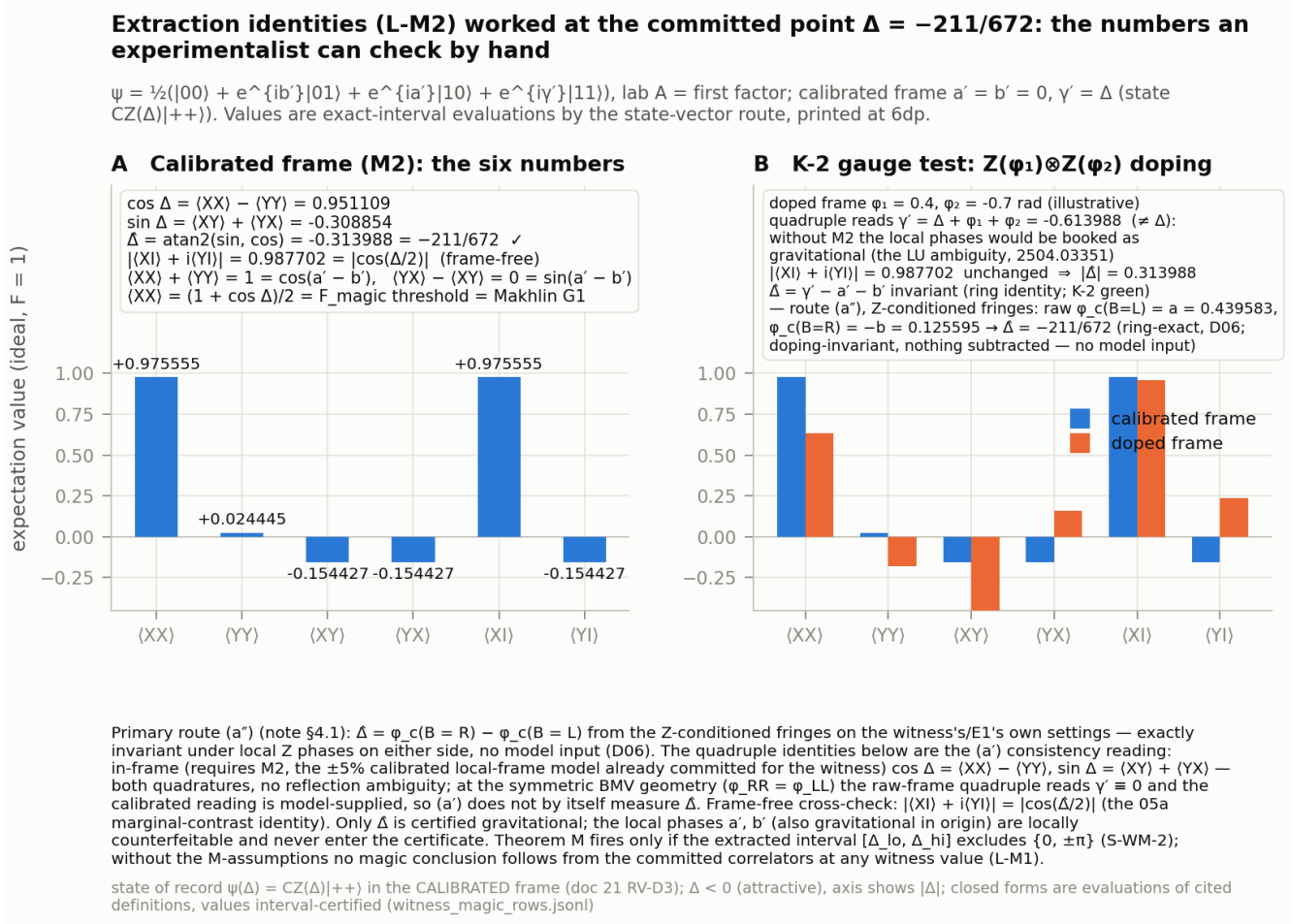


FIG. 4. Extraction identities worked at the committed point (ideal state, $F = 1$). (A) the calibrated-frame expectation values and hand-checkable identities: $\langle XX\rangle = 0.975555$, $\langle YY\rangle = 0.024445$, $\langle XY\rangle = \langle YX\rangle = -0.154427$, $\langle XI\rangle = 0.975555$, $\langle YI\rangle = -0.154427$, $\langle ZZ\rangle = 0$, giving $\cos\hat{\Delta} = 0.951109$, $\sin\hat{\Delta} = -0.308854$, $\hat{\Delta} = -0.313988$, marginal modulus $|\cos(\hat{\Delta}/2)| = 0.987702$. (B) the doping null test: under deliberate local Z phases the quadruple reading moves ($\gamma' = -0.613988$ illustrated) while $\hat{\Delta}$ and the marginal modulus do not; the conditional phases 0.439583 / 0.125595 return $\hat{\Delta} = -211/672$ ring-exactly. With finite visibility every correlator scales by F; the ratio identities are unchanged and only σ_Δ grows. Incidentally $\langle XX\rangle = (1+\cos\Delta)/2$ equals both the invariant G_1 and the threshold F_{magic}.