The closed universe and its internalized boundary: exact constraint-role triviality, the observer repair, and a clock–boundary isomorphism on a lattice pair

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On a finite, exactly solvable pair of lattices, a closed (boundary-deleted) ring and the open chain it is cut from, we compute, in exact integer/rational arithmetic with two-prime certificates and no floating point in any verdict quantity, what imposing the Hamiltonian constraint $H \approx 0$ does to the algebra of observables, and what an adjoined quantum clock restores. Three registered instruments run against a matched quantum-electrodynamics (QED) control on the same closed graphs. (C-A) *Constraint-role triviality:* every populated constrained kernel of the gravity-class models is all-central wherever probed (integer and rational eigenvalues, orders $\nu \leq 2$), with one genuinely structured exception, non-gravitating frozen registers, whose non-central survivors span exactly the compressed register algebra M_2 at rational coupling at three sizes, while QED with H physical keeps the full matrix algebra (M_{256}, M_{4096}) with genuine dynamics on the same graphs, and a role-swap control (QED with H imposed as a constraint) freezes too: triviality is driven by the *constraint role*, and gravity is special only in being forced to put H into that role. (C-B) *The observer repair:* adjoining a matched clock ladder turns the abelian regional algebra nonabelian; the computed shape of the covariant relational algebra is $R_{\mathrm{rel}}^{\mathrm{cov}}(R) \cong \mathrm{alg}(A_{\mathrm{loc}}(R))$, never the registered full matrix algebra, and the repair responds to clock *resolution*, not bare dimension. (C-C) *Time from clocks:* the Page–Wootters conditional-state identity holds exactly (a certified arithmetic identity, tautology-grade given exact projectors), with exact aliasing, detuning, and two-clock tables, and a QED control showing relational time is redundant where H is physical. The flagship result unifies the pair: at $\nu = 0$, in the cut-remembering closure scheme, with the zero-mode anchor re-supplied in the counterterm-site gauge-equivalence class, conjugation by an explicit clock isometry Φ is a $*$-isomorphism taking the closed universe's covariant relational algebra of every probed region onto the open universe's gauge-invariant local algebra of the same region, equivalently, clock-free: the identity holds exactly where the anchored local algebra is stable under the constraint's Heisenberg flow, up to the non-gravitating registers, which neither the constraint killed nor the boundary read. Eight negative controls fail in pre-registered directions and decompose "boundary" into three separately falsifiable ingredients (energy ledger, zero-mode reference, scheme memory of the cut); bounded-below clocks provably never internalize the full boundary. The program tests the constraint mechanism only and adjudicates no path-integral, wormhole, or ensemble statement; every number is reproducible from ancillary scripts, and all refutations of our own pre-registered expectations are reported.

I. INTRODUCTION

A. The closed-universe observables problem

What does a quantum universe without a boundary contain? On an open spatial slice, the gravitational constraints push physical data outward: the ADM energy is a boundary term, holography-of-information arguments locate a copy of the bulk state in any boundary neighborhood [3, 4], and in a preceding series of exact lattice computations we found the same migration mechanism in finite dimensions, order by order, with a matched control [1, 2]. Deleting the boundary deletes the destination. A live continuum dispute asks what is left: arguments from the gravitational path integral conclude that the Hilbert space of a closed universe is one-dimensional or that observables require an explicit observer [5–8, 10], a rebuttal line holds that the apparent triviality is an external-access limitation rather than an internal pathology [9, 11], and observer-rule taxonomies multiply [12, 13].

This paper does not adjudicate that dispute, and the following scoping is binding on every quotation of this work. **Scoping.** *This program tests the constraint mechanism (i) exactly, what imposing $H \approx 0$ does to the algebra of observables on a closed lattice, and says nothing about the path-integral/wormhole/ensemble mechanism (ii) (Refs. [5–8], replica or quantum extremal surface arguments, and the dimension-of-Hilbert-space dispute). The lattices contain*

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no topology change and no wormholes; adjudicating mechanism (ii) is on this program's never-claimable list (Sec. IX). The literature license for this scoping is Wei [15], which separates the two effects in the continuum itself: frozen time comes from summing over metrics (the constraint), one-dimensionality from summing over topologies (wormholes). We compute the first, exactly, with a control.

What the computation *is* load-bearing for, exactly: (a) the folklore constraint claim, "Wheeler–DeWitt alone already kills observables", which circulates inside the dispute but is computed nowhere exactly on either side (approximate constraint-kernel *states* on a finite graph now exist via variational neural-network methods [16, 17]; the physical algebra, its center, and a survivor classification, exactly and with a matched control, still do not); (b) Wei's mechanism-(i) effect, given an exact anchor with a control; and (c) a program-internal retro-validation: whether the boundary-anchoring of the open-lattice papers survives closure or is retro-refuted by it.

B. Results in brief

We work on a pair of finite lattices related by cutting: closed rings (a gravity-class pair lattice $F2c(V)$ and a canonical two-laboratory model $\mathrm{GBc}(N_S)$ with frozen tensor–tensor (TT) registers), and the open chains they are cut from (Sec. II). All arithmetic is exact (integers and rationals; all algebra dimensions certified at two primes; no floating point enters any verdict quantity). Three registered instruments, each with a matched QED control on the same closed graphs, and a two-verifier adversarial record (Sec. VIII):

- **C-A (constraint-role triviality, Sec. III).** In-class closed-universe triviality is exact and *constraint-role-driven*. Closure deletes the ledger's home and forces gravity's H into the constraint role; the populated constrained kernels are all-central wherever probed. QED with H physical on the same closed graphs keeps M_{256} (V=4) and M_{4096} (V=6) with genuine dynamics; QED with H *imposed* as a constraint (the role-swap control) freezes too. One structured exception exists and is the paper's thread: mediation-frozen registers carry non-central survivors spanning exactly the compressed TT-register M_2, at rational coupling, at three sizes, and gravitating that last register splits the doublets and removes them.

- **C-B (the observer repair, Sec. IV).** Adjoining a matched clock ladder repairs the frozen regional algebra from abelian to nonabelian, really, exactly, and contrastively. The registered shape $R_{\mathrm{rel}} = M_{n_T}$ is *refuted*; the computed shape is $R^{\mathrm{cov}}_{\mathrm{rel}}(R) \cong \mathrm{alg}(A_{\mathrm{loc}}(R))$. A degenerate-ladder control shows the repair responds to clock resolution, not dimension; a matter-clock variant shows the repairing readout is the one that does *not* commute with H.

- **C-C (time from clocks, Sec. V).** The Page–Wootters conditional-state identity $\psi_j = U_j\psi_0$ holds exactly at certified integer-split points, tautology-grade given exact projectors, and quoted only as arithmetic integrity, never as a headline, with exact aliasing, detuning, and two-clock distractor tables, and a QED control computing that relational time is redundant where H is physical.

- **The flagship (Sec. VI).** A dressing-map theorem package (T1–T9) plus a ladder-graded computational grid establish, with riders that are part of the sentence: at $\nu = 0$, in the cut-remembering (BC) closure scheme, with the zero-mode anchor re-supplied in the counterterm-site gauge-equivalence class, conjugation by the explicit clock isometry Φ is a $*$-isomorphism from the closed universe's covariant relational algebra of every probed region onto the open universe's gauge-invariant local algebra of the same region, up to the non-gravitating registers (which neither the constraint killed nor the boundary read). Equivalently, clock-free: the identity holds exactly where the anchored seed algebra is ad_H-invariant (criterion T5). Eight negative controls fail in pre-registered directions and decompose "boundary" into three separately falsifiable ingredients, an energy ledger (N2/N3), a zero-mode reference (N1), and scheme memory of the cut site (U4), with a CLPW-shaped no-go: bounded-below clocks never internalize the full boundary (N3).

The verification and prediction record (Sec. VIII) reports every pre-registered outcome, including five refuted registrations of our own, one corrected causal attribution, and one permanently withdrawn row. All scripts, canonical results files, and two independently engineered verifier suites are ancillary (Appendix A).

C. Reading conventions

Verdicts carry certificate classes: [EXACT] (integer/rational identity), [2P] (dimension agreement at two independent primes $> 10^6$, fresh primes $> 2 \cdot 10^6$ for verification), and bracketed provenance labels, [V] marks items first

computed by an independent verifier (all such items are now reproduced in-pipeline by the committed scripts, and re-verified by the ported suites), [U] marks an honest gap with no theorem. "Ladder" grades for the flagship grid: L1 (invariant match) < L2 (constructed isomorphism) < L3 (canonical structure matched); per binding wording obligations, the licensed adverb is "canonically," never bare "naturally," and every L3 label carries the T7 classification of Sec. VI A. All positive observer statements hold at $\nu = 0$ (the chiral point); this rider is part of every such sentence (Sec. IV).

II. THE MODEL PAIR AND THE EXACT METHODOLOGY

A. Closed rings and their open parents

Two model families, inherited verbatim from the open-lattice program [1, 2] so that closure is the only new ingredient:

$F2c(V)$ *(gravity-class pair ring).* V sites on a ring, one binary matter pair variable per site (2^V configurations; $d = 2^V$ for $V = 4$, i.e. $d = 16$, and $d = 64$ at $V = 6$). The gravitational sector is a slaving cascade: order-k potentials are sourced by the matter distribution and slaved to it; on the open chain the cascade solves outward and deposits an additive, boundary-readable ledger $W_{\mathrm{tot}}^{(k)}$ (the ADM analogue of the migration papers). The constrained Hamiltonian H acts on matter configurations with the slaved self-energy diagonal plus hopping; $\nu \in \{0, 1, 2\}$ grades the self-sourcing order retained (at $\nu = 0$, a chiral symmetry $\Gamma H \Gamma = -H$ holds exactly).

GBc(N_S) *(two laboratories plus frozen registers).* The canonical two-lab model of the open program at $d = 18$: two laboratories with pattern variables, a mediating sector, and tensor–tensor (TT) registers that no constraint sources and no boundary reads, "frozen" content. Coupling patterns (a, b, c) switch lab/mediation/register gravitation; $N_S \in \{12, 16, 20\}$ scales the site count.

Closure schemes. Deleting the collar leaves a scheme choice for the ring's zero mode, and the choice is physics, not convention: **BC** ("boundary-condition" or cut-remembering closure) keeps the open chain's kinematic space verbatim and pins the solvability counterterm at the cut site, the ring remembers where the boundary was; **SD** ("self-dual" or homogeneous closure) distributes the counterterm democratically, no site is special. Verdicts are required scheme-invariant unless the scheme difference is itself the reported result (it will be: Sec. VI).

The QED control. On the same closed graphs, a matched compact-QED-class model with Gauss constraints only and H *physical*; and a *role-swap* control in which the same QED H is imposed as a constraint. The pair (control, role-swap) separates "gravity-form" from "constraint-role" explanations of anything C-A finds. Gauge-invariant kinematics for finite-group lattice models are prior art [36, 37]; what is new here is the constrained *algebra* computation with the matched pair.

B. Constraints, zero modes, and handling labels

Closure forces $Q_{\mathrm{tot}} = 0$ (QED) and $E_{\mathrm{tot}} = 0$ at the slaving level (gravity): on the ring the order-k cascade source has $\Sigma\varepsilon > 0$, so slaving is solvable only after projecting the source zero mode, and the open lattice's boundary-migrating ledger $W_{\mathrm{tot}}^{(k)}$ becomes the solvability *obstruction*, absorbed into a global counterterm, a cosmological ledger with no local home (Sec. III A). The Wheeler–DeWitt analogue is the global constraint $H \approx 0$ imposed on the slaved theory; we compute $\ker(H - \lambda)$ across an eigenvalue grid ($\lambda = 0$, the WDW row, always reported first).

Zero-mode handling is labeled per cell and contrasts are made only at identical labels: **Z1** (symmetry-protected $\nu = 0$ point, $\Gamma H \Gamma = -H$ exact); **Z2** (integer-λ kernel scan within sectors, Gershgorin-complete windows); **Z2b** (exact characteristic polynomials over $\mathbb{Z}/\mathbb{Q}$, Faddeev–LeVerrier on scaled integer matrices, complete rational-eigenvalue extraction with exact multiplicities by synthetic division, factor product re-certified, residual certified square-free); **Z3** (ε-window Sturm counts, dimension-only, $d \leq 18$).

C. Exactness discipline

Every verdict quantity is an integer or a rational. Algebra dimensions and commutants are certified at two primes (1000003/999983 for the main run; clock DFT primes $1000033/1000193 \equiv 1 \bmod 32$); the verification suites re-derive at fresh primes above $2 \cdot 10^6$ with independently engineered solvers (Appendix A). On populated kernels we compute the strong Dirac algebra: the compressed algebra generated by PAP for regional gauge-invariant generator sets $A \in A_{\mathrm{loc}}(R)$, its linear span, generated algebra, and center per sector ("lin/alg/cen"), with a frozen survivor taxonomy registered before the run: TRIVIAL-BY-ACCIDENT (empty kernel at scanned couplings, a spectral accident,

never quoted as physics), SUBSTANTIVE-TRIVIAL (populated, all-central at $n \geq 2$), STRUCTURED (non-central survivors localized on registered frozen content), NONTRIVIAL (anything else, the falsifier, which never fired). A dihedral/orbit symmetry classifier guards the NONTRIVIAL label: symmetry-forced orbit content can never raise it.

III. C-A: CONSTRAINT-ROLE TRIVIALITY

A. The vanished ledger re-enters through local data

Before any kernel is imposed, closure itself does something exact. The projected cascade is solvable at orders ≤ 2 on every admissible configuration in every gridded cell; the obstruction, the open ledger $W_{\rm tot}^{(k)}$, is absorbed into the global counterterm. The kinematic question is whether the vanished ledger re-enters through local data. It does, completely: in all 11 cells ($F2c$ $V = 4/6$, both schemes, all discretizations; GBc $N_S = 12$), no two admissible configurations share all projected local profiles while differing in any $W_{\rm tot}^{(k)}$ ($h_{\rm joint} = 1$), and per-order grouping already separates ($h_k = (1, 1)$ everywhere in range; the registered dissent that joint data would be needed is refuted). The QED contrast is analytic: the cascade source vanishes identically at $k \geq 1$ (no self-sourcing in the linear theory), so no obstruction and no ledger exist beyond $Q_{\rm tot} = 0$.

B. Kernel scans and the survivor taxonomy

Table I reports the populated kernels per handling label. Honesty first, as registered: the dominant outcome on the integer grid at $\nu \geq 1$ is *emptiness*, TRIVIAL-BY-ACCIDENT, a spectral accident carrying no physics weight, never aggregated into any headline. (At $V = 4$ BC SITE $\nu = 1$ the verifier upgraded this to theorem strength: the characteristic polynomial is irreducible of degree 16, so the spectrum is simple and every admissible single-eigenvalue handling yields a scalar compressed algebra.) The physics lives in the populated cells:

Regional anatomy. On every populated $F2c$ kernel at every handling label, the $\nu = 0$ binomial ladders, the SD integer cells, the rational $n = 3$ triple, the regional strong Dirac algebras $A_S(L0)$ and $A_S(L0{+}L1)$ collapse to scalars per sector: SUBSTANTIVE-TRIVIAL. The GBc $(1, 0, 1)$ cell is all-central at $n = 4$ (the second substantive $n \geq 2$ exhibit). Two registered expectations of our own *failed* here and are reported as refuted in Sec. VIII: the registered "rich non-central regional algebra at the $\nu = 0$ point" is regionally refuted ($A_S(L0)|_{\rm ker}$ is abelian in both gauges, span$\{I, X_{L0}\}$ at the site anchor, scalars at the mean), and global non-centrality is bookkeeping, not physics: globally $A_S(\text{global})|_{\rm ker} = M_{\dim \ker}$ for every populated cell by the compression theorem, *global non-centrality is kernel degeneracy.* All C-A physics sentences are regional.

C. The structured exception: frozen registers

One class of cells breaks the all-central pattern, and it is not noise: the mediation-frozen GBc cells. At $(1, 0, 0)$, $\lambda \in \{-1, 0, 1\}$ (region $L2{+}$TT: lin 12 / alg 4 / cen 1) and, decisively, at the H_A $(1, 1, 0)$ rational doublets $\lambda =$ 767/9, 749/4, 1509/5 at $N_S = 12/16/20$, the non-central survivors span *exactly* the compressed TT-register M_2: register-image rank 4, exact span equality, stable under adjoining all 12 strong operators, at $\nu = 1$ (rational λ), at three sizes. This is the run's own in-pipeline classification (a register-M_2 test; the original probe tested containment in alg$\{\sigma_{\rm TT}^x\}$ and mislabeled these cells, a recorded defect, repaired at source). The closed universe's only observables are its non-gravitating registers. And the control on the control: gravitating that last register, the $(1, 0, 1)$ pattern, which adds the register term ω, splits the TT doublets and removes even those survivors. *Gravitating the last register removes the last observables.*

D. The QED contrast and the role-swap verdict

On the same closed graphs, with the same handling labels: QED with H physical keeps M_{256} ($V = 4$) and M_{4096} ($V = 6$), with $[H_{\rm QED}, X_{L0}]$ having 16/64 nonzero entries, genuine dynamics, no hidden triviality (single-entry basis verified). QED with H *imposed* freezes: the role-swap SD kernel $\{4{:}2\}$ carries no non-central survivors, all-central, like gravity, and the BC rational cell $(5x - 576)^6$ (dim 6) carries only certified D_V-orbit survivors, the registered symmetry-diagnostic branch (ROLE-SWAP-DEGENERATE: a control diagnostic, never a gravity verdict, and barred from NONTRIVIAL by the orbit guard). The mechanism verdict:

TABLE I. C-A kernel scans (populated $\{\lambda : \dim\ker\}$ per handling label; $\lambda = 0$ is the WDW row). All quoted dimensions two-prime certified; [V] = first found by the independent verifier, now in-pipeline (Z2b handling).

family/cell	handling	populated cells	taxonomy
$F2c$ V=4/6/8 BC+SD, all discs, ν=0	Z1	λ=0: dim = $\binom{V}{V/2}$ = 6/20/70; V=10/12: 252/924	regional anatomy below
$F2c$ V=4 BC, all discs, ν=1, 2	Z2	empty at every scanned λ	accident (theorem-strength at SITE ν=1)
$F2c$ V=4 SD SITE ν=1/ν=2	Z2	{10: 1} / {20: 1}	all-central (dim-1 vacuous)
$F2c$ V=4 SD SPLITL ν=1/ν=2	Z2	{8: 1} / {16: 1}	all-central (dim-1 vacuous)
$F2c$ V=4 SD SITE ν=1	Z2b [V]	$(20x-133)^3$: λ=133/20, dim 3	**SUBSTANTIVE-TRIVIAL** (n=3, all-central)
$F2c$ V=6 BC+SD $\nu\geq1$; V=8 BC $\nu\geq1$ and SD ν=1	Z2	empty (V=8 SD ν=1: verifier-extended; V=8 SD ν=2 not scanned)	accident
GBc $(1,1,1), (2,1,1), (1,2,3), (1,1,0)$, N_S=12/16/20	Z2	empty at every integer λ	accident
GBc H_A $(1,1,0)$	Z2b [V]	$(9x-767)^2$, $(4x-749)^2$, $(5x-1509)^2$: dim 2 at N_S=12/16/20	**STRUCTURED** (TT doublets, Sec. III C)
GBc H_B $(1,1,1)$	Z2b [V]	767/9, 776/9: dim 1 (splitting to 749/4+753/4, 1509/5+1514/5 under $+\omega$)	all-central
GBc $(1,0,0)$ / $(1,0,1)$	Z1-adjacent	{0:2, ±1:2} / {0:4, 1:4, −1:2, 2:2}	STRUCTURED / SUBSTANTIVE-TRIVIAL
role-swap QED V=4	Z2/Z2b	BC integer-empty, $(5x-576)^6$ dim 6; SD {4: 2}, $(5x-19)^3$	SD all-central; BC D_V-orbit (ROLE-SWAP-DEGENERATE)
QED V=4/6 Gauss-only (control)	,	H physical, never imposed: full M_{16} (dim 256) / M_{64} (dim 4096)	dynamical: $[H_{\rm QED}, X_{L0}]$ nnz = 16/64

In-class closed-universe triviality is constraint-role-driven. Closing the lattice deletes the ADM ledger's home, so gravity's H is forced into the constraint role; QED's H is forbidden it (the cascade source vanishes identically at $k \geq 1$, conservation is a global symmetry, not gauge). The role-swap control seals the attribution: put QED's H into the constraint role and it freezes too.

The residual gravity fingerprint is licensed only in multiplicity form, at $V = 4$ only, with no trend claim: gravity's near-injective self-sourced diagonal leaves dim-1 kernels where the role-swap's dihedral-orbit-degenerate Coulomb diagonal leaves dim-2 (the dim-2 role-swap kernel is a certified D_V-orbit doublet under the symmetry classifier; the populated SD gravity integer cells are dim-1 dihedral-scalar). The registered strong form of this dissent, non-central orbit survivors in the gravity-matched comparison, is refuted at the registered SD cell; the BC diagnostic branch above is recorded and quarantined by the orbit guard.

Two further controls: ring duality $A(R) = \text{commutant}(A(\bar{R}))$ holds in all four probe cells (8/8, 4/4 on $F2c$; 9/9, 10/10 on GBc); and the taxonomy class is invariant across zero-mode conventions ($\Sigma\pi = 0$ vs $\pi(0) = 0$). Scheme-level *integer* differences between BC and SD populated cells are real and legitimately scheme-dependent; classified verdicts are scheme-invariant. Finally the retro-validation: the closed lattice trivializes exactly where the open one migrated. The boundary was load-bearing; the open-program anchoring is not retro-refuted but confirmed.

IV. C-B: THE OBSERVER REPAIR

Binding rider. Every statement in this and the next section holds at the chiral point $\nu = 0$. At $\nu \geq 1$ the matched kernel is empty in-window at these sizes (Sec. V, K3 as corrected), so no self-gravitating observer statement of any kind is made; the single instantiable $\nu = 1$ observer cell appears only as negative control N7 (Sec. VI B).

A. Construction

Adjoin a clock ladder $H_C = -\mathrm{diag}(0, \ldots, T-1)$ (or the symmetric window variant) and impose $H_{\mathrm{tot}} = H \otimes I + I \otimes H_C \approx 0$. The kernel is the matched-integer-level space $\ker H_{\mathrm{tot}} = \bigoplus_E \ker(H - E) \otimes |\tau{=}E\rangle$, the finite shadow of "ideal clocks need continuous spectrum," stated as such. On it we compute the covariant relational algebra: the algebra generated by the clock-shift dressings $B_{A,k} = P\,(A \otimes S^k)\,P$ for regional generators A and all shifts k (verified to coincide exactly with the phase-state family), against a literal energy-basis projector family kept as the registered comparison.

B. The repair is real; the registered shape was wrong

At the registered site-anchored cell ($V = 4$, $T = 8$ low window): the clockless regional algebra $A_S(L0)|_{\ker}$ is abelian (lin 2); the band-dressed covariant $R_{\mathrm{rel}}^{\mathrm{cov}}$ has dimension 5, nonabelian, on $n_T = 11$ matched levels. The repair exists, exactly and contrastively (registered prediction confirmed). The kernel staircase confirms the registered law $[(T, n_T)$: $(2,6), (4,10), (8,11), (16,11), (32,11)]$, saturating at the certified level count; the symmetric window reaches $n_T = 16 = d$ at $T = 16$.

The registered *shape* claim $R_{\mathrm{rel}} = M_{n_T}$ is refuted: the full-matrix flag is false at every computed T. The computed shape is the paper's pivot:

$$R_{\mathrm{rel}}^{\mathrm{cov}}(R) \;\cong\; \mathrm{alg}(A_{\mathrm{loc}}(R)), \tag{1}$$

dimension-for-dimension at the full-window site-anchored cell ($\dim R_{\mathrm{rel}}^{\mathrm{cov}}(L0) = 4 = \dim \mathrm{alg}(A_{\mathrm{loc}}(L0))$), robust at unseen $T = 32$ and $T = 64$ (dimension stays exactly 4; n_T stays 11/16) [V]. The relational algebra of a region is the open lattice's gauge-invariant local algebra of that region, the observation the flagship section turns into an isomorphism with an explicit map.

Three sharpenings, each from a registered control:

- *Resolution, not dimension.* The degenerate-ladder control doubles every clock level: the kernel doubles (22, 84) but $R_{\mathrm{rel}}^{\mathrm{cov}}$ is unchanged ($\otimes I_2$, the dressing blind to the copy index). The repair responds to the clock's resolution of the level set, never to bare Hilbert-space dimension.
- *The anchor story, corrected.* The registered candidate "the observer needs a gauge anchor" is refuted as a nonabelianness claim: the mean-gauge covariant repair is nonabelian too (dim 34 at $T = 8$; dim 80 at $T = 16$ symmetric). What is anchor-conditional is not the repair but the *identity* (1): at the mean gauge $R_{\mathrm{rel}}^{\mathrm{cov}}(L0) = 80$ overshoots $\mathrm{alg}(A_{\mathrm{loc}}(L0)) = 8$; only the site anchor matches $4 = 4$. The flagship section explains this (the anchor is the boundary's scar).
- *A bad clock is one aligned with the constraint.* The matter-clock variant (an internal pair as clock, no new register) repairs through the pattern-pointer (σ^z) readout, dim 6, nonabelian, i.e. the readout that does *not* commute with H; the H-commuting σ^x readout is spectral and yields only the abelian family (dim 4).

The literal projector family, registered as the abelian distractor, is block-diagonal in clock time and is abelian at the site anchor but nonabelian at the mean gauge (multiplicity-carried), the registered blanket "abelian everywhere" is refuted as stated, and both registrations are quoted. QED adjunction adds nothing (the control is already M_d with physical dynamics; carried by the control rows). Disclosed caps: $V = 6$ covariant closures capped at $n_T > 24$ (commutant dimensions 225/356/651 reported, closure not certified, no shape claim quoted from those rows); the $V = 6$ generator family is reduced, disclosed per row.

V. C-C: TIME FROM CLOCKS

All at $\nu = 0$; the Sec. IV rider is binding.

The conditional-state identity, with its honest grade. At certified integer-split points, with rational spectral projectors and mod-p DFT at two clock primes, the Page–Wootters conditional state obeys $\psi_j = U_j \psi_0$ for all j, $V \in \{4, 6\}$, $T \in \{4, 8\}$, as an identity in the group ring $\mathbb{Z}[\mathbb{Z}_T]$. Both verifiers grade this tautology-adjacent given exact projectors, the kernel construction post-selects resonance, and it never headlines alone: it is the trinity reduction theorem [18, 19] in exact arithmetic; its value is arithmetic integrity of everything else in the pipeline.

Knob tables (all exact). Fidelity versus window is a step, not a continuous clock-quality curve (registered exemption; the clock-quality literature [42, 43] owns the latter): exact dropped-weight rationals, j-independent, $V = 4$: $3/13 \to$

$7/13 \to 9/13 \to 1$ at $T = 16 = \text{spread} + 1$; $V = 6$: $3/29 \to 15/29 \to 1$ (per-row kernel representatives disclosed, the F values depend on the registered representative). Aliasing: at $T = 4$ under a spread-8 ladder the collision pairs $E \equiv E' \pmod T = [(0,4),(2,6)]$ develop no relative phase at any j (weights $\{0{:}8, 2{:}16, 4{:}32, 6{:}64\}$). Detuning (+diag/2) thins the matched kernel $5 \to 1$ populated levels, with exact conditional dynamics on the survivor. Two clocks (Kuchař's multiple-choice shadow): matched non-interacting clocks condition to the same flow; the registered naive-double-conditioning distractor weights level E by split multiplicity $m_E = \{0{:}1, 2{:}3, 4{:}5\}$, distortion exact unless m_E is constant.

The gravitating clock, corrected. The registered K3 cell (clock sourcing gravity) produced a table of zero survivors at every coupling τ; the registered causal reading ("backreaction destroys the resonance") was confounded and is withdrawn: $H^{(\tau=0)} \equiv H_B(1,1,1)$ exactly, whose unclocked scan is already empty at every integer λ. The corrected verdict, verbatim by both verifiers: *no matched resonance in-window with or without clock sourcing, K3 not instantiable at these sizes.* The interacting-clock deviation form of Smith–Ahmadi [40] remains the registered target for sizes where multi-τ kernels survive.

The QED control. H_{QED} is physical and clocklessly dynamical ($[H_{\mathrm{QED}}, X_{L0}]$ nnz $= 16$); the identical PW machinery on the same hop operator reconstructs exactly that flow by the same arithmetic. Licensed sentence: relational time is redundant where H is physical, computed as a pair of integers on the same pipeline.

Kuchař ledger (nothing beyond). K1 frozen formalism: answered exactly (frozen without a clock, dynamical relationally with one); K2: computed table; K3: not instantiable as corrected; K4: moot in finite dimensions (stated); K5: out of class (never claimed); K6: partial via the matter clock.

VI. THE FLAGSHIP: THE CLOCK IS THE BOUNDARY REBUILT INSIDE

Sections III–V computed three facts that want one mechanism: the closed constraint kills exactly what the open boundary read (C-A + the open program); a clock restores a regional algebra of exactly the open lattice's local shape (C-B); and the restoration is anchor-conditional. This section states the mechanism at theorem grade where it is a theorem, and at certified-grid grade where it is empirical, with the failure taxonomy as part of the result.

A. The dressing map and its hypotheses

Definition 1 (dressing map and covariant bands)**.** With $\mathrm{spec}(H) = \{E_1, \ldots, E_m\}$ certified and Π_E the exact spectral projectors, define

$$\Phi : \mathcal{H}_{\mathrm{kin}} \to \ker(H_{\mathrm{tot}}), \qquad \Phi\psi = \sum_E (\Pi_E \psi) \otimes |\tau{=}E\rangle, \qquad \chi := \Phi^\dagger(\,\cdot\,)\Phi, \tag{2}$$

and the clock-shift bands $B_{A,k} = P(A \otimes S^k)P$, so that $R^{\mathrm{cov}}_{\mathrm{rel}}(R) = \mathrm{alg}\{B_{A,k}\}$. On the system side, $A_k = \sum_E \Pi_{E+k} A \Pi_E$ denotes the k-th band of A in the energy grading ($[H, A_k] = kA_k$, $\sum_k A_k = A$). The zero-mode anchor enters only through the seed generators $A_{\mathrm{loc}}(R)$, never through the map.

Hypotheses, named once: **H-pop** (the matched kernel is populated, the only place $\nu = 0$ enters; at $\nu \geq 1$ integer couplings it fails at these sizes, which is instantiability, never an isomorphism failure); **H-cov** (window covers the spectrum); **H-alias** ($T \geq 2 \cdot \text{spread} + 1$, the corrected sufficient bound; the naive bound $T > E_{\max} - E_{\min}$ is insufficient, and the registered instruments U1 (T=16, glued difference class $\{8\}$), U3 (glued $\{4\}$) and U2 (five glued classes) are *edge-aliased*: at those cells the conclusions below are carried by direct two-sided membership certificates, not by the theorem; theorem grade holds verbatim at the unaliased $T = 32/64$ cells); **H-mult** (simple clock levels; degeneracy handled by T8).

Theorem 1 (dressing-map package; proofs in the ancillary note)**.** *All parts verifier-checked [V]; N8 has no theorem [U].*

T1. *Under H-pop + H-cov:* $\Phi^\dagger\Phi = I$, $\Phi\Phi^\dagger = P$, *and* χ *is a unital* $*$*-isomorphism* $B(\ker H_{\mathrm{tot}}) \to B(\mathcal{H}_{\mathrm{kin}})$ *preserving products, adjoints, spectra, centers, central idempotents, block decompositions, and commutants; injectivity of* χ *on subalgebras is unconditional.*

T2. $\chi(B_{A,k}) = \sum_{E'-E \equiv k} \Pi_{E'} A \Pi_E$*; under H-cov + H-alias this equals* A_k*, and* $\chi(R^{\mathrm{cov}}_{\mathrm{rel}}(R)) = \mathrm{alg}\{A_k\} =: \mathfrak{D}(A_{\mathrm{loc}}(R))$*, well-defined, computable, clock-free.*

T3. $\mathfrak{D}(A_{\mathrm{loc}}(R))$ *is precisely the* ad_H*-closure of* $\mathrm{alg}(A_{\mathrm{loc}}(R))$ *(Vandermonde on distinct band indices); membership certificates run by iterated integer commutators, no cyclotomics needed.*

TABLE II. The unification grid (outcome O1 with an F-7 scheme restriction: all positive cells BC-only). "a/b" = closed relational vs open local dimensions; brackets = certificate class. Every L3 carries the T7 classification; center-dim-1 idempotent legs are vacuous and so read.

axis	cells	verdict
U1 ($F2c$ V=4 BC site-anchor)	$L0$, $L1$, $L0{+}L1$, comp($L0$); unseen comp($L0{+}L1$) [V]; T=32/64	**L3 [EXACT]**: 4/4, 4/4, 16/16, 64/64 (16/16 [V]); centers 1/1; blocks equal
U2 (V=6, full generator bases)	$L0$, $L1$, $L0{+}L1$, comp($L0$) d=64	**L3 [2P+exact-subset; comp 2P capped, disclosed]**: 4/4, 4/4, 16/16, 1024/1024; centers 1/1
U3 (GBc N_S=12 TT-bearing; N_S=16 [V])	$L1$, $L2{+}$TT, comp($L1$)	**L3 [EXACT]**: 10/10, 40/40, 40/40; centers 4/4; element-wise block/charge match, the axis carrying the real naturality content
U1g/U2g/U3g (globals)	global algebra	**L2 [EXACT]** 256, 4096, 324: structurally forced ($A_{\mathrm{kin}}(\mathrm{global}) = M_d$), consistency, not evidence
U4 (SD scheme spot)	$L0$ site-anchor	**FAIL** (the F-7 flip): overshoot 80 vs 8, identical to unanchored numbers, no counterterm site ⇒ no scar (Sec. VI C)

T4. *Under H-cov + H-alias:* $\mathrm{alg}(A_{\mathrm{loc}}(R)) \subseteq \chi(R^{\mathrm{cov}}_{\mathrm{rel}}(R))$ *always. Corollaries: at full-window non-aliased cells, a dimension match forces algebra equality (the L1→L2 upgrade, with no dimension-only shortcut at aliased cells, ever); and full-window failure can only be overshoot, undershoot only via partial window, aliasing, or single-level collapse, every registered failure direction fixed in advance.*

T5. *(The clock-free criterion.) Under H-cov + H-alias:*

$$\chi(R^{\mathrm{cov}}_{\mathrm{rel}}(R)) = \mathrm{alg}(A_{\mathrm{loc}}(R)) \iff \mathrm{alg}(A_{\mathrm{loc}}(R)) \text{ is } \mathrm{ad}_H\text{-invariant.} \tag{3}$$

The clock is eliminable: the identity holds at a cell precisely when the anchored seed is stable under the constraint's discrete Heisenberg flow, a clock-free property of open-side data, verified independently on two engineering stacks.

T6. *(One closure functor, two seeds; the unconditional half.) The closed-plus-clock construction is seed* $\mapsto \mathrm{ad}_H$*-closure evaluated on* $A_{\mathrm{loc}}(R)$*; the open program's boundary-generated algebra* $D(n)$ *is the same functor on the collar seed. Adjoining a matched full-window clock implements on the closed ring exactly the operation boundary dynamics implements on the open chain; the sides differ only in their seeds.*

T7. *(Naturality classification, binding on every L3 label.) (a) automatic from T1/globality: centers, central idempotents, block multisets, commutants, region-inclusion compatibility, duality transport (pipeline consistency, not independent physics); (b) canonical: conserved-charge transport* ($[Q,H]=0 \Rightarrow \chi(P(Q\otimes I)P) = Q$)*; (c) tautology-adjacent: PW-flow intertwining (at* $\nu = 0$ *all flows are trivial, kinematic-identity checks, never dynamical evidence); (d) genuinely empirical: the element-wise named-charge match, carried by U3's three regions (centers of dimension 4) plus B1. Idempotent-matching legs at center-dimension-1 cells are vacuous and so marked.*

T8. *Degenerate clock ladders give* $R^{\mathrm{cov}}_{\mathrm{rel}} = (\text{nondegenerate } R^{\mathrm{cov}}_{\mathrm{rel}}) \otimes I_\mu$*: multiplicity blindness (control N5 forced in advance).*

T9. *Single populated in-window level ⇒ all band offsets vanish and* $\chi(R^{\mathrm{cov}}_{\mathrm{rel}}(R))$ *is the clockless compressed algebra: a one-letter energy alphabet supports no ledger (control N7 forced in advance).*

B. The ladder-graded grid

Certification ladder: L1 invariant match < L2 constructed isomorphism (explicit χ, two-sided membership) < L3 canonical structure matched per T7. Table II is the outcome and Table III the negative pattern; the run is 27/27 checks, exit 0, with a full-grid independent port at reserved primes 2000039/2000081 producing zero verdict differences, and a second committed verifier suite re-deriving spectra, isometry certificates, seed/closure dimensions, and the clock-built band algebras at fresh primes (Appendix A).

TABLE III. Negative controls: every failure in its pre-registered direction [V at fresh primes, plus unseen siblings]. "rel vs kin" = closed relational vs open local dimension.

control	construction	outcome
N1 (mean anchor)	same map, same clock, same window	one-sided overshoot everywhere: 80/8, 80/8, 160/18, 256/40 (V=4); 25/9, 100/36 (GBc, incl. the initially omitted comp($L1$) cell, disclosed [V]); ($\supseteq$) exact everywhere; every band piece incl. k=0 leaves the seed algebra
N2 (partial window)	T=8 low	undershoot(kernel): rel 5 vs 4; n_T 11 < 16; missing levels $[-4,-2]$, deficit = out-of-window bands, printable in advance
N3 (bounded-below ladder)	low window, any T	structural undershoot: saturation 11 < 16 at every T, the CLPW-shaped no-go (Sec. VI F)
N4 (aliased gluing)	T=4 under spread 8	undershoot(kernel): rel 6; n_T 10 < 16; glued classes as pre-printed
N5 (degenerate ladder)	τ-doubling	PASS-with-multiplicity: $R^{\mathrm{cov}}_{\mathrm{rel}}$ unchanged, $\otimes I_2$ (T8-forced), resolution, not dimension
N6 ($\nu\geq1$ integer cells)	V=4/N_S=12	NOT-INSTANTIABLE (empty matched kernels; never aggregated)
N7 (first ν=1 observer cell)	H_A doublet λ=767/9, ×9-scaled clock, T=1601	FAIL by single-level collapse (T9-forced): n_T=2, all offsets k=0, R_{rel} = the clockless compressed TT M_2 (dim 4) vs 36, an instantiability artifact of the one-letter alphabet, *not* evidence that self-gravitation breaks the identity; ν-rider mandatory
N8 (matter clock)	σ^z-readout family	NO-ISO, repair without identity (rel 6 vs kin 4); no theorem [U], honest gap

C. The boundary's scar: why the anchor

Why does the identity hold at the counterterm-site anchor and fail at the mean gauge? The bridge computation B1 compares the closed ring's local algebras with the open cut-chain's, region by region, on independent engines: at the site anchor the ring local algebras are *verbatim* the open chain's (generator sets identical in all four regions; at profile level the ring−chain difference is one configuration-independent counterterm vector); the mean gauge matches nowhere (five configuration-dependent tail vectors); SD matches nowhere. *The anchor is the boundary's scar*: the site-anchored gauge is the one the open collar's boundary condition induces, so the anchor-conditionality of the identity is explained, not merely observed.

The scheme leg was then closed by a dedicated sweep rather than left as an inference: under SD, *every* zero-mode anchor fails, all 42 cells (site anchors $\pi(s)=0$ for $s=0..19$ plus the mean, regions $L0$ and $L1$) violate the clock-free criterion (3), with seed algebras 8/12/16 flow-closing to 80/160/256, "no SD anchor works" is computed, not inferred. The BC control sweep produced a refinement we did not expect and report as found: the identity holds at every site anchor *except* the four pattern-sensitive lab-middle sites $\{2,7,12,17\}$ (where the anchored zero mode is itself dynamical: $s=2$ collapses the seed $2\to4$; $s=7/12/17$ inflate it $8\to16$) and the mean gauge, and every passing anchor carries the same seed algebra as the counterterm-site anchor (union-algebra certificate, both primes). The scar licenses an anchor *equivalence class*: "anchored at the counterterm/cut site" is to be read as "anchored in the counterterm-site gauge-equivalence class," with the sufficiency of the literal counterterm site unchanged.

D. Frozen registers: the rider that closes the loop

The bridge leg B2 splits the last rider: the closed side carries the TT register through Φ (forced by T4 at full window, an outcome otherwise would have been a stop-the-line arithmetic event), while the open boundary's $D(n)$ never reads TT at any order (an anchor of the open program, inherited here, not new evidence). The two sides differ exactly on the frozen registers because the *seeds* differ there: the clock's seed is local kinematics (contains the non-gravitating registers); the boundary's seed is the collar ledger (never reads them). This derives the flagship's final rider and closes the loop with C-A: the only observables the closed constraint leaves (Sec. III C) are exactly the registers the clock–boundary identity exempts. The exception recurs in four independent computations (the C-A STRUCTURED row, U3's carried register, B2's split, N7's collapse onto exactly the TT M_2) and would self-delete under exact universality of the gravitational coupling.

E. The licensed statement

The following is the flagship sentence at exactly its licensed strength, every rider fused, quoted no wider anywhere in this paper; its hypotheses and certificate grades are those of Theorem 1 and Table II:

> *"In this finite slaved-lattice class, at $\nu = 0$, in the boundary-condition (cut-remembering) closure scheme, with the zero-mode anchor re-supplied at the counterterm/cut site, conjugation by the explicit clock isometry Φ is a $*$-isomorphism taking the closed universe's covariant relational algebra of every probed region onto the open universe's gauge-invariant local algebra of the same region, canonically: one global map, restriction-compatible, matching central idempotents and named-charge blocks element-wise where centers are nontrivial, equivalently, clock-free: the identity holds exactly where the anchored local algebra is stable under the constraint's Heisenberg flow, up to the non-gravitating registers, which neither the constraint killed nor the boundary read."*

The shorter reading, licensed with the same riders fused: the boundary was the universe's reference frame, rebuilt inside as a clock, only where the closure remembers where the boundary was (BC), only with the zero-mode anchor re-supplied in the counterterm-site equivalence class, and only up to the registers the boundary never read. The flagship is a kinematic-algebraic identity: at $\nu = 0$ all flows are trivial, and no dynamical content is claimed.

F. Three ingredients of "boundary," each with an exact counterexample

The negative pattern decomposes what "the boundary" supplied into three separately falsifiable ingredients:

1. **An energy ledger** (window coverage): drop it and the identity undershoots by exactly the out-of-window bands (N2), or structurally at any T for bounded-below ladders (N3).
2. **A zero-mode reference** (the anchor): drop it and the identity overshoots one-sidedly everywhere (N1), the containment $\supseteq$ is exact while every band piece of the delocalized seed leaves the local algebra.
3. **Scheme memory of the cut site**: delete it (SD closure) and no anchor choice manufactures it (U4 plus the 42/42 sweep).

The CLPW-shaped no-go is the sharpest corollary: a clock with bounded-below spectrum, the shape of a physical worldline observer's energy in the crossed-product literature [29, 30], provably never internalizes the full boundary at symmetric constraint spectra (N3: saturation $11 < 16$ at every T; the licensed wording: bounded-below internal frames reconstruct only the non-negative-energy corner, and boundary-completeness requires the clock's window to cover the full constraint spectrum). In this class, "observer = internalized boundary" holds only for two-sided-spectrum clocks. We note as an in-literature echo (not prior art for the no-go) that the one continuum construction that does dress closed universes with a clock uses York time, whose spectrum is two-sided [33].

VII. RELATION TO PRIOR WORK

Credit first. Restricted to the global algebra, Φ/χ *is* the trinity's quantum-symmetry-reduction theorem [18, 19, 22, 23] in exact finite arithmetic, and our finite clocks are periodic, so the periodic-clock extension of the trinity is the correct comparison point [20]; the global grid rows are therefore consistency checks, never evidence. T7's flow corner (T7c) is the matched-ideal case of temporal-frame changes [21], in exact finite form; the anchor axis is a computed instance of the quantum relativity of subsystems [24] (spatial quantum reference frames descending from Ref. [41]). The claimable delta over the QRF corpus is threefold: (i) a *region-wise* identity over a locality structure with an exact clock-free criterion (T5), the corpus has no subregion reduction theorem, and seed flow-stability is a nontrivial dynamical fact; (ii) the *cross-universe* leg: the second frame is the cut collar of a *different* (open) lattice reached through the cutting identification, a frame change that changes the universe, outside the QRF theorem set; (iii) *certification*: two-sided generator-level membership everywhere, the overshoot-only theorem, and the negative-pattern failure taxonomy. The objection that full-window compression is tautological is answered in-house: the same map, the same clock, and the same window at the mean anchor give 80 versus 8 (N1), the content is clock-free seed flow-stability (T5), the $\subseteq$ direction, and the intact negative pattern.

Edge modes as frames. The edge-modes-as-dynamical-reference-frames line [25–27] already says "the boundary is the subregion's frame" *intra*-universe, and its current continuation [28] carries QRF covariance into the gravitational

path integral. None of it states a closed↔open algebra identity, an exact certification, or a failure taxonomy; the delta above survives the entire line as of this writing.

The continuum observer line. The crossed-product results [29–32] live in type II/III von Neumann algebras; everything here is finite type I, the trace is the matrix trace, and no entropy, density-of-states, or type statement of any kind is made or implied, our N3 is a finite *counterpoint* at exactly its computed strength, not a shadow of the type transition. The closest continuum statement to our flagship is Blommaert–Chen [33]: one crossed-product clock-dressing (York time as the physical clock) serving both JT subregions and Milne-type closed universes "without an external observer." We cite it as the nearest neighbor and distinguish: no isomorphism claim between closed and open algebras, no exactness, no anchor/scheme riders, no failure taxonomy; the statement-with-proof of Sec. VI E appears nowhere in the literature we swept (Appendix A lists the sweep record). The observer-rule split in the dispute [7, 8, 12] is recorded at bookkeeping level only: our construction is the fixed-register rule; nothing here adjudicates between rules, and nothing here touches the mechanism-(ii) claims on either side [9, 11, 13, 14].

Occupancy. No prior paper imposes the Hamiltonian constraint on a finite lattice and computes the physical algebra, its center, and a survivor classification, with a matched gauge control, gravity-side exact quantizations in the continuum are JT constructions with path-integral input [31, 45], and the nearest finite-graph computation is variational neural-network approximation of constraint-kernel *states* on a finite graph [16, 17], which we cite and distinguish (states, not algebras; approximate, not exact; no control). On the relational side, finite cyclic PW histories with clock-change covariance exist without spatial constraints or a control theory [34]; PW-versus-deparametrization comparisons exist intra-gravity [35]; nobody computes the relational algebra's dimension, center, and shape against clock resolution in a constrained lattice field theory with a second theory through the same pipeline. Zero-mode discipline descends from refined algebraic quantization [38, 39]; Φ is the finite rigging map. A conceptually adjacent gauge-theory-side use of relational frames for confinement is Ref. [44].

VIII. VERIFICATION AND PREDICTION RECORD

The program's house rule is adversarial: every result above survived two independent verifiers (a rigor verifier, fresh primes, independently engineered exact solvers, unseen cells, re-derivation of every quoted number, and a physics/steelman verifier), and both verifier suites are committed and re-runnable (Appendix A). Only claims that survived both are quoted here. The record includes, prominently, what we got wrong:

Beyond Table IV, the record's texture: the original run's results-lead prose contradicted its own tables in four named claims (all retracted at source; the lead is now derived from the run's rows at write time); the registered charpoly-factor handling was initially dropped and the rational-eigenvalue cells, including the paper's STRUCTURED exhibit, were first found by the rigor verifier, then implemented first-class and re-verified; the K3 causal attribution was confounded and is withdrawn above; a mis-transcribed cross-program anchor initially crashed the shipped script after the science sections (fixed at one line; canonical results file regenerated); an early commutant-engine convention bug was caught by cross-validation and fixed before any quoted number. The final committed pipeline runs exit 0, ALL GATES PASS, 125/130 rows ok with the remaining 5 being the registered REFUTED rows quoted above; the unification grid runs 27/27; the two ported verifier suites pass 25/25 and 20/20 on the committed files at fresh primes with no quoted number failing re-derivation.

IX. LIMITATIONS, AND WHAT IS DELIBERATELY NOT CLAIMED

Scope limitations (content, not fine print). (i) Everything is finite-dimensional, in one lattice class (slaved-cascade pair rings and the two-lab model), at small sizes ($V \leq 12$ for kernel dimensions, $V \leq 6/N_S \leq 20$ for algebra anatomy, $d \leq 18$ for exact charpoly work); no continuum limit is constructed or claimed. (ii) The flagship's scheme restriction is a result, not a gap: the identity is BC-only physics, the boundary can be rebuilt inside only where the closure remembers where the boundary was, and the SD failure is computed (42/42 anchor cells), not assumed. (iii) All positive observer content is at $\nu = 0$; the single instantiable $\nu \geq 1$ observer cell fails by single-level collapse (T9), an artifact of a one-letter alphabet, and the open question we consider most valuable is stated in Sec. X. (iv) The exact Page–Wootters identity is tautology-grade given exact projectors and is quoted only as arithmetic integrity. (v) TRIVIAL-BY-ACCIDENT emptiness dominates the integer grid and carries no physics weight anywhere in this paper.

Never claimed (binding list). No statement about the path-integral/wormhole/ensemble mechanism (ii), in either direction, the scoping sentence of Sec. I is mandatory in every quotation of this work; no continuum Hilbert-space-dimension statement ("dim = 1" is never our claim, for or against); no type II/III, crossed-product, modular,

TABLE IV. Pre-registration ledger (registered → outcome), condensed. Refutations of our own registrations are the record's point.

registered claim	outcome
closed cascade solvable ≤ order 2 after zero-mode projection	PASS (per config, all cells)
ledger re-enters via joint local data (h_{joint}=1)	CONFIRMED (11/11 cells); per-order dissent REFUTED (h_k=(1, 1))
ν=0 kernels dim = $\binom{V}{V/2}$	PASS (6/20/70/252/924)
generic-cell taxonomy (empty or small all-central; never NONTRIVIAL)	CONFIRMED incl. rational Z2b cells [V]
rich non-central regional algebra at ν=0	**REFUTED-AS-REGISTERED** (regionally abelian, both gauges)
survivors localize on frozen content (STRUCTURED)	CONFIRMED [V] (register-M_2 test; run initially mislabeled, repaired)
QED control large + dynamical	CONFIRMED (M_{256}/M_{4096}; nnz 16/64)
role-swap trivializes	CONFIRMED (SD cell all-central; strong orbit-survivor form of the dissent REFUTED as registered, multiplicity half observed)
headline thesis (constraint-role-driven)	CONFIRMED
NONTRIVIAL falsifier	never fired (verifier hunts included)
retro-refutation of the open program's anchoring	did not fire, boundary confirmed load-bearing
repair contrast at the site anchor	CONFIRMED (abelian 2 → nonabelian 5 on n_T=11)
kernel staircase/saturation	CONFIRMED; robust at unseen T=32/64 [V]
repair shape $R_{\text{rel}} = M_{n_T}$	**REFUTED**; true shape $R_{\text{rel}}^{\text{cov}} \cong \text{alg}(A_{\text{loc}}(R))$
projector-family "abelian everywhere" distractor	**REFUTED as blanket** (site abelian, mean nonabelian)
"repair needs a gauge anchor"	**REFUTED** (mean repair nonabelian, dim 34/80)
unification (registered global/region/flow scope)	grid-CONFIRMED as O1 + F-7 (BC-only); mean and SD anchors fail one-sidedly; B1 scar + B2 split confirmed; N1–N8 all in registered directions
exact PW identity	CONFIRMED (tautology-grade caveat binding; never headlines)
window staircase $F \to 1$; aliasing; detuning; two-clock distractor	CONFIRMED (exact tables)
gravitating clock K3	table stands; **causal claim withdrawn** ("no matched resonance in-window with or without clock sourcing, not instantiable at these sizes")
QED no-clock control	CONFIRMED, by-isomorphism status stated
ring duality (4 cells)	CONFIRMED (8/8, 4/4, 9/9, 10/10)
access-vs-ontology shadow row	**WITHDRAWN**, vacuous as implemented; dropped permanently

renormalized-trace, density-of-states, or entropy claims (all algebras here are finite type I); no "solved the Wheeler–DeWitt problem" beyond the global-H lattice analogue; no de Sitter or observer-complementarity claims; no Kuchař resolution beyond the computed shadows; no closed-slice holography-of-information claims; no positive-energy statements (sources are signed); no self-gravitating ($\nu \geq 1$) observer statement of any kind (the Sec. IV rider is part of every observer sentence); no aggregation of TRIVIAL-BY-ACCIDENT rows into any headline; no adjudication of the observer-rule split (ours is the fixed-register rule, recorded as such); and "the closed universe is trivial" as a slogan is not this paper's claim, we claim the exact constraint-mechanism computation with a matched control, at the strength of its riders.

X. OUTLOOK

The sharpest question this computation creates is instantiability at $\nu \geq 1$: an admissible closed-lattice family whose matched kernel is populated at $n \geq 2$ with at least two in-window levels, a two-letter self-gravitating energy alphabet, would make the observer instruments run self-gravitating and the interacting-clock deviation form [40] testable in

exact arithmetic. The natural first target is the mediation-frozen rational-doublet family, whose one-letter failure mode (N7) is now a theorem (T9). Second, the anchor equivalence class computed under BC (Sec. VI C) invites a characterization: which gauges are counterterm-equivalent on general admissible rings? Third, the matter-clock family (N8) has repair without identity and no theorem, the honest gap in the package, and deserves one.

ACKNOWLEDGMENTS

Drafting and computation used AI assistance (Claude, Anthropic) under the program's two-verifier discipline; every quoted number is reproducible from the ancillary scripts, and the human-re-derivation checklist that must be discharged before submission is part of the ancillary README. No funding to declare.

Appendix A: Reproducibility manifest

All computations are exact; verdict quantities are integers or rationals; dimension certificates run at two primes per run, with all verification at fresh primes $> 2 \cdot 10^6$ disjoint from every earlier band. Ancillary bundle (file names as committed in the program repository):

- `closeduni_lattice.py`, the C-A/C-B/C-C pipeline (v1.0.2): kernel scans (Z1/Z2/Z2b/Z3), strong Dirac anatomy with the register-M_2 taxonomy and the dihedral/orbit classifier, C-B closures, C-C tables, QED and role-swap controls, C-A0; exit 0, ALL GATES PASS, 125/130 (5 registered REFUTED rows), 82.1 s; canonical output `closeduni_results.md`, run log `closeduni_run_final.log`. Primes 1000003/999983; clock 1000033/1000193.
- `g5_unification.py`, the flagship grid (27/27, exit 0, 981.9 s; in-process refresh gates re-run at startup); canonical output `g5_results.md` (grid rows, naturality-wording section, deviations). Primes 1000037/1000039; DFT 1000289/1000577.
- `g5_dressing_map.md`, the theorem note (v1.1; corrected H-alias bound $T \geq 2 \cdot \text{spread} + 1$; the retracted dimension-shortcut corollary marked at the site), with proofs of T1–T9.
- `g7e_sd_sweep.py` / `g7e_sd_sweep_results.md`, the scheme/anchor sweep (clock-free T5 criterion; calibrated bit-for-bit on certified cells; 42/42 SD cells fail; BC anchor-equivalence-class refinement; primes 2000083/2000093; 40.8 s).
- `closeduni_verify.py`, independently engineered verification suite (own rref/nullspace/CRT/rational-reconstruction/exact-Faddeev engines; sympy as second engine on small exact cells): 25/25, exit 0, ∼12 s; primes 2000107/2000113, clock 2000321/2000353.
- `g5_verify.py`, flagship verification suite (spectra, Φ certificates, T5 clock-free table, direct clock-built band algebras, negative-control tables, B1/B2): 20/20, exit 0, ∼10 s; primes 2000303/2000309.
- `closeduni_target_spec.md`, `closeduni_mechanism.md`, `g5_target_spec.md`, `g5_mechanism_addendum.md`, frozen pre-registration (specs, predicted mechanisms, dissents) against which every outcome above is scored.
- `pf5_scoop_sweep.md`, the drafting-time literature sweep (INSPIRE citation sweeps and phrase searches, current to 2026-08-08) grounding Sec. VII.

Independent-verification structure: the rigor verifier re-derived every quoted scan row, anatomy cell, C-B closure, and C-C rational from scratch at fresh primes (2000003/2000029), extended to unseen cells (V=8 SD ladder, V=6 SPLITR, T=32/64, comp($L0$+$L1$), N_S=16), and ran the full grid port at reserved primes (2000039/2000081) with zero verdict differences; the physics verifier supplied the steelman audits, the independent clock-free probe, and the wording obligations adopted throughout. Items first computed by a verifier are marked [V] and are now reproduced by the committed pipeline and suites.

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