Holography of information on exact lattices: migration, irreducibility, and a controlled QED contrast

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Holography of information, the claim that in quantum gravity all bulk information is available in an arbitrarily small time band of boundary data, because the Hamiltonian is a boundary term, exists as a continuum argument; no exact, finite-dimensional instantiation with dynamics and controls has existed. We construct one. In a constrained one-dimensional lattice class with an integer slaved field, we compute the boundary algebra $B_{\rm bd}^{(n)}$ of collar data through second order in the coupling, the algebra $D^{(n)}$ generated by its Hamiltonian translates, and the commutant $C^{(n)} = D^{(n)\prime}$, exactly, in integer arithmetic with dual-prime certificates, together with a matched QED control run through the identical pipeline at every order in α. Findings: (i) a migration theorem package, proven order-blind, each order's constraint deposits that order's total interaction energy and its dipole at the collar (a lattice ADM identity and filtration), while at linear order the boundary is dynamically dead; (ii) migration does *not* imply holography, the pre-registered dynamics is refuted by a symmetry-forced all-orders bulk survivor, and under generic repaired dynamics the commutant collapses to scalars per boundary-readable block exactly on the sector the boundary translates reach, with every survivor proven to be a register the translate algebra never reaches (one survivor is enrichment-absolute); (iii) in the matter-only enlarged class the commutant equals the boundary-readable superselection center, $C^{(n\geq 1)} = Z$, on all 74/74 admissible cells and for every constraint-compatible dynamics tested; (iv) QED, computed not assumed, never generates: its commutant is constant at the charge-superselection block algebra at every order, and a distractor control shows the pipeline separates constraint-forced migration from bookkeeping. The exact pair, conservation-protection versus dissolution-migration, is a lattice sharpening of the continuum theorems, decides the algebraic half of a cousin mechanism, and leaves the analyticity half untouched by construction.

I. INTRODUCTION

A. The claim, and what has never existed

The principle of *holography of information* (HoI) asserts that in quantum gravity the information available in an arbitrarily small time band of boundary data is the full information of the bulk state. The mechanism narrative is due to Marolf [1, 2]: on the constraint surface the gravitational Hamiltonian is a boundary term, so the boundary algebra at one time equals the boundary algebra at all times ("boundary unitarity"), and boundary observables are complete. The Raju school then proved sharp versions at leading order in G in the continuum: states of massless matter coupled to gravity that agree near the boundary for an infinitesimal time agree everywhere; the split property, which holds in quantum field theory and in QED, fails in gravity; and a boundary observer's recovery protocol that succeeds in gravity "fails completely" in non-gravitational gauge theories, where the boundary sees only the Gauss charge [3–6], with extensions to flat space, de Sitter, finite-radius balls, and two-dimensional models [7, 8, 10], and a massive-gravity control [9].

The continuum statements are contested exactly where they are strongest. Bahiru, Belin, Papadodimas, Sarosi, and Vardian construct state-dressed operators commuting with the boundary Hamiltonian and all asymptotic charges to all orders in $1/N$ on states of large energy variance, a published counter-theorem to naive all-orders holography [11, 12]. Giddings disputes that perturbative dressing implies holography [13]. In the algebraic line, constraints alone can leave a nontrivial center or commutant unless an observer is adjoined [14–16]; and, most recently, Jensen, Raju, and Speranza showed that the strict continuum time-band algebra has *no* commutant at all, products escape the band, so that even defining the object under dispute requires a coarse-graining relative to an observer [24].

What has never existed, on either side of this dispute, is an *exact instantiation*: a finite-dimensional constrained model class in which the boundary algebra, its dynamically generated extension, and the commutant are *computed*, order by order in the coupling, with certificates rather than arguments, together with a matched gauge-theory control

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run through the identical pipeline. Exact commutant computations exist in solvable models, but in a different regime: the JT and double-scaled SYK algebras are infinite-dimensional type II factors answering factor-duality questions, not boundary-band determination, with no order-by-order structure and no gauge control [16, 25, 26]; the topological-holography program computes exact lattice boundary algebras whose mechanism is topological order and quantum error correction, with no constraint dynamics [27–29], as with holographic codes [23]; and Soni's type-I crossed-product analogue is, by his own explicit disclaimer, kinematic, it contains no real-time dynamics [17]. This paper supplies the missing object and reports what it decides.

B. Results and claims

We work in the exactly solvable constrained lattice class of the companion program [33]: matter pattern registers on a one-dimensional chain, an integer field slaved to the matter by a discrete constraint with gravity's affine kernel, a corrected-field cascade generating the order-k field from the order-$(k-1)$ energy density, and a boundary "collar" of field data near the outer end. The boundary algebra $B_{\mathrm{bd}}^{(n)}$ consists of the physical operators diagonal in the joint collar label through order n; the dynamical boundary algebra $D^{(n)}$ is generated by all ad_H-translates of $B_{\mathrm{bd}}^{(n)}$ (H fixed, never adjoined); the deliverable is the commutant $C^{(n)} = D^{(n)\prime}$ with a constructively exhibited basis, in exact integer arithmetic with two-prime modular certificates. Because the slaved boundary algebra is abelian, the correct success criterion is not triviality of $C^{(n)}$ but

$$C^{(n)} \;=\; Z^{(n)}, \tag{1}$$

the boundary-readable block center: "boundary data determines the state up to the boundary's own superselection labels." The residual center is forced by the class and is the lattice Birkhoff/Gauss statement, not a tolerated error.

The findings, at exactly the strength the verification record supports:

C1. First exact finite-dimensional instantiation of the dynamical boundary-commutant question, with integer certificates, in a class where the constraint, not an isometry, not topological order, does the work. Four axes jointly distinguish it from all adjacent exact computations: finite-dimensional; boundary-translate-generated; order-by-order in the coupling; integer-certified, plus the matched gauge control. (In finite dimensions $D^{(n)\prime}$ exists unconditionally; contrast the continuum time band [24].)

C2. The migration theorem package (M1) (Theorems 1 and 2), an order-blind lattice ADM identity (the collar outward difference of the order-k field equals the total order-k absorbed energy, exactly, config-by-config, at $k = 1, 2$, every discretization, every tested collar placement); a filtration identity (each order deposits a fresh energy monopole–dipole pair on differences); a no-screening statement (derived, not searched); and assignment invariance. Proven in-class; the program's strongest deliverable.

C3. Hypothesis anatomy: migration does not imply holography. Migration (M1) and dynamical irreducibility (M2) are separate lattice theorems; M2 is independent, violable, and fails in computable ways. The spec's pre-registered Hamiltonian is refuted by an exact accidental symmetry (an all-orders bulk survivor on the dynamically active sector); the generic repaired dynamics confines all survivors to registers the boundary-translate algebra never reaches, and one such survivor is *enrichment-absolute*, removable by no admissible dynamics whatsoever. Every failure localizes at an energy the constraint does not source or at a register the translates never reach; these are necessity counterexamples (constraint-forced migration never closes a reach gap), not an adjudication of the continuum dispute.

C4. The matched QED pair through one pipeline at every order in α (Proposition 2): the corrected source vanishes identically (computed, not assumed), the commutant is constant at the charge-superselection block algebra, the interior dressed hopper survives forever, and a labeled distractor (Coulomb moments hand-added to the boundary label) fakes gravity-like shrinkage, the pipeline provably separates constraint-forced migration from bookkeeping. Framed strictly as a lattice sharpening of the Raju-school contrast [4, 5], never as "gravity holographic, QED not" simpliciter.

C5. The clean theorem, unconditional within the class (Theorem 4): in the matter-only enlarged models the commutant collapses fully to the boundary-readable center, $C^{(n\geq 1)} = Z$, on every admissible scheme $\times$ depth $\times$ size $\times$ variant cell tested (74/74) and for every constraint-compatible dynamics tested, including $H = 0$. Its content is therefore M1 separation + a linear-order room-to-fail guard + center collapse + per-block generation; we never quote it as evidence that dynamical generation is the discriminating mechanism.

Never claimed (binding scope, stated once): evidence for HoI proper (the positivity/analyticity half of the continuum argument is untested here by construction); "gravity holographic, QED not" as a statement about continuum theories; continuum split-property statements; type or nuclearity statements; adjudication of the Giddings–Raju dispute.

C. Organization

Section II defines the class, the boundary algebras, and the certification pipeline, and cedes the kinematic block/center bookkeeping to the literature where it belongs. Section III proves the migration package. Section IV presents the anatomy of dynamical irreducibility: the linear-order dead boundary, ignition, the refutation of the pre-registered dynamics, the repaired generic dynamics and its frozen fibers, the enrichment controls, the fiat-adjunction diagnostic, and the failure-localization results. Section V states the matter-sector clean theorem at the strength of its robustness sweep. Section VI presents the QED control. Section VII positions every claim against the literature. Section VIII is the verification and prediction record, including all errata. Section IX states the limitations and the honest gap to continuum holography.

II. THE MODEL CLASS, BOUNDARY ALGEBRAS, AND PIPELINE

A. Geometry, matter, and the slaved field

The canonical geometry is a chain of $N_{\rm s} = 15$ sites $\{0, \ldots, 14\}$: two three-site matter "labs" $L_1 = (3, 4, 5)$ and $L_2 = (7, 8, 9)$ separated by a fence site $F = \{6\}$, a buffer $\{10, 11\}$, and a boundary collar $\{12, 13\}$ read at depth $\delta = 2$ (both sites) or $\delta = 1$ (outer site only). Each lab is in one of three integer occupation patterns

$$A = (0, 2, 0), \qquad B = (1, 0, 1), \qquad C = (2, 0, 0),$$

all with the same lab energy but with A, B forming an equal-(E, D) "breathing pair" (equal energy *and* dipole: degenerate at linear order, the lattice shell-versus-point pair) and C distinct. A two-level register ("TT"), attached at the mediation region, stands in for an unsourced tensor-mode polarization: it carries energy only through an optional diagonal term and *never enters the constraint source*. The canonical physical space has $d = 3 \times 3 \times 2 = 18$ states. Enlarged variants (Sec. V) replace the pattern sets and drop or keep the TT register; sizes $N_{\rm s} \in \{15, 19, 23, 27, 31\}$ stretch the buffer.

The linear-order field π^0 is *slaved* to the matter through the discrete constraint with gravity's kernel: with the right-tail Green function $G(i, j) = \max(i - j, 0)$ (one-sided convention, left end pinned, a declared artifact of the class),

$$\pi^k(i) \;=\; \sum_j G(i, j)\, \varepsilon^{(k)}(j), \qquad \varepsilon^{(1)} = \text{matter energy density}, \tag{2}$$

and the *corrected-field cascade* extends this order by order: $\varepsilon^{(k)}$ for $k \geq 2$ is the order-$(k-1)$ field-energy density in the chosen discretization, built from $\pi^0, \ldots, \pi^{k-1}$, the lattice image of solving the constraint iteratively in G, $\mathcal{C}_{\rm lin}[h^{(n)}] = \varepsilon^{(n)}[h^{(<n)}]$. Everything is integer-valued after a fixed global doubling. The *discretization axis* {SITE, SPLITL, SPLITR, SPLITC} is the operator-ordering/point-splitting scheme axis of the quadratic density; the *assignment axis* (how regional energy is apportioned) provably never touches the field profile and hence never touches any boundary object, verified as a computation, closing the boundary against the interior scheme-dependence of the energy's regional bookkeeping [33].

Because the kernel of the adjoint constraint operator is affine, $\{\mathbf{1}, x\}$, the depth-2 collar at linear order reads exactly the affine pair: $\pi_i^0 = i\, E_{\rm tot} - D_{\rm tot}$ at collar sites, so the linear boundary label is $(E_{\rm tot}, D_{\rm tot})$, conserved Tier-0 charges. The QED control (Sec. VI) runs the identical pipeline with the order-1 kernel (kernel = $\{\mathbf{1}\}$): its collar reads the cumulative Gauss charge $Q_{\rm tot}$ and nothing else.

B. Boundary algebras and the commutant deliverable

Definition 1 (Boundary label and boundary algebra)**.** $\Lambda^{(n)}$(config) is the joint tuple of collar values of $(\pi^0, \ldots, \pi^n)$ at the declared depth. $B_{\rm bd}^{(n)}$ is the algebra of physical operators diagonal in $\Lambda^{(n)}$ (constant on $\Lambda^{(n)}$-classes). At $n = 0$

this reproduces, matrix-by-matrix, the constraint-key local algebra of the collar region, a required regression, passed exactly.

Definition 2 (Dynamical boundary algebra and commutant)**.** $D^{(n)}$ is the $*$-algebra generated by all ad_H-translates of $B_{\mathrm{bd}}^{(n)}$, with H fixed and *never adjoined as a generator*. The deliverable is $C^{(n)} = D^{(n)\prime}$, with a constructively exhibited basis. In finite dimensions $\mathrm{span}\{e^{iHt}Be^{-iHt}\} = \mathrm{span}\{\mathrm{ad}_H^k(B)\}$, so "an arbitrarily small time band" equals "all times" automatically, which is precisely why the analyticity step of the continuum argument is not tested here (Sec. IX).

Definition 3 (Success predicate)**.** $Z^{(n)}$ is the abelian span of the dynamical-block projectors of $D^{(n)}$ (blocks = the finest partition jointly refined by $\Lambda^{(n)}$ and H-connectivity). *Holography-in-class* means $C^{(n)} = Z^{(n)}$ with every block boundary-readable (distinct blocks have distinct $\Lambda^{(n)}$ value-sets). A *bulk survivor* is any element of $C^{(n)}$ not a function of readable block labels. Kinematic singleton blocks and Tier-0 total-charge multiplets belong to Z by construction: no physical H connects total-charge sectors.

The kinematic skeleton, a physical algebra decomposing as a direct sum of blocks with center generated by boundary superselection labels, is standard and is not claimed: it is the electric-center structure of lattice gauge theory [19–21] and the general direct-sum-of-type-I-factors-with-center setting in which Soni builds the crossed-product analogue, with the center read as the area operator in holographic codes [17]. We make no type or crossed-product claims. Everything in this paper beyond that skeleton is dynamical: the order-by-order behavior of $C^{(n)}$ under a translate-generated algebra, which no kinematic framework contains, Soni's construction has no real-time dynamics by his own verbatim disclaimer ("despite the name, X is not a Hamiltonian governing the real-time evolution of any system…") [17].

C. The dynamics family

The declared family is

$$H \;=\; a_1 X_{L_1} + a_2 X_{L_2} \;+\; \kappa\left(|B\rangle\langle B|_{L_1} \otimes \sigma^x_{\mathrm{TT}}\right) \;+\; \nu \sum_{i\,\mathrm{int}} \varepsilon_i^{\mathrm{disc}}(\pi^0) \;+\; \omega\, n_{\mathrm{TT}}, \tag{3}$$

with $X_L = |A\rangle\langle B|_L + |B\rangle\langle A|_L$ the breathing hop (lab-local and physical at linear order), the conditional TT gate a hermitian quasilocal element of the fence-adjacent local-algebra product, the diagonal field-energy term in the run's own discretization, and n_{TT} the TT number operator. All coefficients are integers; every member is hermitian and commutes exactly with E_{tot} and D_{tot}; the locality pedigree of every term is verified in code. Two named members matter: H_A ($\omega = 0$), the *pre-registered* Hamiltonian of the computation spec, and H_B ($\omega = 1$), a declared repair variant. Section IV shows H_A sits on a measure-zero symmetry locus of the family and H_B is generic. The matter hop energy is added by hand and never enters the constraint source, a declared class artifact, honestly weaker than gravity's universality, and (Sec. IV G) precisely where the failures localize.

D. Certification

All results are exact. Commutants are solved inside an exact block parametrization; nullspaces are computed mod p at two independent primes (base pair 1000003 and 469762049; the $d = 50$ systems of the robustness sweep also use 999983), lifted by CRT with rational reconstruction, and the resulting basis is verified generator-wise against the full translate list in exact integer arithmetic (on the largest, $d = 50$, systems: at two further fresh primes 998244353 and 2113929217, with the verification mode recorded per row). Every headline claim is an executable assertion: the base pipeline passes 46/46 checks, an adversarial rigor verifier rebuilt with zero shared code passes 38/38 with fresh primes and its own engines, and the two gate scripts pass 25/25 and 19/19 (Sec. VIII). We flag once, prominently: *these counts include refutation-asserts*, checks that a pre-registered claim fails in a specific way, and must never be read as "all conjectures confirmed." All scripts and result tables are ancillary files [34].

III. THE MIGRATION THEOREM PACKAGE (M1)

Doc-level seed, proven in the companion program [33]: the $O(G)$ interaction energy has no invariant interior address, every regional assignment is superpotential bookkeeping, movable across any interior boundary by an admissible bounded divergence transport, while its *total* is exactly readable at the collar. The thesis under test reads that

dissolution as a mechanism: the same divergence freedom that erases every interior address telescopes the energy data to the boundary at every order. The results of this section establish exactly that, order-blind.

Theorem 1 (Order-blind lattice ADM identity, M1a)**.** *For every discretization scheme, every tested geometry and size, and $k = 1, 2$: the collar outward difference reads the total order-k absorbed energy,*

$$\pi^k(N_s{-}1) - \pi^k(N_s{-}2) \;=\; W^{(k)}_{\rm tot}, \tag{4}$$

exactly, configuration by configuration. Consequently the order-corrected total energy ledger $H^{(n)}_{\rm ADM} = E_{\rm tot} + \sum_{k\le n} g^k W^{(k)}_{\rm tot}$ is an element of $B^{(n)}_{\rm bd}$ at every order: on physical states, the sourced part of the total energy is a boundary operator, order by order.

The derivation is one line and is the point: the step-difference of the right-tail Green function is the indicator of the entire interior, so the outward difference of Eq. (2) is $\sum_j \varepsilon^{(k)}(j)$, the linear operator, not the source, carries the divergence structure, which is why the identity is order-blind. The identity holds verbatim at a collar moved into the mid-buffer $\{10, 11\}$ and at depth 3 (verifier runs): *any* exterior cut suffices. Presentational caveat, disclosed: the site $N_s{-}1$ lies outside the declared collar; membership of $W^{(k)}_{\rm tot}$ in $B^{(n)}_{\rm bd}$ is nonetheless exact via the class-spread computation, $W^{(1)}_{\rm tot}$ has positive spread over $\Lambda^{(0)}$-classes (85 SITE, 90 SPLITL/R, 104 SPLITC) and exactly zero spread over $\Lambda^{(1)}$, at both depths, all sizes: the interaction energy enters the boundary algebra exactly at its own order.

Theorem 2 (Filtration identity, M1–L2)**.** *For two configurations with equal lower-order field tails, the collar difference at order k is affine in the site index,*

$$\Delta\pi^k(i) \;=\; i\,\Delta W^{(k)}_{\rm tot} - \Delta D^{(k)}_{W}, \tag{5}$$

with slope the order-k total-energy difference and intercept the order-k energy-dipole difference: each order deposits a fresh (energy monopole, energy dipole) pair, reproducing the $n = 0$ kernel structure at every order. Depth 1 reads one affine combination per order, a genuinely poorer boundary.

This is the lattice-exact, integer-arithmetic image of the finite-ball order-by-order claim of Ref. [10]. Two registered wrinkles: the order-k *source* is not compactly supported (the quadratic density grows along the tail), so collar *values* are whole-interior moments and grow quartically in N_s (fourth differences exactly constant); only *differences* at equal lower-order tails are affine, and all separation statements are difference statements. The separating invariant pair for the canonical breathing pair is $(\Delta W, \Delta M) = (1, 4)$, exactly N_s-constant from 15 through 31; the enlarged model's second pair carries $(4, 16)$, likewise constant. This N_s-independence is *structural* in the one-dimensional right-tail kernel, the size axis is a consistency check, not independent capacity evidence; the genuine capacity axis is matter content (Sec. V).

Proposition 1 (No screening, M1b, a derivation, not a search)**.** *The breathing flip's order-1 field difference reaches the collar at every size and discretization with $\Delta W^{(1)}_{\rm tot} \neq 0$, while its linear-order difference stays lab-confined. Hence no operator acting as the hop X_L on patterns can commute with the collar $\Lambda^{(1)}$ data: the $O(g)$-dressed hop has support reaching the boundary, there is no interior-supported physical completion, i.e., no gravitational screening in-class. QED contrast: the dressed hopper is interior-completable.*

We label this honestly: it is proved as a derivation from computed exhibits (the difference-tail support and $\Delta W_{\rm tot} \neq 0$ are computed exactly at every size and scheme), not by the exhaustive minimal-support search the pre-registration promised; the registered prediction PR10 is carried in the scorecard as "relabeled" (Sec. VIII). It is the lattice-exact form of dressing-to-the-boundary [13, 22].

Mechanism summary, at exactly the supported strength. Three structural drivers, derived: the ADM surface integrand is linear in the field, so migration is order-local; the linearized constraint is a total divergence with affine kernel, so the surface reading of the order-n field equals the total order-n energy inside, at every order for the same reason; and universality, gravity's constraint is sourced by the stress of everything, including the field itself, keeps the induction running with no exceptional order. The three-way split $H = \sum_{k\le n} G^k \oint \partial h^{(k)} + (\text{constraint terms, zero on } \mathcal{H}_{\rm phys}) + O(G^{n+1})$ holds with the remainder pure truncation residue. Where the class is honestly weaker than gravity: the by-hand hop and TT energies are not constraint sources, so the *full* H is never in $B_{\rm bd}$ at any order, only the ledger $H^{(n)}_{\rm ADM}$ is. That is a strictly weaker premise than Marolf's "H is a boundary term," it is why the generation question is nontrivial rather than definitional, and it is exactly where the failures of Sec. IV will localize.

TABLE I. Canonical geometry ($d = 18$, $N_s = 15$, SITE, depth 2; identical verdicts at all sizes 15–23, schemes, and depths). $\dim C$ = commutant dimension; $\dim Z$ = its center; "nodyn" = commutant of $B_{\rm bd}^{(n)\prime\prime}$ without translates (M2b control). H_A = pre-registered dynamics ($\omega = 0$); H_B = repaired ($\omega = 1$). From `holography_lattice_results.md`.

system	n	Λ-classes	$\dim C^{(n)}$	$\dim Z^{(n)}$	blocks	nodyn
H_A or H_B	0	3	132	3	18	132
H_A (pre-registered)	1, 2	9	12	6	6	36
H_B (repaired)	1, 2	9	10	4	6	36
QED (canonical)	0, 1, 2	1	324	1	18	324
QED (charge-varying, $d = 32$)	0, 1, 2	3	472	3	32	472
QED distractor ($n = 1$)	1	8	20	5	8	44
H_B, $\kappa = 0$ ablation	1, 2	9	16	4	8	36
Enlarged (matter-only, $d = 25$)	0	8	89	8	25	89
Enlarged (matter-only, $d = 25$)	1, 2	25	**9**	**9**	9	25

IV. IGNITION AND IRREDUCIBILITY: THE ANATOMY OF M2

We now turn the dynamics on. Pre-registered outcome first, as the fidelity review requires: *the spec's success predicate, Eq. (1) with readability, is False on every canonical-geometry row of the main tables, including under the repaired dynamics.* Everything "clean" below is a derived sector statement, admissible only because the frozen sectors were themselves pre-registered predictions (PR6), found exactly where predicted. The full-space refutation is a finding, not an accident to be excused; this section is its anatomy. Table I collects the canonical numbers.

A. Linear order: the boundary is dynamically dead

At $n = 0$, $[H, B_{\rm bd}^{(0)}] = 0$ exactly, for every member of the family: the linear collar reads the conserved totals $(E_{\rm tot}, D_{\rm tot})$, so $D^{(0)} = B_{\rm bd}^{(0)}$ and $C^{(0)}$ is huge, dimension 132 on $d = 18$, containing the breathing hop and the full TT algebra. *Linear-order holography fails*: gravity at $O(G)$ is QED-shaped, consistent with the linear-order split-property survival of the companion program [33]. This is simultaneously the trivial-fake guard: every geometry is certified to have room to fail at $n \geq 1$.

B. Ignition (M2a) and generation (M2b)

The one-commutator discriminator of the whole program:

$$[H, B_{\rm bd}^{(0)}] = 0 \text{ (both theories)}; \qquad [H_{\rm grav}, B_{\rm bd}^{(1)}] \neq 0 \text{ (max integer entry 1)}; \qquad [H_{\rm QED}, B_{\rm bd}^{(n)}] = 0 \ \forall n. \tag{6}$$

Gravity's order-1 boundary datum is *non-conserved* interaction energy, the hop moves the W-moments, so the dynamics writes to the boundary and generation ignites at $n = 1$. QED's boundary datum is exactly conserved charge, generation never starts. Ignition is *forced* in-class: any H that moves patterns within an (E, D)-class fails to commute with $\Lambda^{(1)}$, because $\Lambda^{(1)}$ separates the moved configurations (Theorem 2); the only ignition-free dynamics is pattern-diagonal, i.e., none.

Generation is measured against a no-dynamics control: without translates, $B_{\rm bd}^{(n)\prime\prime}$ has commutant of dimension ≥ 36 versus 10 with translates, an abelian boundary reading alone does nothing. We adopt the steelman finding of the program's fidelity review and state it against ourselves: once labels separate and the orbit graph is connected, generation is near-generic finite-dimensional linear algebra, the QED distractor proves any separating label shrinks the commutant. *The discriminating physics is M1 plus the ignition commutator*; only the cascade (the label computed from the theory's own source) distinguishes constraint-forced labels from fiat ones. Every novelty claim in this paper is located accordingly.

C. The pre-registered dynamics is refuted: a symmetry-forced all-orders survivor

Under the pre-registered H_A, $[H_A, \sigma^x_{\rm TT}] = 0$ is an exact accidental $\mathbb{Z}_2$ symmetry (the TT vertex is a single abelian coupling; H_A has three degenerate adjacent eigenpairs where H_B has none). Hence $\sigma^x_{\rm TT} \in C^{(n)}$ for every n, at every scheme, size, and depth, a genuine bulk survivor *on the dynamically active sector* under the registered dynamics: verdict pair $(\dim C, \dim Z) = (12, 6)$. This is the honest headline of the canonical model: **pre-registered dynamics refuted**. Migration is intact (M1 is H-independent); determination fails. Dynamical irreducibility (M2) is thereby demonstrated to be a load-bearing, violable hypothesis with a computable failure mode, a lattice-exact, finite-dimensional analogue of the all-orders dressed commutants of BBPSV [12] and of the constraints-leave-centers phenomenon [15, 16], arising here organically from a symmetry of a pre-registered H rather than by construction.

The robustness sweep sharpens this to a family statement (script `paper_c_gate_sweep.py`): in the declared family (3), all ten tested grid points with $\omega \neq 0, \kappa \neq 0$ and hops on, including $\nu = 0$, reproduce the identical repaired verdict; every failure lies on an identified locus with a constructive witness: $\omega = 0$ (the exact $\mathbb{Z}_2$; and adding a *second* σ^x-type gate instead of $n_{\rm TT}$ stays on the locus, the failure is the symmetry, not a lack of coupling strength), $\kappa = 0$ (TT disconnection: the full TT algebra survives, the calibrated M2c signature), $a_1 = 0$ (a dropped hop that the gate cannot re-ignite: $(\dim C, \dim Z) = (24, 6)$ with a contaminated active sector, dynamical connectivity, not migration, is what fails). H_A **is a measure-zero symmetry locus;** H_B **is generic.** "Repaired dynamics" means: off the symmetry and disconnection loci.

D. Repaired dynamics: exact holography on the reached sector, survivors on frozen fibers

Under H_B, $\dim C$ shrinks $132 \to 10$ at $n = 1$ and stays (the cascade *saturates* at order 1: $\Lambda^{(1)}$ already separates every separable configuration, the small model reaches its resolution limit, so the order-by-order shrinkage is one nontrivial step). But $\dim C = 10 > \dim Z = 4$: the literal full-space predicate fails. The survivors are constructively exhibited and *proven complete* by the adversarial verifier's exhaustive restricted solve: they are exactly the TT-register M_2 algebras on the two mediation-frozen L_1=C fibers, the (C, C) pattern block and the $\{CA, CB\}$ block, where every translate acts as (pattern-op) $\otimes\, I_{\rm TT}$. On the mediation-active sector the commutant is *scalars per boundary-readable block, exactly*, at $n = 1$ and 2. Sector decomposition of $C^{(1)}$: (active, $\{CA, CB\}$, (C, C)) $= (2, 4, 4)$.

Both fibers were pre-registered predictions (PR6: "the run must find it, or the pipeline is wrong"), found exactly as predicted. The invariant statement, and the wording matters, see the next subsection, is a dichotomy:

bulk commutant content $\iff$ *bulk registers the boundary-translate algebra never reaches.*

Reached $\Rightarrow$ scalars; unreached $\Rightarrow$ a full surviving register. This is essentially von Neumann's theorem unpacked; the content is the exhibits, their completeness proof, and the controls that follow.

E. Enrichment controls: reach, not connectivity; and an enrichment-absolute survivor

The ablation $\kappa = 0$ calibrates the disconnection signature (full TT survivor, $\dim C = 16$), but it only tests the removal direction. The converse control, pre-registered as an open gate, varies the *enrichment* (script `paper_c_gate_controls.py`): add to H_B a C-conditioned gate and ask whether the frozen-fiber survivors ignite. Four admissible enrichments were run (all hermitian, integer, charge-conserving, quasilocal, pedigrees verified): the uniform mirror gate $E_1 = |C\rangle\langle C|_{L_1} \otimes \sigma^x_{\rm TT}$; the label-correlated gate $E_2 = |C\rangle\langle C|_{L_1}|B\rangle\langle B|_{L_2} \otimes \sigma^x_{\rm TT}$; their sum; and a coefficient sweep, plus depth-1 and $N_{\rm s} = 19$ spot checks.

Converse confirmed. Under E_2 (and E_3, and $\mu = 3$) the translates reach the $\{CA, CB\}$ register: both masked TT witnesses die, the fiber's commutant contribution collapses $4 \to 1$, $\dim C : 10 \to 7$, sector decomposition $(2, 4, 4) \to (2, 1, 4)$. Reached $\Rightarrow$ no survivor, the causal direction the ablation could not test.

An exact counterexample corrects the registered signature. E_1, the literal "any C-conditioned gate" of the pre-registration, H-connects *both* frozen fibers, yet every translate still acts as (pattern-op) $\otimes\, I_{\rm TT}$ on both: $\dim C$, the sector decomposition, and both witnesses are *identical* to unenriched H_B. Root cause in one commutator: on the L_1=C sector the uniform gate restricts to $I \otimes \sigma^x$, so $[\,\text{gate}, [H, P_{\rm class}]\,] = 0$ exactly, the hop-then-conditional-gate path that feeds the TT register into the translate algebra has no analogue. The registered parenthetical "ignites if enriched by any C-conditioned gate" is thereby *refuted as stated* (erratum carried in Sec. VIII); the corrected signature is: the survivor collapses iff the enrichment couples the register to *boundary-separated labels* (gate condition non-uniform on the fiber), label-correlated gating, strictly stronger than H-connectivity. Accordingly, survivor attribution throughout this paper says "registers the translate algebra never reaches," never "dynamically disconnected registers": E_1 exhibits connected-but-unreached.

TABLE II. The fiat diagnostic PR11, run as registered (canonical geometry, exact; `paper_c_gate_controls_results.md`). $F' = \mathrm{alg}\{B_{\mathrm{bd}}^{(0)} \cup \{H\}\}'$.

H	$\dim F'$	$\dim C^{(0)}$	$\dim C^{(1)}$	$\sigma^z_{\mathrm{TT}} \in F'$	$\sigma^x_{\mathrm{TT}} \in F'$	$H \in F'$	$C^{(1)} \subseteq F'$	$F' \subseteq C^{(1)}$
H_A	24	132	12	no	yes	yes	yes (proper)	no
H_B	18	132	10	no	no	yes	no	no

Theorem 3 (Enrichment-absolute survivor)**.** *The $(C,C)\times\mathrm{TT}$ fiber is exactly one $(E_{\mathrm{tot}}, D_{\mathrm{tot}}) = (4, 20)$ superselection class with one-dimensional pattern space, and its Λ-class is a TT-blind singleton at every order, scheme, and depth. Consequently, for every admissible dynamics (H hermitian, $[H, E_{\mathrm{tot}}] = [H, D_{\mathrm{tot}}] = 0$), every ad_H-translate of every class projector restricts to a scalar on the fiber, so $D^{(n)}|_{CC} = \mathbb{C}\, I$ and the masked $M_2(CC)$ lies in $C^{(n)}$ for every n. The survivor is removable only by removing the register or changing the matter content, never by enriching the dynamics.*

Proof. Write $\mathcal{H} = V \oplus V^{\perp}$ with V the fiber. By the first premise V is a full joint eigenspace of $(E_{\mathrm{tot}}, D_{\mathrm{tot}})$, so every admissible H and every Λ-class projector P preserves V; hence every $\mathrm{ad}_H^k(P)$ is block-diagonal with respect to $V \oplus V^{\perp}$, and restriction to V commutes with products and commutators. By the second premise $P|_V \in \{0, I_V\}$ is a scalar, so $\mathrm{ad}_H(P)|_V = [\,H|_V, P|_V\,] = 0$, and inductively $\mathrm{ad}_H^k(P)|_V = 0$ for all $k \geq 1$. Since $B_{\mathrm{bd}}^{(n)}$ is generated by the class projectors, every generator of $D^{(n)}$ restricts to a scalar on V: $D^{(n)}|_V = \mathbb{C}\, I_V$. For any W supported on V and any generator $T = T|_V \oplus T|_{V^{\perp}}$, $[W, T] = [\,W|_V, T|_V\,] \oplus 0 = 0$; so the masked $M_2(CC)$ lies in $C^{(n)}$ for every n and every admissible H. □

Both premises are verified in code, and the theorem was attacked with three pseudo-random charge-conserving Hamiltonians (arbitrary within (E, D) blocks): every CC matrix unit commutes with every translate in every case. This is the sharpest object in the paper's anatomy: an *unsourced register on a Tier-0 singleton*, the exact in-class form of the decoupled systems that power the BBPSV and CLPW phenomena [12, 15]. The matter-only enlarged model removes the register, and its clean theorem (Sec. V) is the consistency anchor: removal, not repair, is why $C = Z$ becomes reachable there.

F. The fiat diagnostic: what refusing to adjoin H protects

The pipeline never adjoins H to the boundary algebra. The pre-registered diagnostic PR11 quantifies why that refusal is load-bearing: compute $F' = \mathrm{alg}\{B_{\mathrm{bd}}^{(0)} \cup \{H\}\}'$, the "fiat" commutant obtained by declaring H boundary by hand at linear order, and compare (Table II). Run exactly as registered, confirmed on both clauses and both Hamiltonians: $\dim F' = 24$ (H_A) and 18 (H_B) against the honest $C^{(0)} = 132$, fiat adjunction fakes most of the $n \geq 1$ shrinkage at an order where the honest boundary is provably dead ($[H, B_{\mathrm{bd}}^{(0)}] = 0$ is the *reason* $C^{(0)} = 132$); it smuggles the manifestly bulk, hop-containing operator H itself into the "boundary" commutant ($H \in F'$, $H \notin C^{(1)}$); and it still cannot kill the symmetry-forced survivor ($\sigma^x_{\mathrm{TT}} \in F'$ under H_A). The containment pattern is decisive: $F' \not\subseteq C^{(1)}$ always, and under H_B neither contains the other, the fiat object is not an approximation of the translate commutant; it is a different object.

G. Persistence under enlargement, failure localization, and a bookkeeping regularity

Both halves of the anatomy persist when the model is enlarged (variant E4: two breathing pairs per lab *plus* the TT register, $d = 50$). H_A-enlarged: $[H, \sigma^x_{\mathrm{TT}}] = 0$ persists exactly and the survivor contaminates the active sector, $(\dim C, \dim Z) = (30, 12)$. H_B-enlarged: the active sector is exactly clean and the survivors are the TT registers on the now-larger family of mediation-frozen fibers, $(\dim C, \dim Z, \mathrm{dup}) = (27, 9, 6)$, invariant across the (κ, ν, ω) grid and the scheme spot; the loci reproduce their calibrated signatures at $d = 50$.

Failure localization, sharpened. Freezing a *gravitating* register does not break holography-in-class: dropping a hop in the matter-only model leaves the frozen pair boundary-readable, because the constraint sources it, and $C = Z$ holds with more blocks ($\dim C = \dim Z = 12$). Freezing or symmetrizing the *unsourced* TT register is what breaks it. Across every run of the program, every failure localizes at an energy the constraint does not source or at a register the translates never reach, the model's non-gravitating components speaking. This is the strongest lattice form of

the program's closing sentence: where the model stops gravitating, holography stops with it. In full general relativity universality is exact; that this closes the gap is a continuum conjecture shape, not a lattice theorem.

Observation 1 (Bookkeeping regularity, not a theorem). On every TT-carrying run of the program, generic, symmetry-locus, and disconnection-locus alike, $\dim C - \dim Z = 3 \cdot \mathrm{dup}$, where dup counts the unread TT doublings (block pairs with identical boundary data): each contributes exactly one M_2 register ($4 - 1 = 3$). Seven verdict tuples instantiate it: $(10, 4, 2)$, $(12, 6, 2)$, $(24, 6, 6)$, $(27, 9, 6)$, $(30, 12, 6)$, $(26, 14, 4)$, $(36, 9, 9)$. Stated as an observed regularity of the sweep; the natural conjecture is that it is the register-bookkeeping identity for any set of boundary-blind qubit fibers.

V. THE CLEAN THEOREM: THE MATTER SECTOR, AT SWEPT STRENGTH

The matter-only enlarged models remove the unsourced register: every degree of freedom left either gravitates (sources the constraint) or is a pattern the constraint reads. Three variants were swept (script `paper_c_gate_sweep.py`): E1, two breathing pairs plus a singleton per lab, $d = 25$; E2, a third equal-(E, D) partner forming a three-element kinematic class $\{(0, 4, 0), (2, 0, 2), (1, 2, 1)\}$ with hop path $D \leftrightarrow E \leftrightarrow F$, $d = 36$; E3, a different geometry entirely (four-site labs, two *inequivalent* breathing pairs, energy-degenerate content), $d = 25$.

Theorem 4 (Clean theorem, unconditional within the class)**.** *On every admissible cell of the robustness battery,* $74/74$ *across the three variants* $\times$ *sizes* ($N_\mathrm{s} \in \{15, 19, 23\}$ *for E1; spot-covered for E2, E3)* $\times$ *four discretizations* $\times$ *both depths* $\times$ $n \in \{1, 2\}$*, and for every constraint-compatible dynamics tested (coefficient grids,* $\nu = 0$*, dropped hops, diagonal decoration, mismatched* H*-scheme versus* Λ*-scheme, order-corrected energy, and* $H = 0$*):*

$$C^{(n \geq 1)} \;=\; Z^{(n \geq 1)} \;=\; \textit{the boundary-readable block center},$$

with the linear-order room-to-fail guard (bulk survivors at $n = 0$*, all hops in* $C^{(0)}$*) passing on every cell. The verdict tuple is exactly invariant cell-to-cell: E1,* $(89 \to 9)$*; E2,* $(204 \to 9)$*; E3,* $(161 \to 9)$*.*

The theorem's honest content, and this attribution is part of the claim, adopted from the program's steelman review and now carried by a constructive exhibit, is the following. With $H = 0$ the predicate still evaluates true ($\dim C = \dim Z = 25$ singleton blocks, all readable), because $\Lambda^{(1)}$ *already separates every configuration*: the constraint-forced order-1 boundary data is a complete state label, kinematically, on every cell tested. Given that separation, $C = Z$ is forced for *any* constraint-compatible dynamics; the dynamics only sets how far the center collapses (25 singleton blocks $\to$ 9 orbit blocks under the full family) and supplies full per-block generation (translate span $= \sum_{\mathrm{blocks}} (\dim \mathrm{block})^2 = 81$; E2: 196). **The discriminating physics of the clean theorem is M1 separation plus the** $n = 0$ **guard; it must never be quoted as evidence for dynamical generation as a mechanism.** What the theorem *does* establish, robustly: in a class where everything gravitates, the constraint alone pushes a complete label to the boundary order by order, and no admissible dynamics can hide a bulk degree of freedom from it, while one unsourced register suffices to break this absolutely (Theorem 3). The finite-size discipline holds throughout: separating invariants exactly N_s-constant, $(1, 4)$ and $(4, 16)$ for the two E1 pairs.

VI. THE QED CONTROL

The identical pipeline, same geometries, same cascade code path, same commutant certification, no shortcuts, with the order-1 kernel and QED's own corrected source, which the cascade computes to be identically zero at every order ($\pi^k \equiv 0$, $k \geq 1$, all four discretizations): the photon is uncharged, the Gauss law stays linear at every order in α, the induction of Sec. III fails at its first step. Hence $\Lambda^{(n)} = \Lambda^{(0)}$, $[H_{\mathrm{QED}}, B_{\mathrm{bd}}^{(n)}] = 0$ at every order (the boundary datum is exactly conserved charge), $D^{(n)} = B_{\mathrm{bd}}^{(0)}$, and:

Proposition 2 (QED constancy)**.** $C^{(n)}$ *is constant in* n *at every order computed:* $\dim C = 324 = 18^2$ *on the canonical geometry (a single charge class, the boundary is literally blind there), and* $472 = 18^2 + 12^2 + 2^2$ *on the charge-varying geometry (*$d = 32$*, three* Q_{tot} *sectors): the commutant is the charge-superselection block algebra, and the boundary's total information content is* Q_{tot}*, superselection data only. The interior dressed hopper is a bulk survivor at every order (the split-property witness, in its finite-type-I sense); the Coulomb energy has positive spread inside charge classes forever, exhibit pair: patterns* $(1, 0)$ *versus* $(0, 1)$*, equal* $Q_{\mathrm{tot}} = 1$*, identical collar data at every order,* $\sum \pi^2 = 7$ *versus* 6*. The boundary never reads energy.*

Scope, disclosed: the statement "the boundary reads exactly the Q_{tot} sectors" is carried by the charge-varying run (independently verified, one configuration); the canonical run's constancy is verified across all sizes and both depths.

The *distractor* control completes the pair: pushing Coulomb moments into the QED boundary label *by hand* fakes gravity-like shrinkage, $324 \to 20$, the pipeline provably distinguishes "the constraint put it there" from "I put it there." That distinction, **conservation-protection versus dissolution-migration**, is the quotable form of the pair, at the boundary-algebra level, in one pipeline, at every order. It is a lattice sharpening of the Raju-school contrast [4, 5], and it is never quotable as "gravity holographic, QED not" simpliciter: in 3+1 dimensions the exterior Coulomb field reads all multipoles, and the reason readable-but-unforced multipoles fail to deliver the state rests on the continuum split property [32], which the one-dimensional kernel forecloses on both sides, the lattice pair tests "the constraint forces only Q into the exterior," not the multipole question. The gravitational Birkhoff caveat is carved out as the center Z, exactly where it must appear: spherically-symmetric (here: Tier-0) data is superselection, and both theories' residual centers are instances of sectorwise HoI in the sense of Ref. [18].

VII. RELATION TO THE LITERATURE

Mechanism origin. The narrative "H is a boundary term $\Rightarrow$ boundary algebra time-independent $\Rightarrow$ boundary complete" is Marolf's [1, 2]; his completeness assumption is precisely what the commutant computation decides in-class, and his toy-model theorem is the continuum M2 via Reeh–Schlieder. Our class differs on both sides of his ledger: only the sourced ledger part of H is a boundary operator (weaker premise), and the boundary is slaved-abelian, so dynamics must do all the non-abelian work (harder generation).

Raju school. The $O(G)$ determination theorems, the QED/split-property contrast, and the recovery protocol are theirs [3–6]; the conceptual gravity-versus-gauge contrast is scooped physics and is presented here only as a theorem-grade lattice pair (C4). Our filtration identity is the lattice image of the finite-ball claim [10]. The continuum time-band subtlety of Ref. [24], the strict band algebra has no commutant; an observer is needed for a coarse-grained one, strengthens the finite-dimensional half of C1 (here $D^{(n)\prime}$ exists unconditionally and is computed) and is directly adjacent to the frozen-fiber anatomy: their observer-relative commutant and our unreached registers are two faces of "commutant content is relative to what the boundary algebra can reach."

BBPSV and Giddings. The lattice $C^{(n)}$ is state-independent; the BBPSV commutants are compressions PMP on code subspaces [11, 12], the two coexist without contradiction, and the lattice localizes the continuum dispute at the compression step, the one step it cannot see. What the lattice contributes is the state-independent dichotomy (Sec. IV D) with completeness proofs and causal controls, and *necessity counterexamples*: constraint-forced migration never closes a reach gap (H_A; E_1; Theorem 3), which partially vindicates the Giddings side [13] at the algebraic level. An earlier program claim that the lattice "adjudicates" the Giddings–Raju dispute is *withdrawn*: in-class, perturbative equals exact by construction, the dressing is unique and hardcoded, and order-by-order shrinkage is one nontrivial step (saturation at order 1); no adjudication is claimed.

Observer/crossed-product line. The frozen-fiber survivors are the finite-dimensional organic instance of the constraints-leave-centers/observer-needed phenomenon [14–16]; the $\kappa = 0$ ablation is its by-construction twin, and the label-correlated gate that ignites a fiber is the "observer coupling" in-class. The kinematic block/center scaffold is ceded to Refs. [17, 19–21] (Sec. II B); our class instantiates Soni's general direct-sum setting trivially, his semiclassical machinery (isomorphic infinite-dimensional sectors, e^{-S} spacing, microcanonical peaking) does not apply to it, and no type statement is made. Edge modes [22] run in the opposite direction (added to restore factorization); the tension between the two directions is exactly the M1/M2 split.

Exact-boundary-algebra adjacencies, distinguished. Holographic codes achieve bulk reconstruction by isometry [23]; the topological-holography program computes exact lattice boundary algebras from topological order [27–29]; the JT/DSSYK line computes exact von Neumann algebras and commutants in solvable constrained models, infinite-dimensional, answering factor/duality questions [16, 25, 26]; observer algebras in de Sitter continue the crossed-product line in the continuum [30], and superselection-theoretic treatments of QED boundary data remain kinematic [31]. None combines finite dimensionality, a translate-generated boundary algebra, order-by-order coupling structure, integer certificates, and a matched gauge control; C1 rests on exactly that conjunction, graded against this list.

Finite-dimensional warnings. Kumar's result that finite-dimensional tensor networks cannot support Reeh–Schlieder cyclicity, and that with superselection sectors HoI holds only sectorwise [18], is *evaded, not contradicted*: the lattice mechanism (constraint-forced separation plus reach) needs no vacuum, no entanglement resource, and no cyclicity, the slaved sector has neither, and the residual sectorwise structure appears here exactly as his theorem requires, as the center Z.

VIII. VERIFICATION AND PREDICTION RECORD

A. Pipeline and adversarial verification

Four self-testing scripts carry every number in this paper; all are deterministic, exit 0, and are provided as ancillary files with their full result tables [34]: the base pipeline (`holography_lattice.py`, 46/46 checks, 141 systems); an adversarial rigor verifier (`holography_lattice_verify.py`, 38/38) rebuilt with zero shared code, own configuration enumeration, own forward-recursion field solver, own cascade/translate/commutant engines, fresh primes, center computed exactly over $\mathbb{Q}$, which reproduced every headline number exactly, extended sizes to $N_s \in \{27, 31\}$, added the SPLITC scheme, depth 3, and a moved collar, and ran two counterexample hunts that came back *empty* (exhaustive restricted solve on the active sector: scalars only, classification complete; QED boundary provably a function of $Q_{\rm tot}$ alone, orders 0–2, depths 1–3); the robustness sweep (`paper_c_gate_sweep.py`, 25/25, 146 systems, engine anchored against the published numbers before any new claim; the seven-tuple table behind Observation 1 is in its results file); and the controls script (`paper_c_gate_controls.py`, 19/19, baseline re-asserted before any new run). A full-text fidelity review independently re-derived the $d = 18$ block bookkeeping by hand (6 blocks; 10/4; 12/6), matching exactly, and its steelman findings are adopted in the text where they bite (Secs. IV B, V).

B. Pre-registered predictions: the scorecard, with errata

Twelve predictions (PR1–PR12) were registered in the mechanism document before any commutant was computed, several with exact hand-derived integers. Outcomes, honestly, including everything that went wrong:

- **PR1** (M1a at $k = 2$): confirmed exactly. **PR3** ($n = 0$ regression): confirmed. **PR5** (generation; TT reached at ad-order ≥ 2 via the hop-then-gate path): confirmed. **PR7** (QED shape): confirmed. **PR8** (scheme robustness): confirmed; the registered R2 failure shape did not occur. **PR9** (finite-size laws): confirmed exactly (values quartic, separations affine, invariants constant).
- **PR2, PR4** (filtration integers; ignition integers): confirmed *in structure*, with a $\times 2$ integer erratum, the registered values misapplied the doubling convention (registered $\Delta W^{(1)} = -2$ SITE; verified -1; registered max ignition entry 2; verified 1). Direction, affine law, separation, and the forced-ignition rider all stand. Erratum applied at source.
- **PR6** (the frozen-fiber survivor, the sharpest registration: predicted, then found, both fibers, with the completeness proof): headline *stands*. Its parenthetical signature ("ignites if enriched by *any* C-conditioned gate") is *refuted by exact counterexample* E_1 (Sec. IV E); corrected signature: label-correlated gating. Erratum applied at source.
- **PR10** (no-screening): *relabeled*, delivered as a derivation from computed exhibits, not the promised exhaustive minimal-support search (Prop. 1).
- **PR11** (fiat diagnostic): registered, initially unrun (a scorecard defect flagged in review), then run *exactly as registered* and confirmed on both clauses and both Hamiltonians (Sec. IV F). Nothing struck.
- **PR12** (depth-1 resolution chain): *wrong as a partition statement*, the depth axis is vacuous on the canonical matter content ($E_{\rm tot} = 4$ for every configuration), a geometry fact the registration missed; verdict-neutral (real placement robustness comes from the verifier's depth-3 and moved-collar runs).

No prediction was struck; two carry integer errata, one a refuted parenthetical, one a relabel, one a vacuity finding, all disclosed above and in the ancillary errata. The pre-registered success predicate itself evaluated *False* on every canonical-geometry row (Sec. IV); the refutations R1–R5 registered against the program fired exactly once, under the pre-registered H_A, and that firing leads Sec. IV as the honest headline.

IX. LIMITATIONS: THE HONEST GAP TO CONTINUUM HOLOGRAPHY

What the lattice does not and cannot test, verbatim from the program's scope ledger:

1. *The small-time-band/analyticity step.* Finite dimension makes every orbit entire: "small band = all times" is automatic, and **positivity of H is never used anywhere in the pipeline**, while the continuum proof is false without it. The lattice can neither support nor falsify that step.

2. *The premise "H is a boundary term."* In-class, only the constraint-sourced ledger $H^{(n)}_{\mathrm{ADM}}$ is boundary; the hop and TT energies are unsourced by hand. The lattice premise is strictly weaker than Marolf–Raju's.

3. *Reeh–Schlieder.* Replaced, not finitized: the working mechanism is constraint-forced label separation plus translate reach, no vacuum, no entanglement resource. A lattice success is evidence for a *different sufficient mechanism*, evading rather than contradicting the finite-dimensional no-go of Ref. [18].

4. *Type-III structure, the continuum split property proper, radiative data, state-dependence, resummation.* All finite type I here; "split survives in QED" means only finite-type-I commutant nontriviality; the boundary is slaved-abelian and radiation-free; formal g, no resummation claims.

The refutation force is asymmetric and we state it in both directions: an in-class success is *conservative* evidence (even an impoverished, abelian, radiation-free boundary suffices for the algebraic half), while an in-class failure could in principle be a one-dimensional artifact, which is why the failures reported here are anatomized (localized at unsourced/unreached components with causal controls) rather than headlined as continuum claims. The lattice decides the algebraic half of a *cousin* mechanism: constraint-forced migration plus dynamical reach, with superselection centers where they must be. The named next calculation, for both branches: a richer boundary, one non-slaved boundary mode or a radiative register (two-sided chain or 2D strip), where in-class failure would stop being a plausible 1D artifact, the frozen-fiber phenomenon meets genuine boundary dynamics, and the missing analyticity half could first begin to bite.

X. CONCLUSION

The program asked whether the boundary residue of dissolved interaction energy, plus dynamics, is the whole state. The answer, exactly and with integer certificates: it is, precisely on the sector the boundary-translate dynamics reaches, and up to the boundary's own superselection labels; migration is proven order-blind and is the strongest deliverable; irreducibility is a separate, violable hypothesis whose every failure localizes at a component the model exempts from gravitating, including one survivor no admissible dynamics can remove; in the class where nothing is exempt, the collapse $C = Z$ is unconditional; and QED, run through the identical pipeline, never starts. Where the model stops gravitating, holography stops with it, and both halves of that sentence are now theorems with certificates.

Code and data availability. Ancillary files: `holography_lattice.py` (46/46), `holography_lattice_verify.py` (38/38), `paper_c_gate_sweep.py` (25/25), `paper_c_gate_controls.py` (19/19), with result tables `holography_lattice_result` `paper_c_gate_sweep_results.md`, `paper_c_gate_controls_results.md`. All runs deterministic; every number in this paper appears in a result table or ledger line [34].

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[34] Ancillary files with this submission: `holography_lattice.py`, `holography_lattice_verify.py`, `paper_c_gate_sweep.py`, `paper_c_gate_controls.py`, and the result tables `holography_lattice_results.md`, `paper_c_gate_sweep_results.md`, `paper_c_gate_controls_results.md`. Deterministic; python3.12; exact integer arithmetic with two-prime certificates (construction primes 1000003, 469762049, and 999983; verification primes 998244353, 2113929217).