What gravitationally induced entanglement certifies: mediation semantics and a Bell-style theorem

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Proposals to entangle two masses through gravity (Bose et al.; Marletto and Vedral) are widely described as tests of the quantum nature of gravity, but the field has no theorem stating what a positive result would certify, from which assumptions, with which alternatives surviving. We supply one. The central missing ingredient is semantic: a formal, theory-independent definition of "the entanglement was *mediated* by the gravitational field." We prove that circuit topology alone cannot provide it, every bipartite gate decomposes exactly into mediator-local rounds, and construct a five-clause definition (interposition, autonomy, locally factorized coupling with a cut criterion, causal propagation, carriage) with three description-invariant operational signatures (retardation, interposed blockability, autonomy response). The definition is adequacy-tested against every configuration in the recent classical-gravity-produces-entanglement debate, is provably non-circular (an explicitly classical theory satisfies every clause and provably cannot entangle), and yields a slaving dichotomy: a candidate mediator is either constraint-slaved (no independent data, no dead time, fixed operator-Schmidt-rank-2 gate) or autonomous (retarded), with the two indistinguishable in all mass-side data. Around this definition we assemble a Bell-style certification theorem from three lemmas with stated scopes: a mediation definition adequacy lemma; a per-run Bell/steering inertness theorem for charge-transfer coherences, presented as a constrained-field-theory instantiation and strengthening of Baldijão et al., including a multi-copy activation boundary we exhibit exactly (joint processing of two runs restores Tsirelson-value violations, which forces an i.i.d. per-run clause into the certified statistic); and a continuum classical-mediator no-go, presented as a dynamical, instrument-level strengthening of Ludescher et al., covering back-reacting instruments with feed-forward over uncountable outcome spaces, continuous-time limits, and the Oppenheim classical–quantum framework, for which the anticipated "small" entanglement is shown to be exactly zero in the locally coupled and Diósi–Penrose classes. The resulting theorem: under six explicitly priced assumptions, a certified positive result implies that the degree of freedom coupling the two labs is non-classical in the state-space sense, either a non-classical mediator, or a nonseparable direct quantum gate with no carrier, the two separated in principle only by retardation. Both disjuncts are non-classical; every classical-gravity model in the current literature is refuted on its own formulation, while prequantum hybrids (van Hove) are correctly classified rather than excluded. We state the null-result converse, price its linear-additivity caveat, compare the theorem honestly with Bell's, and give experimental guidance, including a visibility-scan certificate for the closed-lab assumption. All model-level claims are verified in deterministic ancillary scripts.

I. INTRODUCTION

Two adjacent matter-wave interferometers, each holding a mass $m \approx 10^{-14}\,\mathrm{kg}$ in a spatial superposition, are predicted to become spin-entangled through their mutual gravitational interaction [1, 2]. The experiment (BMV, or gravitationally induced entanglement, GIE) is routinely glossed as a test of "the quantum nature of gravity." The gloss rests on a family of no-go results, if gravity is a classical mediator, no entanglement can be generated [2–4], and on a growing adversarial literature disputing either the no-go premises or their application [5, 6, 9–11].

What the field lacks is a *theorem for the experiment*: a single statement, in the style of Bell's theorem, listing explicit assumptions under which an observed, certified entanglement statistic implies a stated conclusion about gravity, together with the sharpest surviving alternative or a proof that none survives. The absence is not rhetorical. Three structural facts make the inference genuinely harder than Bell's, and each has been exploited in print.

First, *the antecedent "gravity mediated it" is not self-certifying.* The Newtonian entangling phase is produced by two mathematically identical descriptions, a mediated (polaron-type) exchange and a direct diagonal two-body gate, which agree on all mass-side data. Nothing in the observed state says which occurred. Any certification must therefore consume a *definition* of mediation, and the published candidates are either deferred to unpublished work

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[13], or reduce to circuit-decomposition conditions that we prove below are vacuous (Sec. III), a point sharpened by Di Biagio's observation that mediation in the circuit sense is gauge-relative [11, 12].

Second, *the protocol is deeply timelike*. The hold time $\tau \approx 2\,\mathrm{s}$ exceeds the light-crossing time $d/c \approx 0.7\,\mathrm{ps}$ by twelve orders of magnitude. No spacelike-separation argument is available anywhere in the analysis; the protections that in Bell tests are enforced by protocol must here be proved inside specific physics.

Third, *constrained gravity composes badly*. Linearized gravity fails local tomography strictly more severely than QED (the failure directions are coherences of the boundary Poincaré charges, including a gravity-only tier in which position itself is charged), and at order G the interaction-energy bookkeeping that would label branches is scheme-dependent [30]. Entanglement witnesses built from local observables can, in such theories, be blind to, or in engineered variants faked by, state components invisible to products of local observables [25–27, 29].

This paper supplies the missing theorem. Its lead contribution is the missing definition.

A. Contributions

(1) A formal definition of mediated entanglement (Sec. III). We first prove a trivialization lemma: in any compositional theory, *every* entangling bipartite gate admits an exact decomposition into mediator-local rounds (SWAP $\cdot$ U_{MB} $\cdot$ SWAP), so circuit topology alone, the reading of Galley–Giacomini–Selby's Condition 2 [3], and the "mediation" node of Di Biagio's trichotomy [11], cannot distinguish mediation from a direct interaction. Definition 3 then anchors mediation in spacetime: five clauses M0–M4 (interposition; autonomy; locally factorized coupling with a cut criterion; causal propagation at finite c_M; carriage), certified by three description-invariant signatures S1–S3 (retardation; interposed blockability; autonomy response). The definition passes adequacy controls on every configuration in the current debate (quantized exchange in; every classical configuration out, each at an identified clause); it supports a slaving dichotomy (Theorem 2): in constrained-Hamiltonian presentations a candidate mediator is either *slaved* (elliptic constraint; no independent data; entanglement onset at $t = 0^+$; fixed operator-Schmidt-rank-2 gate) or *autonomous* (hyperbolic; dead time $\geq d/c_M$), the two agreeing on all final-time mass-side data; and it is provably non-circular (Theorem 3): a fully classical theory (a classical delay line; physically, the Kafri–Taylor–Milburn channel [23]) satisfies every clause and every signature, and provably cannot entangle. Mediation is thus certified by falsifiable physics, and non-classicality then follows by theorem, not by definition.

(2) A per-run inertness theorem for transit coherences, and its activation boundary (Sec. IV). For the constrained models relevant to GIE we prove that the sector-dephasing map $\mathcal{D}$ (the conditional expectation onto the commutant of the region charges) controls everything physically measurable locally: on each experimental run, charge-transfer ("transit") coherences contribute exactly zero to every physical Bell functional and steering assemblage, statically, adaptively, one-sidedly, and after arbitrary physical local processing (a distillation no-go). We present this as a constrained-field-theory instantiation and strengthening of Baldijão et al.'s tomographically-nonlocal-entanglement propositions [25], with an explicit dictionary ($\mathcal{D} = \Pi_{\mathrm{TL}}$). We then exhibit the exact boundary of the protection: *jointly processing two runs* with strictly physical (charge-conserving) observables activates the transit coherences to the Tsirelson value $2\sqrt{2}$, standard superselection-rule entanglement activation [41, 42], here quantified for the gravitational models. This finding forces a load-bearing clause into the definition of a certified positive result: *the certified statistic must be a per-run i.i.d. correlator*. To our knowledge neither the activation boundary at GIE certification nor the per-run clause appears elsewhere.

(3) A continuum classical-mediator no-go, as a strengthening of Ludescher et al. (Sec. V). Ludescher, Loveridge, Galley, and Müller proved that a classical system modeled as a commutative unital C*-algebra, of any cardinality, cannot mediate entanglement under predetermined channel sequences [4]. Our Lemma C (the collective name we use for Theorems 6–7 and their corollaries, Sec. V) is the dynamical, instrument-level counterpart: classical–quantum (CQ) states as operator-valued measures on an arbitrary standard Borel configuration space (distributional field configurations included), with back-reacting instruments, feed-forward, arbitrary Markov-kernel classical processing and postselection over uncountable outcome spaces, and all trace-norm/weak-$*$ limits, including continuous time (Trotter–Kato–Chernoff) and mass-losing dynamics. Neither result contains the other (theirs is stronger on one axis: arbitrary C*-algebra locals). Specialized to Oppenheim's CQ gravity [6], the framework's footnote-level LOCC argument becomes a theorem, and the Diósi–Penrose-type Hermitian-commuting cross-kernel class yields *exactly zero* entanglement (Proposition 2); we reconcile this with Trillo–Navascués's result that the DP model *predicts* GIE [9] by an exact term split: their entangler is an outcome-feedback direct pair Hamiltonian, a statespace violation in Di Biagio's classification [11], and their Hermitian-commuting sector contributes exactly zero.

(4) The witness theorem (Sec. VI). Under six assumptions W1–W6, each stated with its physical meaning, verification status, and operational certificate, a certified positive result implies: the degree of freedom that coupled the two labs is non-classical, exhaustively, either (I) a mediator satisfying Definition 3, which is then provably not classical in any state-space sense (finite GPT-classical, continuum CQ, c-number, or locally-classical non-tomographic

at the regime), or (II) a nonseparable direct quantum gate with no carrier. Both disjuncts are non-classical; every classical-gravity model in the corpus is refuted on its own formulation (Corollary 4); the two readings are separated in principle only by retardation (Sec. VII). The certified conclusion is *non-classicality of the mediator in the state-space sense*, never "gravity is quantized general relativity": the hybrid van Hove model of Ulbricht, Bermúdez Manjarres, and Reginatto [10], whose mediator has exactly Liouville-classical marginal statistics but non-commuting observables, is *correctly classified* by a positive result, not refuted by it (Sec. VI C).

(5) The null-result converse, priced (Sec. VIII). At sufficient sensitivity a null certifies unconditionally that the gravitational channel imparts no second-difference phase; under linear phase additivity (Marletto–Vedral's own caveat) it refutes the standard quantized-coupling conjunction, with no intrinsic-decoherence escape, the invariant gravitational record is $N_g \approx 10^{-57}$ quanta, a floor independently corroborated at exact BMV parameters by Matsui [55]. A null never certifies "gravity is classical." A companion visibility protocol (E1) splits the null outcome space between the classical-channel class and modified-coupling quantum models [31].

(6) An honest Bell comparison (Sec. IX): four points where the analogy holds, four permanent disanalogies, engaging Di Biagio's opposite-moral comparison [11] point by point.

Every model-level claim in this paper is verified in deterministic ancillary scripts (Table III); bracketed tags [V] throughout indicate machine-verified statements, [cited] literature consumed as-is, and [U] declared open items (Sec. XI).

B. Relation to prior and concurrent work

The debate this paper enters moved quickly through 2025–2026; we state our position relative to each strand and return to each in context.

No-go theorems. The finite-dimensional GPT no-go is Galley–Giacomini–Selby [3]; the infinite-dimensional commutative-C* no-go is Ludescher et al. [4], which predates this work and holds the "first continuum no-go" priority; Sec. V states exactly what our Lemma C adds (instruments, feed-forward, continuous time, CQ-states-as-measures, the Oppenheim corollary, a sharpness suite) and what it does not (arbitrary C*-algebra locals). Angeli and Carlesso [32] gave the quantitative Gaussian-sector dividing line (an exact Kossakowski condition) a year before our instrument-level treatment; Proposition 2 is the measure-theoretic, exactly-zero statement on the Hermitian-commuting side of their line.

Mediation semantics. Marletto, Oppenheim, Vedral, and Wilson's comment [13] asserts a rectified mediation criterion but defers it to a reference in preparation; at the time of writing it has not appeared. Di Biagio [11] decomposes the GGS locality assumption into statespace/mediation/classicality and observes that every GIE-predicting classical model violates statespace or mediation, never classicality alone, a conclusion our scoreboard reproduces from the other direction (every such model lands in our transfer/modulation/direct-gate bins, Sec. III C). His "mediation," however, is the bare circuit-decomposition notion that Lemma 1 proves vacuous, he notes himself, via Baker–Campbell–Hausdorff, that Trotterization produces tripartite terms, without drawing the conclusion, and his framework contains no autonomy, cut, propagation, or carriage clause, no operational signatures, and no non-circularity theorem. His gauge-relativity analysis (Coulomb vs. de Donder) is the published form of exactly why our signatures are required to be description-invariant. Marletto and Vedral [14] use the polaron identity in the same argumentative role we do. Schneider, Huggett, and Linnemann [15] circle the constraint character of the Newtonian sector; Theorem 2 is the constrained-Hamiltonian statement with verified fingerprints.

The Aziz–Howl dispute. Aziz and Howl claim a c-number gravitational field produces GIE in relativistic QFT [5]. The diagnosis of that claim is now community consensus, published independently by Gundhi–Infantino–Bassi [16] (the diagonal-amplitude projection), Tang et al. [17] (cancellation failure; binding-potential suppression with $\kappa = \sqrt{2mE_b}/\hbar$), and Tibau Vidal–Iyer [18] (on-shell forcing; the carrier is the quantized matter field); see also [19, 20]. We claim no diagnosis priority. What this paper contributes on that front is rigor: a theorem-grade transport bound (Sec. VI A, W2) with $|\varepsilon| \le C(1 + V_0 t/\hbar)e^{-\kappa_{\rm eff}(d-2R)}$ and a relativistic bridging identity, closing the claim end-to-end in the static configuration with no unverified step, and the formal counterexample to Aziz–Howl's switch criterion (Sec. III C).

Tomography and readout. Baldijão et al. [25] proved at GPT level that tomographically-nonlocal entanglement yields no Bell violation or steering, and identified the multi-copy activation boundary abstractly (their Props. 12–15); Sec. IV instantiates and strengthens this in constrained field theory and quantifies the activation at the gravitational models. The tomography classification for linearized gravity that our assumptions W1/W3/W5 consume is the companion Letter [30]; Boulle and Franzmann [29] raise the subsystem-independence threat qualitatively; Spalvieri–Garmier–Giacomini [28] treat QED and name the gravity extension as open.

Retardation. That the graviton-consistency argument survives only if retardation in GIE is detected was published by Mitrakos, Papageorgiou, Perche, and Christodoulou [33], with earlier scoping by Fragkos, Kopp, and Pikovski

[34]. Our contribution is the placement of retardation *inside* a formal mediation definition as its unique in-principle discriminator (Sec. VII), with the direct gate as the unfalsifiable-in-the-limit $c_M \to \infty$ point, and the 0.7 ps feasibility pricing; we argue in Sec. VII that proposed alternative diagnostics [35] do not evade the polaron identity.

Hybrids. Trillo–Navascués [9] (with Feng–Vedral–Marletto [37] reaching the same moral) and Ulbricht–Bermúdez Manjarres–Reginatto [10] publish prima facie counterexamples to no-go slogans; Secs. V and VI C adjudicate both by name: neither touches any theorem proved here, and each lands exactly where the theorem's classification says it must.

II. SETTING: REGIME, PROTOCOL, AND WHAT A POSITIVE RESULT IS

A. Protocol class and regime

Two masses $m \approx 10^{-14}$ kg are held in adjacent Stern–Gerlach matter-wave interferometers: trap separation $d = 450\,\mu$m, spin-dependent superposition size $\Delta x = 250\,\mu$m, closest branch approach $d - \Delta x = 200\,\mu$m (the Casimir floor), hold time $\tau \approx 1$–2.5 s [1]. Branch phases are $\phi_{\xi\xi'} = Gm^2\tau/(\hbar\, s_{\xi\xi'})$ with $s_{\xi\xi'}$ the branch separations; the entangling second difference is $\Delta = \phi_{LL} + \phi_{RR} - \phi_{LR} - \phi_{RL}$. At the design geometry the branch values are $\Delta\phi_{RL} = 0.4395$ and $\Delta\phi_{LR} = -0.1256$ [V; this corrects the rounded values $0.7/-0.2$ of the original proposal's text, and is corroborated by the published witness value $W \sim 1.16$ [1] and consistent with the independent variant analysis of [46]], giving $\Delta = -0.314$ and concurrence $C = |\sin(\Delta/2)| = 0.1564$ [V, exact].

All claims are at the experiment's own order: matter-source/linear order in the mediator degrees of freedom, $O(G)$ in the phase. The run is deeply timelike: $\tau \approx 2$ s versus $d/c \approx 0.7$ ps. This drives disanalogy 2 of Sec. IX.

B. Certified positive result

Definition 1 (certified positive result)**.** A positive GIE result is the observation, above the certified noise/systematics budget, of a spin–spin entanglement statistic satisfying all three of:

(i) *Sound separable bound.* The statistic is the two-setting witness $W = |\langle\sigma_x \otimes \sigma_z\rangle + \langle\sigma_y \otimes \sigma_y\rangle|$ with separable bound 1 and design value $1 + C = 1.1563$ [V], or any physical Bell-grade statistic (the recombined BMV state supports physical spin-CHSH $= 2\sqrt{2}$ in one charge sector, with observables verified inside the lab algebras [V, Sec. IV]).[1]

(ii) *Budgeted electromagnetic exclusion.* The excess over the electromagnetic channel is budgeted explicitly: the Casimir–Polder/gravity ratio ≈ 0.05 at $200\,\mu$m is order-of-magnitude only [V], so certification requires either a stated margin on the EM contribution or the scan discriminator, multi-point (d, τ) families separate the gravitational $1/r$ phase scaling from the Casimir $1/r^4$ scaling within the planned campaign at no extra cost [31].

(iii) *Per-run statistic.* The certified statistic is a per-run (i.i.d.) correlator: each run is prepared, evolved, and read out independently, and the statistic is estimated across runs. No joint quantum processing of multiple runs enters the certificate. This clause is load-bearing: see Theorem 5.

C. Classical, and mediated

Definition 2 (classical)**.** Three senses, each excluded separately by Theorem 8: **(K)** kinematic/GPT, simplex state space, hypercube effects, standard composition (GGS Definition 1 [3]); **(D)** dynamical c-number, gravity a function with no state space; **(O)** ontological hybrid, classical phase-space variable with quantum back-reaction (CQ master equations, the Oppenheim class [6, 7]). The classical measure-space mediator of Sec. V (standard Borel Ω, CQ instruments) covers (O) and the stochastic completions of (D) at any cardinality. Throughout, "classical" means classical in this *state-space* sense; theories whose "classical" sector carries non-commuting observables (prequantum hybrids, van Hove [10]) are outside it by construction (Sec. VI C).

"Mediated" is Definition 3 below; stating it properly requires the negative result that motivates it.

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[1] The operator pair above is the one printed in the version of record [1] ("$W \sim 1.16$"). The widely downloaded arXiv-v1 preprint prints a different pair whose separable bound is $\sqrt{2}$ while its design value is 0.19–0.33; that pair does not certify. Teams working from the journal text need no correction; teams working from the preprint do. See also [45].

III. THE MEDIATION DEFINITION

The witness inference is a modus tollens: *if* gravity mediated the observed entanglement and gravity is classical, *then* no entanglement. Observation refutes the conjunction. To conclude non-classicality one must independently certify the other conjunct: that gravity *mediated* it. Three facts make this the bottleneck. (1) The antecedent is not self-certifying: the polaron identity [V], the Newtonian phase produced identically by a mediated description and a direct diagonal gate, means mediation cannot be read off the produced state. (2) The naive criterion is insufficient: Aziz–Howl's switch criterion ("H_0 alone does not entangle until $H_{\rm int}$ is on") is satisfied by configurations that are not gravitational mediation (Sec. III C). (3) The published formal criterion is deferred: GGS Condition 2 gives the right skeleton but, as we now show, circuit topology alone is vacuous; the rectification announced in [13] is unpublished.

A. The trivialization lemma

Lemma 1 (trivialization)**.** *In any compositional theory (quantum, classical, GPT) closed under ancillas and swaps, every entangling bipartite gate U_{AB} admits an exact circuit decomposition using only AM and MB gates for some system M: $U_{AB} \otimes \mathbb{1}_M = \mathrm{SWAP}_{A,M} \cdot U_{MB} \cdot \mathrm{SWAP}_{A,M}$ with M an ancilla in any fixed state. Hence "the circuit decomposes into mediator-local rounds" is satisfiable by any channel whatsoever and cannot, by itself, distinguish mediation from a direct interaction.*

The proof is the displayed identity, verified to machine precision for CZ [V]. The compilation is not physics: implementing $\mathrm{SWAP}_{A,M}$ then U_{MB} with A and B a distance d apart requires transporting M, at finite speed, with its own state. Circuit language erases exactly this. Two consequences fix the design: mediation must be defined over descriptions *anchored in spacetime* (systems and generator terms carry spatial support), and the work must be done by clauses the compilation cannot fake at a distance: autonomy, locality of couplings and of M's own dynamics, and carriage. This is also the precise sense in which Di Biagio's gauge-relativity of mediation [11] is correct, and why the operational content must live in description-invariant signatures, not in a favorite gauge.

B. Definition M and its signatures

Work in any compositional probabilistic framework enriched with located systems (A in region R_A, B in R_B, disjoint, separated by d; candidate mediator M) and located dynamics (a generator decomposition or round sequence, each term carrying spatial and system support). A *cut* Σ is a hypersurface separating R_A from R_B; a term *crosses* Σ if its support meets both sides; *severing* at Σ deletes every Σ-crossing term of M's own dynamics. Empirical content is the family of induced channels and correlations under all available interventions; mediation is asserted of the equivalence class of descriptions reproducing it.

Definition 3 (mediated entanglement)**.** Entanglement generated between A and B over $[0, T]$ is *mediated by* M iff the theory admits a description, reproducing its full empirical content, in which:

M0 (*interposition*) A, B, M are distinct systems and $\mathrm{Supp}(M)$ covers every interposing cut;

M1 (*autonomy*) M is a system, not a parameter or slaved functional: it has a state space with at least two states, its state is independent initial data (not fixed by constraint as a functional of A, B), and distinct M-preparations are dynamically admissible with A, B held fixed;

M2 (*locally factorized coupling; cut criterion*) $L = L_A + L_B + L_M + L_{AM} + L_{MB}$, where L_{AM} couples A only to M's degrees of freedom supported in or near R_A (with compact support or Lieb–Robinson-grade exponential tails), and symmetrically for L_{MB}; and for every interposing cut Σ, every Σ-crossing generator term is a term of L_M. Forbidden in one clause: bare A–B terms; M-conditioned A–B gates; classical feed-forward from M triggering a joint A–B gate; couplings of A to M-degrees of freedom at R_B;

M3 (*causal propagation*) L_M is local in M's own geometry with finite propagation speed c_M;

M4 (*carriage*) severing M at any single interposing cut renders the induced A–B dynamics non-entangling for all $t \le T$.

The associated description-invariant signatures are: **S1** (*retardation*) all $A \to B$ influence, including entanglement onset, is delayed by $\ge d/c_M$; **S2** (*interposed blockability*) operations on M alone, confined to the interposed region,

modulate or destroy the entangling power (severing; or continuous dephasing of a cut-set of M in any fixed local basis, which renders the crossing channel measure-and-prepare, hence LOCC, hence non-entangling); **S3** (*autonomy response*) the induced A–B channel depends on M's incoming state, and M carries transient records mid-flight.

Theorem 1 (signatures follow)**.** *M0–M4 ⇒ S1–S3, with S3's channel dependence generic rather than universal. [V at model level: mediator chains of length $k = 1, 2, 3$ give strictly increasing onset times $0.30/0.95/2.40$ with Lieb–Robinson tails; severing kills entangling power to 10^{-30}; cut-set dephasing suppresses by a factor 5700; channel trace distance 0.852 under M-preparations; transient $I(A{:}M) = 0.60$ bits mid-flight [63]. Lattice lift by standard Lieb–Robinson bounds.]*

Proposition 1 (partial converse; what certification means)**.** *Under the background axiom that physics admits some local causal description: if the observed channel family displays S1 with speed c_M, S2 under interventions confined to the interposed region addressing degrees of freedom other than A's and B's, and S3 under preparations of those degrees of freedom, then every description reproducing this content contains a subsystem satisfying M0–M4 with that c_M. Status: reconstruction argument, sound at model level; not proven at full GPT generality [U].*

Theorem 2 (slaving dichotomy)**.** *In constrained-Hamiltonian presentations exactly one of two structures obtains for a candidate M coupled to A and B:*

(a) Slaved (elliptic constraint: $M = F[\text{matter}]$). After reduction M has no independent data (M1 fails); the reduced dynamics contains a direct A–B term (M2 fails; for pairwise potentials, a diagonal gate); the induced channel has no dead time, entanglement from $t = 0^+$ at every separation, onset rate equal to the coupling, and is a fixed gate of operator-Schmidt rank exactly 2 at all times [V; at the corrected BMV phases: rank 2, concurrence $|\sin(\Delta/2)| = 0.1564$].

(b) Autonomous (independent Cauchy data, hyperbolic finite-c_M dynamics). M0–M4 hold for the exchange channel and S1–S3 follow; at intermediate times the A–B dynamics is not a gate at all (M carries A-correlations), and only the $t \to \infty$ compilation is gate-like.

The two structures agree on all final-time mass-side data (the polaron identity [V]) and disagree exactly in S1–S3.

Instantaneity and slaving are one failure seen dynamically and kinematically; retardation and autonomy are one property likewise. The elliptic/hyperbolic split is the standard constraint analysis [cited]; the continuum-gravity instance of the slaved branch (physical field state exactly vacuum in every branch; rank-2 gate) is proved in the companion Letter [30].

Theorem 3 (non-circularity)**.** *There exist theories in which M is classical in the full GGS Definition-1 sense and M0–M4 hold for a nontrivial correlating channel between quantum A and B. Construction [V]: a classical delay line of length L (register contents $\{\pm 1\}^L$, genuine independent initial data), written by a weak measurement at A, transported at one site per step, read by a local kick at B. Verified: retardation exact (influence 0 for $t \le L$, $O(1)$ after); autonomy (M's data acts on B before any signal from A arrives); carriage (severing kills all influence and correlation); correlations genuinely carried ($I(A{:}B) = 0.130$ bits); and entanglement exactly zero (PPT throughout): the channel is measure–transport–feedforward, LOCC across every cut.*

Theorem 3 closes the circularity loop: the definition nowhere encodes quantumness. A classical theory can be retarded, blockable, autonomous, and carrying, the Kafri–Taylor–Milburn channel [23] is this construction with the delay abstracted away, and Oppenheim's CQ theory is its field-theoretic instance, and what a classical M cannot do is *entangle through* that structure: by theorem (Secs. V, VI), not by definition. The witness logic is genuinely two-premise: mediation is certified by falsifiable physics (S1–S3); non-classicality then follows by theorem.

Classicality is a property of the joint class, never of the marginal. A Koopman–von Neumann phase-space mediator can keep its reduced state exactly diagonal at all times (off-diagonal 7×10^{-17}) while entangling $A{:}B$ maximally (negativity 0.5000) [V]: the joint state carries mediator-configuration coherences and the entangling degree of freedom is the KvN phase, which is non-classical; the identical circuit with a genuine classical register gives exactly 0 [V]. Classicality must therefore be judged on the joint state-space/dynamics class (CQ or simplex membership), never on the mediator's marginal or trajectory statistics. The published van Hove hybrid [10] realizes exactly this structure in canonical form (Sec. VI C).

C. Adequacy controls

Table I runs Definition 3 over every configuration in the current debate. Three rows deserve comment.

TABLE I. Adequacy of Definition 3: every configuration in the current debate, classified clause by clause. Evidence tags: [V] = machine-verified in ancillary scripts (Table III) or in the companion Letter's suites [30]; model rows are minimal exact models; magnitudes trace to verified computations.

Configuration	M1	M2	M3	M4	Entangles?	Verdict
Linearized QG, graviton exchange	yes	yes	yes	yes	yes	**mediation** [V-model + cited microcausality]
QED photon exchange	yes	yes	yes	yes	yes	**mediation** [V]
c-number field (Aziz–Howl [5])	no	no	,	,	(claimed)	**modulation** (carrier is matter) [V]
Diósi static background [21, 22]	no	,	,	,	no (exact product)	**no channel** [V]
Mean-field/Schrödinger–Newton	no	no	,	,	no	**modulation (feedback)** [V]
KTM classical channel [23]	yes	yes	yes	yes	no	**classical mediation ⇒ LOCC** [V]
Oppenheim CQ [6]	yes	yes	yes	yes	no	**classical mediation ⇒ LOCC** (Cor. 2)
Constrained direct gate (Newtonian sector)	no	no	no	,	yes	**direct gate** (reading II) [V]
Matter-transfer channel	no	no	,	no	yes	**transfer** (carrier is quantized matter) [V]
Separable-potential models [24]	no	,	,	,	no	**no channel**
Classical-label channels [20]	label only	no	,	,	no	**modulation (classical label)**

Linearized quantum gravity passes, with the subtlety confronted. In its covariant description $h_{\mu\nu}$ carries independent Cauchy data (M1), couples through the local integral $h_{\mu\nu}T^{\mu\nu}$ (M2), propagates hyperbolically at c (M3), and severing kills the channel (M4); microcausality is the exact statement that all influence is retarded [cited]. The same theory in its constraint-solved description presents the Newtonian part as a direct diagonal gate with only the TT sector autonomous. The existence quantifier of Definition 3 resolves the bookkeeping: the theory admits an M0–M4 description reproducing its full empirical content, which includes retarded response. What the quantifier cannot do, honestly, is certify that the *world's* Newtonian channel is that theory rather than a genuinely instantaneous direct-gate theory: by Theorem 2 the two differ only at retardation scale, unprobed at BMV parameters. That residue is Sec. VII.

The c-number field fails at the first gate. Φ enters as an external function: no state space, no preparations (M1). The cut criterion then localizes the would-be channel: every crossing generator term is a *matter* term, so whatever the amplitude arithmetic says, the carrier is quantized matter with Φ a multiplicative dial. The two-line counterexample to Aziz–Howl's switch criterion: $H = \Phi\, X_A X_B$ entangles iff $\Phi \neq 0$ (negativity 0.42 vs. 0 [V]), satisfying the criterion while the entangling operator is the matter coupling. Mediation implies switch-dependence; not conversely. The criterion is necessary, not sufficient, as [13] assert without proof, and as their laser and clock-triggered-harvesting examples illustrate (the Lemma assigns each system its correct role: the modulator is the c-number control; the carrier, where one exists, is the quantum field).

GGS Condition 2, absorbed and repaired. M2's cut criterion is their "sequence of transformations on AG and BG" with their own locality refinement made mandatory and their forbidden diagram excluded verbatim; the repairs are (1) vacuity (Lemma 1: spacetime anchoring becomes definitional), (2) physicality (their assumption that G has a state at all is promoted to clause M1 and made testable, S3), and (3) the spacelike guarantee (correct for the idealized protocol, inapplicable to the timelike BMV runs; S1's dead-time scan is the operative certificate).

IV. PER-RUN INERTNESS OF TRANSIT COHERENCES, AND ITS ACTIVATION BOUNDARY

Constrained gravity fails local tomography; the failure directions are coherences between eigensectors of the region charges ("transit" coherences), and an engineered locally-classical mediator can load them with negativity invisible to, or in bare readouts faked into, local witnesses [26, 27, 30]. Baldijão et al. [25] proved at GPT level, for finite-dimensional prepare-and-measure scenarios, that entanglement lacking a tomographically-local component yields no Bell violation (their Prop. 9) and no steering (their Prop. 10), that the statement is static over all states (their post-Prop.-11 remark), and that multi-copy processing activates it (their Props. 12–15 and Conjecture 1). This section *instantiates and strengthens* those results in the constrained-field-theory setting the witness actually inhabits. The dictionary is exact: the GPT projector Π_{TL} onto the span of product effects is realized as the sector-dephasing conditional expectation $\mathcal{D}(\rho) = \sum_s \Pi_s \rho \Pi_s$ onto the commutant of the region charges; the holistic subspace is the sector-off-diagonal (transit) directions; tomographically-nonlocal entanglement of ρ is entanglement of ρ with $\mathcal{D}(\rho)$ separable. What the constrained setting adds beyond [25]: the instrument-level (adaptive, multi-round, feed-forward) closure, their framework has no transformations; charge conservation as an algebra theorem rather than an assumption; one-sided steering against a trusted party with *unrestricted bare* tomography; LOCC covariance and a distillation no-

go; abelian-lab triviality; the exact activation quantification below; and the gravity scoping of Sec. IV C. We also take up their open problems on physical instantiation (their Sec. VIII) in exactly this setting.

A. Setting and theorem

The model class $\mathfrak{M}$: a physical Hilbert space H (constraints solved exactly); a complete family of orthogonal sector projectors $\{\Pi_s\}$, the joint spectral projectors of commuting region charges $\{Q_r\}$ (gravity: per-lab boundary charges, the 1D shadow of the Poincaré ten-plet; QED: per-lab Gauss charge; fermions: per-part parity); part algebras $A_P \subset B(H)$, mutually commuting, each commuting with all Q_r (charge centrality, in the models a theorem about the algebra, not an assumption [V]). Physical measurements and instruments are POVMs and CP instruments with elements/Kraus operators in the part algebras; classical communication is free. Verified instances: the gravity chains, QED chains, and the five-mode fermionic transport chain of [27]; in each, the span of products of part-local physical operators equals the charge commutant exactly (integer/mod-p rank certificates) [V].

Theorem 4 (per-run inertness; instantiation and strengthening of [25, Props. 9–10])**.** *Let $(H, \{\Pi_s\}, \{A_P\}) \in \mathfrak{M}$ and let ρ be any state. Then, on a single instance of ρ:*

(a) Uniform inertness. The complete outcome statistics of any scenario built from physical local measurements and instruments, with classical communication and adaptive multi-round strategies, coincide exactly for ρ and $\mathcal{D}(\rho)$; the transit component $\rho - \mathcal{D}(\rho)$ contributes exactly zero to every physical Bell functional, steering functional, and witness expectation.

(b) Bell uselessness. If $\mathcal{D}(\rho)$ is separable across a bipartition of the parts, no Bell inequality, in any scenario with any finite numbers of physical settings and outcomes, is violated by ρ: the physical behavior admits an explicit local-hidden-variable model.

(c) Steering uselessness, including one-sided. A physical Alice cannot steer even a Bob who holds unrestricted bare tomography: the full bare assemblage of ρ equals that of the one-sidedly dephased state, and if the latter is separable an explicit local-hidden-state model exists.

(d) No distillation. Physical local Kraus operators commute with $\mathcal{D}$; hence physical LOCC never converts transit coherence into Bell- or steering-usable form on the instance.

(e) Abelian-part triviality. A party whose physical algebra is commutative cannot pose a Bell test at all; single-mass gravity labs below the charge-multiplicity threshold are in this class [V].

Proofs reduce to three lemmas verified exactly in the ancillary suite: $\mathcal{D}$ is the trace-preserving conditional expectation onto the charge commutant; every physical product correlator is blind to the transit component; physical operations commute with $\mathcal{D}$ [64].

Instantiated at the models' mid-protocol transit states: the fermionic $\mathcal{D}(\rho)$ is an explicit two-term product mixture and the gravity $\mathcal{D}(\rho)$ a four-term product mixture [V, exact]; physical CHSH equals 2.000000 (LHV bound, attained) while *bare* (constraint-violating) observables reach $2\sqrt{2}$ exactly, and in the gravity models the enabling bare observable is precisely the dipole-charged, boundary-dressed displacement class, which self-flags in protected exterior data [V]. The delivered state of the fermionic engine has negativity $1.5 = 0.5$ usable (in-sector) $+ 1.0$ transit, with the transit part contributing exactly 0 to every physical correlator [V, exact].

The honest converse is part of the result. In-sector entanglement stays fully Bell-active under physical measurements: the recombined BMV state passes physical spin-CHSH at $2\sqrt{2}$ in one charge sector [V]. Theorem 4 does *not* by itself block the fermionic-style engine (which pumps genuine in-sector entanglement alongside transit coherence); the transport bounds and the engine's precondition failures carry that load jointly (Sec. VI, Step 2). What Theorem 4 establishes is the complementary protection: per instance, no fake living in the failure directions can pass a Bell/steering-grade readout, and whatever passes is in-sector, where sector tomography is exact and the GGS theorem applies with full force. The legitimate signal is maximally certifiable; the fake's resource is maximally uncertifiable.

B. The multi-copy activation boundary

Theorem 5 (multi-copy activation; exact boundary of Theorem 4)**.** *At two or more jointly processed copies the hypotheses of Theorem 4 correctly fail, and the protection is genuinely lost: for the transit states above, $\mathcal{D}_{\mathrm{ext}}(\rho \otimes \rho)$ is entangled (negativity 0.500), and strictly physical, parity-even, charge-conserving cross-copy observables reach*

CHSH $= 2\sqrt{2}$ *[V, exact [66]; controls: single copy* 2.000000*; dephased pairs* 2.000000*; per-copy inertness intact to* 3×10^{-33}*]. Mechanism: the second run supplies the matching coherence that acts as a reference frame, standard superselection-rule entanglement activation [41–44], here exhibited inside the constrained gravitational models.*

Theorem 5 is, to our knowledge, the first statement of this boundary at GIE certification, and it is why Definition 1(iii) exists. The witness is protected three independent ways: (i) the certified statistic is a per-run i.i.d. correlator, so the single-instance quantifier of Theorem 4 applies run by run; (ii) activated entanglement scales as the square of the transit amplitude, doubly re-suppressed under the transport bound of W2 ($\theta_{\text{eff}} \sim 10^{-O(10^7)}$); (iii) a channel pumping $O(1)$ activatable transit coherence is a coherent charge transporter, excluded independently (W2, and the regime supplies no locally-classical carrier). But the hardening claim itself is per-instance forever: abelian labs jointly processed across runs become non-abelian and can pose a Bell test. Standard i.i.d. correlator estimation, what every proposed campaign does anyway, satisfies clause (iii) automatically; exotic joint-measurement readouts would need separate analysis.

C. Scope

The inertness direction needs only charge centrality: continuum-standard for QED and fermion parity [47, 48], linear-order for gravity (lattice theorem plus continuum-native confirmation of charge invariance [30]; the operator-algebra lift is open [U]). The classification "failure directions = exactly the charge coherences" is lattice-exact, continuum-open [U]; only the converse uses it. $\mathcal{D}$-separability is a genuine hypothesis in QFT: local algebras are type III, the vacuum violates Bell inequalities across wedges [49], and separable states across regions need the split property, which fails for gravity nonperturbatively [50]; the LHV/LHS conclusions are conditional exactly there, blindness is not. At $O(G)$ the region charges themselves dissolve [30]: Theorem 4 is a linear-order statement for gravity, complementary to the dissolution mechanism. Non-abelian sector labels (irrep projectors, twirl $\mathcal{D}$) are expected mechanical but not done [U].

V. THE CONTINUUM CLASSICAL NO-GO, AS A STRENGTHENING OF LUDESCHER ET AL.

Ludescher, Loveridge, Galley, and Müller [4] proved the continuum kinematic/channel no-go: a classical system modeled as a commutative unital C*-algebra, of any cardinality, cannot mediate entanglement between quantum systems modeled by arbitrary unital C*-algebras, under countable alternating rounds and weak-$*$ limits. That result predates this work and owns the headline. Lemma C is its dynamical, instrument-level counterpart, and the two are logically incomparable: Lemma C adds (a) back-reacting instruments with feed-forward and outcome-conditioned operations over uncountable outcome spaces, arbitrary Markov-kernel classical processing and postselection (their theorem covers predetermined channel sequences; their appendix treats countable-outcome LOCC with finite-dimensional locals); (b) continuous-time dynamics (Definition 4 below, with Trotter–Kato–Chernoff control); (c) CQ states as operator-valued measures with an operator-valued Radon–Nikodým calculus and a martingale converse; (d) the Oppenheim-CQ corollary machinery; (e) a sharpness suite; (f) the observation that complete positivity is load-bearing (Remark 1). Conversely their result is stronger on one axis: arbitrary C*-algebra locals versus our type-I restriction. Their classicality clause and ours coincide (commutative mediator algebra / CQ state class); their assumed locally-tomographic composition is exactly the clause audited in [30]; and their open problems (operational classicality, multipartite) leave the continuum-GPT residue of Sec. XI open. The quantitative Gaussian-sector precedent of the dividing line below is Angeli–Carlesso [32].

A. Kinematics and dynamics

H_A, H_B are separable Hilbert spaces (type-I lab systems); (Ω, Σ) is an arbitrary standard Borel space, finite sets, $\mathbb{R}^n$, phase spaces, and distributional field configuration spaces $\mathcal{S}'(\mathbb{R}^3)$ (so Oppenheim's $z = (g_{ij}, \pi^{ij})$ is included); no reference measure, topology, or local compactness on Ω is used anywhere. A *CQ state* is a map $\hat{\rho} : \Sigma \to T(H_A \otimes H_B)$, positive, countably additive in trace norm, with $\mathrm{Tr}\hat{\rho}(\Omega) = 1$ (sub-normalized allowed). Since $T(H) = K(H)^*$ has the Radon–Nikodým property, every CQ state disintegrates, $\hat{\rho}(E) = \int_E \sigma(z)\, \mu(dz)$, with μ the classical marginal and $\sigma(z)$ a μ-a.e.-unique family of conditional states [proved]; this is the exact continuum form of the classical resolution of identity that powers the discrete GGS proof, with the measure-theoretic cost carried by the quantumness of A, B, not by any tameness of the mediator. SEP is the trace-norm closed separable cone [58, 59]; the invariant class $\mathfrak{S}_{\text{sep}}$

consists of CQ states with $\sigma(z) \in \mathrm{SEP}$ μ-a.e. (equivalently $\hat{\rho}(E) \in \mathrm{SEP}$ for all E, by a Banach-valued martingale differentiation argument [proved, cited: [60]]).

A *CQ operation kernel* on $(A; Z)$ assigns to each z a countably additive family $E \mapsto \Lambda(E|z)$ of completely positive maps on $T(H_A)$, measurable in z, trace non-increasing; this covers arbitrary classical processing (Markov kernels), z-conditioned local unitaries, and back-reaction instruments with feed-forward (measure A, kick z accordingly), and Oppenheim's finite-time CQ-Kraus maps are a Kraus-decomposable special case [proved by inspection]. A *mediated CQ circuit* is a finite composition of such channels each touching only (A, Z) or (B, Z); excluded is any round acting jointly on A and B.

Theorem 6 (discrete rounds; LOCC form)**.** *Any mediated CQ circuit maps* $\mathfrak{S}_{\mathrm{sep}}$ *into itself; the final AB state is separable and remains so under arbitrary classical postselection and local-instrument postselection. Under Kraus-decomposable kernels the process factorizes over classical trajectories,* $(\Phi\hat{\rho}_0)(E) = \int (\mathcal{E}_A^\omega \otimes \mathcal{E}_B^\omega)(\sigma_0(z_0))\, M(d\omega)$*: mediation is LOCC with a continuous classical communication register. [Proved; the exact continuum analog of GGS Eqs. (16)–(17); verified with no discretization of* z *anywhere in the main suite, max final negativity* 1.7×10^{-16} *over adversarial circuits [65].]*

Remark 1 (complete positivity is load-bearing). The tensor-stability step of Theorem 6 uses complete positivity: for merely positive kernels it *fails* in infinite dimensions (the transpose map has $\|(T \otimes \mathrm{id})|\Phi_d\rangle\langle\Phi_d|\|_1 = d$, unbounded on positives). Any weakening of the CP hypothesis must be re-examined, not assumed.

Theorem 7 (closure under limits)**.** *The separability verdict survives every limit in which the AB marginal converges even only weak-*$*$*, with or without trace conservation:* SEP *intersected with the sub-normalized cone is weak-*$*$ *compact (a local-compression argument), and on normalized states weak-*$*$ *sequential convergence upgrades to trace-norm automatically. Hence grid refinement, Trotter limits, cutoff removal, adiabatic elimination, and heating/mass-losing (non-conservative) dynamics cannot re-open the conclusion; the surviving normalized sector of an exploding dynamics is separable. Declared boundary: finitely additive/singular limit states ("probability at infinity") are outside the countably additive class by fiat, the continuum shadow of GGS's composition clause. [Proved.]*

Definition 4 (mediated continuous time)**.** A family $\{\Phi_t\}$ of CQ channels is mediated by Z if for every t and initial CQ state there are mediated circuits whose AB marginals converge weak-$*$ to Φ_t's. By Theorem 7, mediated continuous-time dynamics never entangles from $\mathfrak{S}_{\mathrm{sep}}$.

Corollary 1 (Trotter)**.** *If* $L = L_{AZ} + L_{BZ} + L_Z$ *is bounded, with each term generating CQ channels, Definition 4 holds unconditionally (Lie–Trotter). For unbounded generators it holds under Trotter–Kato–Chernoff conditions [cited]; for singular unbounded field dynamics the product formula is a checked hypothesis, not an automatic property, formally written pairwise generators do not certify mediation by themselves [U]. Classical-side continuity is free (transition kernels are single legal rounds); unbounded quantum Lindblad operators never obstruct (the theorem consumes finite-time channels, and non-conservative solutions are covered by Theorem 7).*

B. Sharpness, the Diósi class, and Trillo–Navascués

Every hypothesis is load-bearing [V]: the same alternating rounds with one *quantized* mode (the geometric-phase gate) give negativity 0.291 while the identical kicks on classical (x, p) give exactly 0, the entire difference is the symplectic form; a z-conditioned direct $X \otimes X$ gate gives 0.295 (mediation load-bearing); a non-Hermitian cross-Lindblad term (collective decay) gives 0.103 (locality load-bearing). Twenty adversarial three-round feedback-plus-postselection circuits: maximum negativity 2.26×10^{-16} [66].

Proposition 2 (Diósi–Penrose class: exactly zero)**.** *Cross-region pure-decoherence couplings among mutually commuting Hermitian local operators (pair kernels* $D_0(x, y)$ *on mass densities) are mediated in the sense of Definition 4, realized exactly by correlated classical noise on an enlarged register (an exact Gaussian unraveling, verified to* 2×10^{-15}*), hence non-entangling: the anticipated "small effect" is exactly zero. Non-Hermitian cross couplings genuinely entangle but are direct dissipative A–B gates: mediation violations feeding reading (II)'s bin, not classical mediation. Hermitian cross couplings entering drift/diffusion sandwiches are covered at derivation grade only; the non-commuting case is explicitly unclaimed [U].*

Corollary 2 (Oppenheim's CQ gravity)**.** *The CQ framework of [6, 7] (states, master equation, hybrid Kraus theorem, field-theoretic couplings) is a special case of the theorem's dynamical class under (H1) local Lindblad operators, (H2) no A–B cross blocks, including the cluster-separable delta-kernel case, or cross blocks only among commuting Hermitian operators, and (H3) Definition 4 (automatic when bounded). Then no A:B entanglement is ever created from separable data: the framework's own footnote-level LOCC argument becomes a theorem, and its "we would anticipate this to be a*

small effect" is upgraded to exactly zero for the covered classes. If the constraint algebra fails to close (the framework's own flag), coverage is vacuous, not evaded.

Reconciliation with Trillo–Navascués [9]. Their published result, the Diósi–Penrose model of classical gravity *predicts* GIE, is not a counterexample to Proposition 2. Their DP master equation splits exactly into (i) a real double-commutator: Hermitian-commuting dephasing, precisely Proposition 2's class, contributing exactly zero entanglement; plus (ii) an imaginary commutator $i\int[\mu(r)\mu(s),\rho]/\|r-s\|$, an effective direct Newtonian pair Hamiltonian generated by outcome-dependent feedback from pre-correlated weak-measurement instruments. Term (ii) is the entangler (with entanglement iff $d < d_c$, matching the Angeli–Carlesso line [32]), and their dynamics does not factor into pairwise field–mass rounds: it is outside Definition 4's mediator format, a *statespace violation* in Di Biagio's trichotomy [11] whose entangling piece is a direct A–B gate, i.e., the second disjunct of Theorem 8, not classical mediation. Feng–Vedral–Marletto [37] publish the same moral; the increment here is the quantitative term split: the Proposition-2 sector is exactly zero, so *all* of their entanglement is the direct gate.

Where full GPT generality is not reached, exactly: an infinite-dimensional GPT framework with Choquet-simplex classicality; tensor theory for infinite GPTs; a measurable-instrument Radon–Nikodým/disintegration theorem for GPT transformations (the deepest gap, here delivered by the RNP of $T(H)$, with no GPT analog); closedness/compression structure for the GPT separable cone. Independently outside: non-locally-tomographic composition (handled at the regime by [30]), dynamics nonlinear in the state, finitely additive idealizations, type-III locals. No known physical model lives in the continuum-GPT gap.

VI. THE WITNESS THEOREM

A. Assumptions W1–W6, with certificates

W1 (closed labs, with the erasability repair). All branch-dependent operations are generated in each lab's constraint-invariant local algebra; for the real Earth-anchored apparatus, A1′: every exterior branch record is dressed/slaved (recombinable, "false decoherence" [57]) and erased at recombination. *Status:* the closed-lab version is a theorem at matter-source order, interior dynamics conserves every region charge of the Poincaré ten-plet exactly [30]; A1′ is a criterion, [U] as a theorem, with a quantitative floor: the invariant gravitational branch record is the radiated content, $N_g \approx 1.3\times10^{-57}$ (worst-case recoil levers $\leq 6.4\times10^{-45}$), anchored in the Danielson–Satishchandran–Wald decoherence mechanism [53, 54] and independently corroborated at exact BMV parameters by [55].

W2 (transfer exclusion). No coherent matter transport contributes at witness order: $|\varepsilon(d,t)| \leq C(1 + V_0 t/\hbar)\, e^{-\kappa_{\rm eff}(d-2R)}$ with $\kappa_{\rm eff} = \sqrt{2mE_b}/\hbar\,(1 - E_b/4mc^2 + \dots)$, under bound constituents ($E_b \gtrsim 0.1\,$eV, Aziz–Howl's own premise) and static background; numerically $\log_{10}|\varepsilon| \leq -1.9\times10^7$, accessible transfer entanglement $\theta_{\rm eff} \sim 10^{-O(10^7\text{–}10^8)}$ [V, two independent routes]. This is the rigor completion of the published diagnosis [16–18]; the closure is end-to-end with no load-bearing open item *in the static configuration*; the pair-creation/time-dependent channel is sketch-bounded ($\sim 10^{-35}$) and reclassified (any pair-creation entangler is quantized-matter squeezing modulated by Φ, failing M2) [U, declared non-load-bearing]. **W2b:** electromagnetic screening, priced inside Definition 1(ii).

W3 (linear-order composition, $O(G)$ edge dissolved). The protocol composes at the order where sector structure is well-defined and, for the abelian charges actually computed, sector-wise tomography is restored exactly with standard tensor composition; the closed-lab protocol never occupies the failure directions [30]. At $O(G)$ no invariant which-branch energy record exists: admissible energy localizations disagree at $O(1)$ on every region share; only the total and the second difference $\Delta_2 E_{\rm int}$ are invariant, and $\Delta_2 E_{\rm int}$ *is* the phase [30]. *Residue:* non-abelian ten-plet in-sector structure is uncomputed, no branch-dependence mechanism identified, but the absence is an expectation, not a computation [U].

W4 (mediation semantics). "Gravity mediated the entanglement" is read per Definition 3; mediation claims are properties of empirical-content classes, certified by S1–S3; modulation, transfer, and direct gate are the definition-failing alternatives, each fingerprinted (Sec. III).

W5 (subsystem independence). A and B are distinct subsystems: mutually commuting part algebras, independent preparation, free local settings, initial state separable. Exact at linear order (part algebras are commutants of complements, sector-wise standard composition [30]); the *strong* form (local tomography) is known false for constrained gravity and deliberately not assumed. Nonperturbatively "parts" dissolve, the theorem's terms, not its conclusion, fail there [50–52]; cf. [29].

W6 (no superdeterminism; QM/Born for matter). (i) Setting choices statistically independent of the channel microstate; fair sampling. Unfalsifiable in principle [U by nature], the Bell-shared residue. (ii) Standard quantum mechanics, including the Born rule, for the matter and spin readout.

TABLE II. Operational certificates for the assumptions. E1 is the fixed-phase visibility-scan protocol of the companion note [31].

Assumption	Operational certificate	Grade
W1 / A1′	E1 pair/single visibility ratio on fixed-$\Delta\phi$ contours (the certificate is the visibility the campaign measures anyway; conspiracy channel budgeted, fails by > 4 orders)	designed; not yet run
W2 (binding)	material binding spectroscopy, eV scale, trivially satisfied by solids; evasion needs $E_b \lesssim 10^{-14}$ eV	trivial
W2 (static bkg.)	single-mass control interferograms (a driven background shows as which-path decoherence)	standard control
W2b	(d, τ) scan: $1/r$ vs. $1/r^4$ scaling	free within E1
W3	theorem-level only (abelian scope declared); no lab certificate	,
W4 / S1–S3	ps-scale timing, graviton absorption: *infeasible*, declared; hence the conclusion is a disjunction	honest gap
W5	preparation/calibration runs	standard
W6 (ii)	single-mass fringe statistics (Born)	standard
W6 (i)	not certifiable (as in every Bell test)	[U]

B. Statement and proof

Theorem 8 (witness theorem)**.** *Assume W1–W6 at the regime of Sec. II, and suppose the protocol returns a certified positive result (Definition 1, all three clauses). Then the degree of freedom that coupled the two labs is* ***non-classical****, in the following exhaustive sense. Exactly one of two structural readings holds; they agree on all mass-side data and differ only in the S1–S3 signatures (Theorem 2):*

(I) Mediation reading. *The channel satisfies Definition 3 with mediator M (the gravitational degrees of freedom). Then M is:*

(I.1) not a finite-dimensional GPT-classical system, simplex states, hypercube effects, and standard composition cannot mediate entanglement (GGS Theorem 1 [3]; every clause independently load-bearing [V]);

(I.2) not a classical measure-space (CQ) mediator of any cardinality, Theorems 6–7, Corollary 1, Proposition 2, Corollary 2; coverage of Oppenheim's CQ gravity scoped as stated there (bounded, block-local, delta-kernel/cluster-separable, and DP pure-decoherence realizations proved; Hermitian cross drift/diffusion sandwiches at derivation grade, non-commuting case unclaimed; singular unbounded dynamics under Definition 4 as a checked hypothesis) [U at the scoped edges];

(I.3) not a c-number field, doubly: a c-number Φ cannot instantiate reading (I) at all (fails M1; the cut-crossing term is matter's own: modulation, not mediation [V]); and independently its configurations produce no mediated entanglement anywhere in the debate's model class, exact product for the static background [V], the superselection-accessible remnant is gravity-off-surviving transfer at $O(\theta_{\text{eff}})$ [V], and the relativistic magnitude is dead by the W2 theorem [V];

(I.4) not a locally-classical mediator exploiting non-tomographic composition at the regime, the engineered engine needs a locally-classical carrier with ≥ 2 charge states (the regime supplies none: the constraint-slaved sector has no state space, the TT mode is locally quantum), branches leaving one joint sector (excluded by the closed-lab theorem), and sector-pumping local dynamics (charge-conserving physical circuits mediate exactly nothing) [30]; and by Theorem 4 applied run by run, which suffices, by Definition 1(iii), transit coherences contribute exactly zero to every physical Bell/steering statistic per instance, while the multi-copy activation channel (Theorem 5) is excluded by clause (iii) and, independently, doubly suppressed under W2;

(I.5) not classical mediation of any other kind, classical mediation is possible (Theorem 3) but classically mediated channels are LOCC across every cut: verified for discrete records [V], theorem-grade for finite GPT mediators [3] and for continuum CQ mediators (Sec. V). The sole remaining generality gap is the continuum-GPT class, in which no known physical model lives [U].

Hence in reading (I), M possesses independent states, couples locally, propagates causally, carries the entanglement, and no classical state space with any classical dynamics can do what it did. Non-classicality is the GPT-grade conclusion; quantized linearized gravity is the minimal known theory instantiating it, passing all of M0–M4 [V-model + cited microcausality].

(II) Direct-gate reading. *The channel fails Definition 3: constrained-direct-gate semantics, with no mediating degree of freedom at all, the Newtonian sector as pure constraint bookkeeping: no independent initial data (the field*

remains exactly in vacuum [30]), a direct diagonal A–B term after reduction, operator-Schmidt rank exactly 2 at all times, no dead time, the $c_M \to \infty$ point of the timing family. This reading concedes the certified entanglement and is analyzed in Sec. VII.

In either reading: *the observed entanglement is attributable to no classical carrier, and every classical-gravity model in the current literature is refuted on its own formulation (Corollary 4). What a positive result cannot do, from mass-side data alone, is discriminate (I) from (II) [V, polaron identity; principled by Theorem 2].*

Proof (assembly). Step 0 (well-posedness; the non-LOCC certificate). By W5 the $A{:}B$ cut and initial separability are well-defined at the regime; by W6(ii) and Definition 1(i) the certified statistic witnesses genuine final-state $A{:}B$ entanglement. Every entanglement witness is an LOCC monotone and the initial state is separable, so some element of the interposed dynamics implemented a non-LOCC channel across the cut.

Step 1 (localization to the gravitational sector). By W2b and Definition 1(ii), the electromagnetic channel is excluded with stated margin or scan discriminator. By W2, coherent transport contributes $\theta_{\text{eff}} \sim 10^{-O(10^7)}$, over a million orders below the certified $O(0.1)$ statistic. By W1, no boundary-anchored or exterior channel operated (exact charge conservation; A1′ residue $\leq 6.4 \times 10^{-45}$). The non-LOCC element lies in the gravitational coupling.

Step 2 (readout hygiene, per instance). By W1 and W3 all four branches stay in one joint charge sector; the certified entanglement is in-sector. Suppose the statistic were produced by occupation of tomography-failure directions. By Theorem 4(a)–(d) applied run by run, sufficient, since the certified statistic is a per-run i.i.d. correlator, the complete per-instance statistics equal those of the sector-dephased state, and the dephased image of any transit-carried state is separable with an explicit LHV/LHS model: contradiction. The multi-copy channel is excluded by Definition 1(iii) and independently by ε^2 re-suppression under W2. Independently of all of this, the engine's own preconditions fail at the regime [30].

Step 3 (the dichotomy). By W4 the mediation question is Definition 3's. By Theorem 2, in constrained-Hamiltonian presentations the channel is exactly one of slaved (direct-gate fingerprints) or autonomous (M0–M4 hold). The existence quantifier over empirically equivalent descriptions makes the cases exhaustive and exclusive once timing data are included; at BMV, which lacks timing data, both readings of the world's channel remain open, hence the disjunction.

Step 4 (classical exclusion inside reading (I)). Let M satisfy M0–M4 and suppose it classical in any Definition-2 sense. (K) finite: GGS Theorem 1 forbids Step 0's certificate. (O), and continuum-(K) within CQ: Theorems 6–7 with Corollaries 1–2 give separable finals under every admissible limit, contradiction; the DP cross-kernel case is exactly zero (Proposition 2). (D): a c-number has no state space, fails M1 before dynamics is examined; its would-be channel died at Steps 1–2. Nonlinear self-consistent (Schrödinger–Newton class): fails M1, preserves products exactly [V]; excluded on entangling grounds; its GPT shadow only is constrained [U]. Discrete classical records: LOCC by machine-verified theorem (Theorem 3). No classical option remains except the continuum-GPT class [U], with no known inhabiting model. Hence prongs (I.1)–(I.5).

Step 5 (characterization of reading (II)). If the channel fails Definition 3, by Step 3 it is the slaved structure, fingerprint fully verified (vacuum field, rank 2, no dead time, rate = coupling). It is not a classical *mediator* model, it posits no mediator; its gate is a nonseparable quantum interaction on the joint matter system, which is what Step 0 certified. Every corpus classical model asserts strictly more and is excluded by Steps 1–4 on its own terms. Steps 0–5 exhaust the cases. □

Corollary 3 (conditional collapse)**.** *If one additionally accepts the relativistic-embedding argument, a strictly instantaneous direct A–B gate admits no consistent relativistic completion [3, 56], then reading (II) is excluded and a certified positive implies a non-classical mediating degree of freedom. Status: [cited], never re-derived here; the theorem proper does not consume it.*

Corollary 4 (scoreboard)**.** *Under W1–W6, a certified positive result refutes, each on its own formulation: the Aziz–Howl c-number claim [5] (independently closed by W2); the static-background class [21, 22]; mean-field/Schrödinger–Newton; separable-potential models [24]; classical-label channels [20]; the decohered-field benchmark of [1]; Kafri–Taylor–Milburn [23]; Oppenheim CQ gravity (all realizations satisfying its own locality/cluster structure, per Corollary 2); and every finite-GPT-classical mediator [3]. "Classical" throughout means classical in the state-space sense of Definition 2; hybrids whose classical sector carries non-commuting observables are outside it by construction and are classified, not refuted (Sec. VI C).*

C. What the conclusion is not: the van Hove scoping

The hybrid van Hove model of Ulbricht, Bermúdez Manjarres, and Reginatto [10], in which two qubits are entangled through a mediator whose *marginal* statistics are exactly Liouville-classical, does not intersect this theorem's classical

class and does not evade its conclusion. We have reproduced their central calculation exactly (closed forms re-derived by characteristics match direct quadrature to 3×10^{-16} per density-matrix element; concurrence 0.178 at coupling $g^2/m\omega^3\hbar = 1$) [67]: the entanglement is real. But the model's classical sector is *prequantum*: its observables are the Kostant–Souriau prequantization operators, which satisfy $[\hat{O}_q, \hat{O}_p] = i\hbar$ exactly [V], and its states are phase-space wavefunctions whose action phase is physical and is precisely the degree of freedom that entangles, deleting it (passing to Koopman–von Neumann transport of the same densities) yields exactly zero entanglement at all times [V], while its secular entangling rate equals the fully quantum polaron rate [V], so "classicality" does no moderating work in the model's predictions. The model therefore falls outside classicality as defined here (CQ state space), outside GGS Definition 1 (simplex), and outside the commutative-C* class of [4], in each case by the same single clause, the non-classicality of the mediator's joint state space and observable algebra, and not by any failure of complete positivity or of the mediation format (its evolution is unitary and its coupling pairwise; it plausibly satisfies M0–M4). A positive witness outcome accordingly does not refute van-Hove gravity; it *correctly classifies* it, since the theorem certifies exactly the structure this model possesses by construction: a mediator with non-commuting observables carrying coherent phase. The certified conclusion of Theorem 8 is non-classicality of the mediator in the state-space sense, never "gravity is quantized general relativity." Van Hove hybrids show this residual gap is inhabited (their own abstract concedes the mirror-image point: consistent *quantum* theories featuring classical gravity); distinguishing its inhabitants from quantized gravity requires mediator-sector discriminators (product-algebra and uncertainty-relation physics), not the entanglement witness. The qualitative ancestor of this diagnosis is the Hall–Reginatto versus Marconato–Marletto exchange [38–40].

VII. THE SOLE SURVIVING ALTERNATIVE, AND ITS DISCRIMINATOR

Reading (II), exactly. The gravitational interaction at BMV is the constraint-solved Newtonian term: a direct diagonal gate $U = \exp(-i \sum_b \phi_b |b\rangle\langle b|)$ with (i) no independent mediator data, the physical field state exactly vacuum in every branch [30]; (ii) operator-Schmidt rank exactly 2 at all times, concurrence 0.1564 at the corrected phases [V]; (iii) no dead time, onset at $t = 0^+$ at every separation, rate equal to the coupling [V]; (iv) autonomy-free, no S3 response, nothing to sever. It is the $c_M \to \infty$ point of the one-parameter timing family in which mediation occupies every finite c_M.

It is a reinterpretation, not a counterexample, for three verified reasons: (1) it *concedes the entanglement*, the gate is the entangler; it rescues no classical model (every model in Corollary 4 predicts no certified witness value and dies on the observation itself); (2) it supplies no carrier, there is nothing for "classical?" to be asked of; the witness target shifts from "the field is quantum" to "the interaction is a nonseparable quantum gate with no autonomous field degree of freedom at this order"; (3) it is mass-side-identical to mediation (the polaron identity), so the closure is principled, not accidental. We verified the definition is not rigged to exclude it: the direct gate is *admitted and named*, not defined away, and classical autonomous hyperbolic configurations (classical GR waves) satisfy M0–M4 and are excluded by the theorems of Sec. V, never definitionally [V].

The unique in-principle discriminator is retardation (signature S1, Theorem 1). Mediation at speed c_M predicts dead time $\geq d/c_M$ in entanglement onset and all $A \to B$ influence; the direct gate predicts onset at $t = 0^+$, exhibited side by side in one verified run [V]. Experiments bound c_M from below; the direct gate is the unfalsifiable-in-the-limit $c_M \to \infty$ point, the same epistemic structure as any locality test. At $d = 200\,\mu$m the required resolution is $\sim 0.7\,$ps on a $10^{-14}\,$kg interferometer: infeasible. Interposition (no graviton absorber; radiated content 10^{-57}-class) and source/autonomy variation are worse. This convergence, that the graviton-consistency argument goes through only if retardation is detected, is independently the published conclusion of Mitrakos et al. [33], with the sharpening that retarded phases alone are determined by the classical equations of motion; see also [34]. What this paper adds is the theorem-grade placement: S1 is a description-invariant signature *inside* a formal definition whose slaving dichotomy proves no mass-side quantity can substitute for it. In particular, proposed state-side diagnostics (e.g. fidelity-susceptibility measures [35]) cannot discriminate the readings: by the polaron identity the two produce identical mass-side states at all times, so any discriminator must consume timing, interposition, or mediator-state data. The one on-paper decider is Corollary 3's embedding argument, deliberately left outside the theorem proper.

VIII. THE NULL-RESULT CONVERSE

Sufficient sensitivity: the decoherence budget, independently certified by single-mass controls, leaves $1+C = 1.1563$ resolvable with margin; a multi-point (d, τ) scan excludes the measure-zero accident $\Delta = 2\pi n$; sampling is fair (W6).

Theorem 9 (null converse)**.** *At sufficient sensitivity, a null result certifies unconditionally: the gravitational channel imparts no second-difference phase, branch phases compose additively at the probed regime (entanglement $\Leftrightarrow \Delta \neq 2\pi n$ is exact [V]). Interpretation is then conditional on linear phase additivity (Marletto–Vedral's own caveat [U]):*

(i) Under it, the null refutes the conjunction {quantized gravity $\wedge$ linearized coupling $\wedge$ QM for matter at these scales}. Quantized gravity has no intrinsic-decoherence escape: the invariant record is $N_g \approx 10^{-57}$, independently computed at exact BMV parameters by Matsui [55] ($\Gamma_{\rm vac} = 8.7\times10^{-61}$), so a well-budgeted null cannot be blamed on gravitational self-decoherence. A budgeted null genuinely bites.

(ii) Without it, the null is uninformative about mediator quantumness: nonlinearly composing quantum mediators also predict null (cf. [36]). Not closable by the experiment itself.

A null never certifies "gravity is classical": all classical models in Corollary 4 predict null, but so do modified-coupling quantum models; affirming the consequent is unavailable. (Consistently, [9] note in closing that GIE-dynamics tests can falsify specific collapse dynamics; the pricing above is the general form of that observation.)

The contribution of Theorem 9 is assembly and pricing, not a headline; its two legs are anchored in [9, 36, 55]. The companion E1 protocol [31] sharpens the null space: *null + full visibility at design phase* also kills the surviving classical-channel class, Kafri–Taylor–Milburn and Oppenheim-CQ models are forced by the decoherence–diffusion trade-off [8, 23] to an $O(1)$ visibility deficit at $\Delta\phi = O(1)$, cornering both camps and pointing at modified coupling or nonlinear composition; *null + deficit collapsing onto a function of* $\Delta\phi$ is positive evidence *for* the classical-channel class: the first laboratory discrimination between the hypothesis classes.

IX. HONEST COMPARISON WITH BELL'S THEOREM

Where the analogy holds. (1) *Logical form*: Bell, {local causality $\wedge$ measurement independence $\wedge$ fair sampling} $\Rightarrow$ CHSH ≤ 2; observed violation refutes the conjunction. Witness, {W1–W6 $\wedge$ classical gravitational channel} $\Rightarrow$ no certified witness value; observation refutes the conjunction, with Secs. III–V localizing the failure at classicality. (2) *Theory-independence of the excluded class*: all finite GPT-classical mediators, all continuum CQ mediators, all c-number configurations, all locally-classical non-tomographic fakes at the regime, per instance, an enumerated-then-closed class rather than Bell's single stroke, with declared small residue (continuum-GPT [U], no known inhabiting model). (3) *The unfalsifiable escapes*: superdeterminism for Bell; here superdeterminism *plus* the $c_M \to \infty$ direct gate, each the limit point of a family falsifiable at every interior point. (4) *Loophole discipline*: detection/locality $\leftrightarrow$ transfer (closed by theorem); exterior records (A1$'$ + E1); tomography fake (closed per instance by Theorem 4, multi-copy channel excluded by protocol plus suppression); $O(G)$ edge (dissolved [30]).

Where it fails, four disanalogies, permanent. (1) *Bell needs no premise about mechanism; the witness cannot avoid one.* Production is not readable from the produced state (polaron identity), hence W4, with no Bell analog, and hence the direct-gate reading, which has no Bell-side counterpart. (2) *Bell's assumptions are protocol-enforceable; W1–W3 are theorems inside specific physics.* BMV is timelike through and through; no-signaling enforces nothing; the protections are calculated, theory-laden in a way light cones are not. (3) *Bell delivers a verdict; the witness delivers a disjunction.* Both disjuncts are non-classical, Bell-grade *against classicality*, but "a quantum gravitational degree of freedom exists" is certified only up to retardation. (4) *Bell's parts are clean; gravity's are not.* Constrained gravity fails local tomography, sector labels dissolve at $O(G)$, subsystems are scheme-dependent nonperturbatively; the protocol had to be *proved* to avoid the pathological directions [30]; Bell had no such burden.

Di Biagio [11] runs a Bell comparison to the opposite moral: that GIE-predicting classical theories violate statespace or mediation rather than classicality, so the witness targets the wrong assumption. The four disanalogies above are our answer in structured form. His statespace-violation category is exactly where the Trillo–Navascués dynamics lands (Sec. V) and where reading (II) circles; his mediation category, read as circuit decomposition, is vacuous by Lemma 1; once mediation is given operational content (Definition 3), the disjunction of Theorem 8 is precisely the residue of his observation, and both of its branches are non-classical, which is the Bell-grade part of the conclusion. His gauge-relativity analysis is, we emphasize again, the published reason the signatures must be description-invariant, and our Sec. VII agrees with his bottom line to this extent: the ontology question ("is there a quantum mediator?") is settled by a positive result only up to retardation.

Net. The witness theorem stands to "is gravity a classical channel?" as Bell stands to "is nature locally causal?", comparable rigor on the exclusion side, one honest step short on the ontology side: Bell violation refutes a worldview; certified GIE refutes a class of theories and *bounds* the worldview question by a single, sharply characterized, currently untestable alternative.

X. EXPERIMENTAL GUIDANCE

1. *The deliverable of a positive run is the disjunction, not the slogan.* A certified positive (all three clauses of Definition 1) licenses: "no classical degree of freedom carried this entanglement, either a non-classical mediator did, or the interaction is a nonseparable direct quantum gate with no carrier." "Gravity is quantum" additionally consumes either ps-scale timing (infeasible) or the cited embedding argument (Corollary 3), and should be flagged as doing so. Nor does a positive distinguish quantized gravity from prequantum hybrids (Sec. VI C); the certified phrase is "non-classical in the state-space sense."

2. *Run the E1 visibility scan as part of the main campaign* [31]. It is the operational certificate for W1/A1′ (the assumption a referee will attack first), it costs the visibility measurement the campaign performs anyway, its conspiracy channel is budgeted (fails by more than four orders), and its cross-structure is the only laboratory handle on the classical-channel class in the null branch (Sec. VIII); cf. the noise-threshold analysis of [61] and the entanglement-free alternative of [62].

3. *Design the* (d, τ) *scan in from the start.* It excludes the $\Delta = 2\pi n$ accident (needed by Theorem 9), supplies the $1/r$-vs-$1/r^4$ electromagnetic discriminator (now a clause of Definition 1), and is the natural home of any future retardation bound (every finite c_M is falsifiable; only the limit point is not).

4. *Keep the certified statistic per-run.* Definition 1(iii) is load-bearing: joint quantum processing of multiple runs voids the per-instance protection (Theorem 5). Standard i.i.d. correlator estimation satisfies it automatically; exotic joint-measurement readouts need separate analysis.

5. *Use the sound witness pair.* The two-setting witness with separable bound 1 and design value 1.1563, or an in-sector physical CHSH; teams working from the journal text of [1] need no correction, teams working from the arXiv-v1 download do (footnote to Definition 1).

6. *Certification map.* Every assumption that can be lab-certified has its control named (Table II); the two that cannot (W6(i); W4's signatures) are declared, which is the honest analog of Bell's own residue.

XI. LIMITATIONS AND OPEN ITEMS

The theorem is a conditional certification; its strength is exactly its ledger's. Machine verification covers model instances and exact algebraic identities; the generality claims of the three lemmas rest on the proofs summarized here, and their full statements, together with all [V] items, are reproducible from the ancillary suite (Table III). The open items, ranked by what they gate:

1. *Retardation* (the empirical half of the modulation-vs-mediation fork): discriminates readings (I)/(II); infeasible at BMV parameters; on paper only via the cited embedding argument (Corollary 3) [U].

2. *Non-abelian ten-plet in-sector structure*: uncomputed; no branch-dependence mechanism identified, but the absence is an expectation, not a computation. Gates W3's residue and the non-abelian extension of Theorem 4 [U].

3. *Continuum-GPT classical no-go*: the last generality step of prong (I.5); Sec. V lists exactly what a proof would need; no known physical model lives there [U].

4. *A1′ as a theorem*: currently criterion, quantitative floor, and designed certificate (E1). The referee-facing assumption [U].

5. *Lemma-C edges*: Definition-4 conditionality for singular unbounded generators; Hermitian cross drift/diffusion sandwiches at derivation grade, non-commuting case unclaimed [U].

6. *Per-instance scope of Theorem 4 is permanent*: the multi-copy activation is real and blocked at the witness only by protocol plus suppression [U].

7. *Continuum lifts*: gravity charge centrality beyond the lattice; span-equals-commutant in the continuum; $\mathcal{D}$-separability under type-III/split-property failure [50] [U].

8. *Converse of Proposition 1* at GPT generality; *nonlinear-in-state dynamics* (excluded on entangling grounds, GPT shadow only constrained); *no superdeterminism* (unfalsifiable, Bell-shared); *linear phase additivity* (gates only Theorem 9(i)) [U].

TABLE III. Ancillary verification suite (deterministic, `python3.12`, numpy only; byte-identical reruns; exit 0). The base suites of the companion Letter [30] are consumed as cited there.

Script	Self-tests
`lemma_M_checks.py` (mediation models)	23/23
`lemma_T2_checks.py` (inertness; exact fermionic layer)	26/26
`lemma_C_continuum_checks.py` (continuum no-go)	25/25
`attack_suite.py` (adversarial: activation, frames, KvN, feedback)	4 attacks
`van_hove_adjudication.py` (reproduction of [10])	18/18

Items consumed as [cited] and never re-derived here: microcausality of linearized QG; the DSW decoherence anchor [53–55]; the embedding argument [56]; Trotter–Kato–Chernoff; the decoherence–diffusion trade-off [8]. A referee should check that each supports the exact weight placed on it. All models are low-dimensional exact constructions chosen to isolate clauses; every physical magnitude quoted traces to a verified computation, not to experiment; the claims are structural (theorems and fingerprints), not quantitative predictions beyond the protocol's own arithmetic. Finally, the comparison instrument for the announced mediation criterion of [13] is Table I: extensional agreement on every published example is already established for their prose hints; a material divergence, if their criterion appears, would land on the declared residue of Sec. VII, not on the assembly.

XII. CONCLUSION

The question the entanglement-witness experiments were designed to answer has, until now, lacked the theorem that would let a positive result answer it. The bottleneck was semantic, "mediated" had no formal content that circuit topology could not fake, and the repair is Definition 3: mediation as interposition, autonomy, locally factorized coupling, causal propagation, and carriage, certified by retardation, blockability, and autonomy response, non-circular by construction, and equipped with a slaving dichotomy that says exactly why mass-side data cannot settle the question. Around it, three lemmas with stated scopes close the classical class at every level at which any published classical model is actually formulated, and the assembled theorem prices every assumption, names the one survivor, and hands the experiments their deliverable sentence: a certified positive result means *no classical degree of freedom carried this entanglement, either a non-classical mediator did, or the interaction is a nonseparable direct quantum gate with no carrier*. The two are separated by retardation, and by nothing else.

Acknowledgments., Numerical certificates, verification suites, and drafting made use of AI-assisted tools; all results were adversarially verified as described, and the author takes full responsibility for the content.

Data and code availability., The ancillary scripts of Table III reproduce every [V]-tagged claim deterministically and will be posted with the preprint.

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[63] Ancillary file `lemma_M_checks.py` (23/23 self-tests): trivialization lemma; signature suite (onset times, Lieb–Robinson tails, severing, cut-set dephasing, autonomy response); adequacy rows (c-number counterexample, static background, mean-field, transfer); classical delay line (Theorem 3); slaving dichotomy fingerprints.
[64] Ancillary file `lemma_T2_checks.py` (26/26; exact integer/`Fraction`/mod-p arithmetic in the fermionic layer): conditional-expectation lemmas; blindness; LHV/LHS constructions; one-sided steering; distillation no-go; price list (physical 2 vs. bare $2\sqrt{2}$); gravity-chain instantiation; abelian-lab triviality.
[65] Ancillary file `lemma_C_continuum_checks.py` (25/25): exact complex-Gaussian continuum circuits (no discretization of z); grid-refinement enactment; trajectory/LOCC factorization; sharpness suite; Trotter approximants; Diósi-class unraveling identity.
[66] Ancillary file `attack_suite.py` (adversarial): multi-copy activation (Theorem 5) with single-copy and dephased-pair controls; shared-reference-frame lift with physical-frame control; Koopman–von Neumann mediator; twenty feedback-plus-postselection CQ circuits (max negativity 2.26×10^{-16}).
[67] Ancillary file `van_hove_adjudication.py` (18/18): exact reproduction of [10] by characteristics; CCR of the van Hove operators; resource localization (Koopman–von Neumann transport of the same densities: exactly zero); quantum control.