Local tomography of linearized gravity under its constraints

David Vávra[1, *]

[1] *Independent Researcher, Prague, Czech Republic*

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Proposed tests of gravitationally induced entanglement (GIE) infer a non-classical mediator from local spin correlations. This inference assumes local tomography: that products of local observables separate states of the composite. Constrained theories can violate it, and an explicit fermionic protocol shows the violation can fake entanglement witnesses through a locally classical mediator. We settle, in exactly solved models, the status of this assumption for linearized gravity. In constrained lattice models, with all algebraic claims in exact integer arithmetic certified modulo two primes, the product-observable span equals the commutant of a per-region charge multiplet fixed by the kernel of the adjoint constraint map: the boundary Poincaré ten-plet $\{E, K_i, P_i, J_i\}(R)$, versus QED's single Gauss charge $Q(R)$, including a tier with no electromagnetic analog in which position itself is charged. Within a joint charge sector, tomography and standard tensor composition are restored exactly. Interior-generated dynamics conserves every region charge, so a closed-lab GIE protocol never occupies the failure directions at matter-source order: the constraint structure polices its own loophole. At order G, where branch interaction energy equals the witness phase, the would-be refinement dissolves, the interaction-energy assignment is scheme-dependent, the corrected commutant moves with the scheme, and no invariant branch splitting survives the admissible scheme family, leaving as invariant record only radiated gravitons, $N_g \sim 10^{-57}$. QED protects mediation witnesses by conservation; linearized gravity protects them by dissolution. This supplies the explicit construction named as a central open goal by Spalvieri, Garmier, and Giacomini.

Introduction., The Bose–Marletto–Vedral (BMV) proposal [1, 2] would certify a non-classical mediator from gravitationally induced entanglement between two matter interferometers, via general no-go theorems for classical mediators [3, 4]. Every such theorem carries a composition assumption, *local tomography*: products of local observables separate states of the composite [3, 5]. The assumption is not innocent. Fermionic parity superselection already violates it [6], and Tibau Vidal, Marletto, Vedral, and Chiribella [7] exhibited a protocol in which a mediator that is classical in every local sense delivers negativity 1.5 by exploiting exactly this violation. Constraints (Gauss laws) are symmetry restrictions, and restricting a theory to symmetry-covariant processes makes tomographic failure generic [5]; a bare statement that "gravity's constraints break local tomography" is therefore nearly content-free. The questions with content are: *which* directions of state space become invisible to local products, and *does the GIE protocol populate them*? For QED, a framework for gauge-invariant local operations without a local tensor product was recently constructed by Spalvieri, Garmier, and Giacomini [8], who state that the extension to linearized gravity "remains a central open goal" and that an explicit construction is lacking. Here we give that construction, classify the failure exactly, and prove that the BMV protocol never occupies it at matter-source order, then show that at $O(G)$, the first order at which the gravitational analogy to charge might bite, the threat dissolves rather than being merely bounded.

All results below are theorems in exactly solved finite models of the constrained theories (three QED control geometries; four lattice models of linearized gravity), with every span and commutant computed in exact integer arithmetic and dimensions certified by modular rank at two primes; continuum statements are labeled as such. Every number quoted is produced by a deterministic, self-testing ancillary script (Table I); two independent adversarial verification suites re-derived closed forms, upgraded floating-point claims to exact arithmetic, and searched for the results that would have overturned the conclusions (Supplemental Material (SM), §I, §IX–X).

Setup., For a region R, the local physical algebra $\mathcal{A}_R$ is the algebra of constraint-invariant operators supported on R (for fermions, additionally parity-invariant): each "part" is matter plus its local field, populated by Dirac/ADM-dressed operators. Two hygiene facts hold exactly in every model: $\mathcal{A}_R$ equals the commutant of its complement's algebra (lattice Haag duality, so localization is construction-independent), and "products separate states" coincides with "local algebras generate the global algebra" (SM §II). Local tomography in the unrestricted (GPT) sense [3, 7] is then

$$\mathrm{span}\{AFB : A \in \mathcal{A}_1,\, F \in \mathcal{A}_F,\, B \in \mathcal{A}_2\} \stackrel{?}{=} \mathcal{B}(\mathcal{H}_{\mathrm{phys}}), \tag{1}$$

and sector-wise tomography is the same test restricted to a joint eigensector of the per-region constraint charges. Following [10], entanglement splits into tomographically local (TL) and tomographically nonlocal (TNL); at GPT level TNL entanglement is unusable for Bell-type and steering readout. The BMV signal is a local spin-correlation witness, so this taxonomy is the relevant discrimination tool (instantiated for our constrained models in SM §VII).

The loophole engine of [7], reproduced exactly in our

formalism (algebra dimensions 8/2/8; the paper's witness value $-1/2$ recovered), needs three ingredients: (N1) a locally classical mediator, a commutative local physical algebra with at least two locally distinguishable charge states; (N2) a populated non-tomographic resource, physical coherence between values of a joint sector label; (N3) sector-pumping local dynamics. Its anatomy is sharper than previously noted: of the delivered negativity 1.5, only 0.25 survives a local particle-number twirl, 1.25 is charge-transfer coherence, and the protocol's own product-observable witness $\langle \hat{X}_1 \hat{X}_2 \rangle = -1/2$ is reproduced *entirely* by the sector-dephased state [38]: where the engine exists, the loophole fakes even product witnesses, so a rescue of the BMV inference must show gravity lacks the structure at the protocol's regime, not that the structure is benign. A transport-necessity theorem closes the third ingredient: mediator-charge-conserving physical circuits through a locally classical mediator mediate exactly nothing (maximum negativity 2×10^{-15} over 40 random circuits) [38].

QED control., With the Gauss law imposed exactly and no gauge fixing, every geometry gives, exactly,

$$\mathrm{span}\{AFB\} \;=\; \{Q_1,\, Q_2\}', \tag{2}$$

the commutant of the region charges [36]. Every deficit of Eq. (1) is a coherence between superselection-type labels: total-charge coherences, inter-region charge coherences, the direction [7] exploits, with local parity replaced by local Gauss charge, and pure spatial topology, absent in $\mathbb{R}^3$ and in BMV geometry (tier structure, Wilson-string witness pair, and the Gauss-center edge-mode restriction [11, 17, 18] in SM §III). Within any joint (Q_1, Q_2) sector, parts compose as a standard tensor product and arbitrary in-sector states, including matter–photon entangled states, are reconstructed exactly by linear inversion from product correlations; a two-lab mediation protocol coupled only through the field delivers lab–lab negativity 0.474 while remaining exactly in one charge sector [36]. The control verdict: constraint-induced non-tomography per se does not gut a witness, QED has it, and photon-mediation witnesses are fine, because the failure directions are exclusively charge-transfer coherences and a mediation protocol never excites them. This poses the sharp question for gravity: what are the per-region boundary charges, and do the four BMV branches differ in any of them at witness order?

Classification for linearized gravity., We discretize the linearized Hamiltonian constraint in the form $(\partial_i\partial_j h_{ij} - \nabla^2 h)/16\pi G = \rho$ (as in [14]), together with the momentum constraint, verbatim on the lattice. The charge content is fixed by the kernel of the adjoint constraint map: smearing functions f with $(\partial_i\partial_j - \delta_{ij}\nabla^2)f = 0$ and vector fields ξ with vanishing symmetrized gradient. QED's first-order Gauss law has kernel = constants, hence the single charge $Q(R)$. Gravity's second-derivative constraint has kernel = affine functions $\{1, x^i\}$ (rank exactly 4 in 3+1, by direct computation), giving region energy $E(R)$ *and* the energy dipole/boost triple $K_i(R)$; the momentum constraint adds the six flat Killing vectors, giving $P_i(R)$ and $J_i(R)$: the matter Poincaré ten-plet, each element a linearized-ADM surface integral [15, 16, 37] (a scalar Poisson-type toy would over-protect; the tensor structure protects exactly the ten-plet).

Theorem 1 (Failure classification)**.** *In each model, exactly,*

$$\mathrm{span}\{AFB\} \;=\; \{E(R_a), K_i(R_a), P_i(R_a), J_i(R_a)\}'_{a=1,2}, \tag{3}$$

with every part-local physical operator conserving all region charges, part algebras equal to the commutant of their complement, and all charges exterior-collar field operators on $\mathcal{H}_{\mathrm{phys}}$ *[37].*

Linearized gravity therefore fails local tomography *strictly more* than QED. Against the QED tiers: Tier 0 becomes superselection of total $\{E, K\}$, "the Hamiltonian is a boundary term" at the algebra level [14, 16]; Tier 1 becomes region-energy coherence invisibility, with clock superpositions playing the role of charge superpositions; and there is a gravity-only tier with no QED analog:

Position is charged. A single mass in a superposition of two interior positions is invisible to every product of strictly local physical observables (deviation 0 at trace distance 1), detectable only by a dressed displacement whose dressing reaches the boundary. A compactly supported dressing exists iff the monopole *and* dipole of the rearrangement vanish (verified exhaustively over 125 source rearrangements; in QED, monopole only), the lattice-exact content of the gravitational no-screening property of Donnelly and Giddings [12].

Corollary 1 (Classicalization of small systems)**.** *A region too small to host internal multiplicity at fixed charges (charge-degenerate pairs, spins) has an abelian physical algebra: constrained linearized gravity classicalizes isolated small systems [37].*

Sector restoration. Within a joint eigensector of all region charges, composition is a standard tensor product and tomography is exact: the product span is complete (576/576 in the largest model) and arbitrary in-sector states, including a field-excited, matter–field-entangled state ($\langle n \rangle = 0.149$, negativity 0.335), are reconstructed by linear inversion to machine precision [37, 38]. At linear order the field carries a genuine noncommutative local invariant algebra (the transverse-traceless (TT) sector, the analog of $F_{\mu\nu}$), which is what makes the mediator a self-contained tomographic part; beyond linear order no local invariants exist at all [12, 13], so this certificate is linear-order-only. The no-go theorem of [3] then applies *in-sector* with full force: in our models a classical mediator delivers negativity 4×10^{-16} and a quantum mode mediator 0.500 [37, 38].

Non-occupancy., The classification would threaten the witness only if the protocol populated the failure directions. It does not:

Theorem 2 (BMV non-occupancy)**.** *Every part-local physical operator conserves every region charge exactly (two-line proof: region charges are boundary terms plus constraints, and interior-supported invariant operators commute with both; SM §V). Hence all branch-dependent operations generated in the labs' interiors keep the four BMV branches* $\{LL, LR, RL, RR\}$ *in a single joint charge sector: at matter-source order the protocol never occupies the non-tomographic directions, and the certification runs entirely in a sector where tomography and tensor composition hold exactly [37, 38].*

The constraint structure that creates the failure directions is the same structure that polices them. Three supporting facts. (i) The coherence the witness actually uses, each mass's left/right path coherence, is locally visible (interference contrast 2.00), i.e. TL, not the Vidal resource [38]. (ii) The worry that region *energy* superselects operationally proves too much: a referenceless energy superselection rule kills ordinary gravity-free Ramsey fringes ($0.454 \to 0$ in the model); interior clock coherences live at fixed region charge via internal reference frames [19–21], and unlike Gauss charge, regional energy is pumped by the joint coupling itself ($[V_{\mathrm{Newton}}, H_{\mathrm{lab}}] \neq 0$), a structural disanalogy cutting toward *less* usable sector structure (SM §V). (iii) The in-sector phase is real, not an artifact of protection: branch pairs recoil-compensated to exactly equal (E, K) per lab still acquire a second-difference phase through unprotected higher multipoles in three dimensions (-1.7×10^{-4} against an uncompensated reference -7.2×10^{-4}; the one-dimensional affine kernel gives exactly 0, a declared artifact of that toy) [42].

Engineered transit self-flags. Branches engineered to differ in a region charge do transit the failure directions: mid-protocol, the entangling coherences are invisible to every product of local physical observables while the product-visible state carries zero entanglement [38]. But such transit broadcasts a classical which-branch record into protected exterior data; its split/recombine unitaries are boundary-anchored strings, not lab-local; and its mid-flight coherences are TNL, hence unusable for the Bell/steering-grade readout the certification needs [10], while the final BMV entanglement is TL (residuals and details in SM §VII). The unique $O(1)$-transport channel on the BMV composite is coherent matter transfer through the field region, a channel that survives with gravity switched off, hence certifies nothing about the mediator, and is exponentially suppressed by tunneling bounds through the inter-lab barrier [29, 30]; we assume it excluded at that level (assumption A2 below).

Real apparatus. Laboratory Stern–Gerlach splittings are Earth-anchored: recoil compensation lodges outside any lab-sized region, so real branches are weakly dipole-distinct, formally in the weak transit class. The operative protection is then *erasability*: the exterior record is dressed and slaved (recombinable, false decoherence in Unruh's sense [28]), not radiated or amplified, the distinction made sharp by Danielson, Satishchandran, and Wald [24]. Observed $O(1)$ interference visibility certifies operationally that no resolvable record formed. We state this as a criterion (A1′), not a theorem.

The $O(G)$ dissolution., At second order the Hamiltonian constraint's charge stops being matter-only: it absorbs field interaction energy, and the four branches differ in candidate region energy by $\delta E \sim Gm^2/d \approx 3.3 \times 10^{-35}\,\mathrm{J}$ ($m = 10^{-14}\,\mathrm{kg}$, $d = 200\,\mu\mathrm{m}$), i.e. $\delta E\, t/\hbar \approx 0.6\,\mathrm{rad}$ at $t = 2\,\mathrm{s}$. The marginality is an *identity*: the witness phase *is* the branch interaction-energy difference times $t/\hbar$, so any run tuned to $\Delta\phi = O(1)$ sits at $O(1)$ would-be sector-splitting action by construction, unimprovable by parameter choice. If region energy superselected at $O(G)$, the witness would be compromised at its own operating point, always. It does not, for a structural reason:

(i) No invariant assignment exists (exact, bookkeeping level). Admissible energy localizations, densities differing by superpotential divergences [32, 33], agree exactly on the total $-Gm_1m_2/d$ and disagree at $O(1)$ on every region share: the field scheme ($-|\nabla\phi|^2/8\pi G$ cross term) assigns each lab ball exactly $\max(0, 1 - d/a)$, i.e. zero for every lab-sized ball, with the share beyond radius r falling as d/r (half of δE beyond $r = 2d \approx 1\,\mathrm{mm}$); the source scheme $\frac{1}{2}(\rho_1\phi_2 + \rho_2\phi_1)$ puts half in each lab; an asymmetric scheme all in one lab; a stress-inclusive (Tolman) scheme routes it through the holding hardware (exact static virial $\int T_{kk} = +Gm_1m_2/d$); and an explicit bounded improvement current ($|f| \leq 1.2 \times 10^{-28}\,\mathrm{J/m^2}$) transports the entire δE across any lab boundary at zero total change [39, 43]. A quantity with no scheme-independent region-level value is not a region observable of the constrained theory; a non-observable cannot superselect. Only two energy objects survive every admissible reassignment: the asymptotic charges, branch-*independent* for the closed protocol, and the second difference $\Delta_2 E_{\mathrm{int}}$, which *is* the witness phase (0.63 rad at the parameters above; 0.31 rad for the published BMV geometry [1], branch phases $\Delta\phi_{RL} = 0.439$, $\Delta\phi_{LR} = -0.126$, consistent with the $W \sim 1.16$ quoted there).

(ii) The failure directions themselves dissolve (exact, operator level). Extending the lattice models to $O(G)$, the corrected constraint solved exactly at first order in formal g, with the quadratic term discretized three admissible ways and its energy redistributed four admissible ways, gives: the linear-order span-equals-commutant theorem re-proves exactly (dimension 100); the *corrected* commutant is 36 under every interior scheme but 100 under the boundary scheme, the commutant itself moves with the scheme; a pre-derived witness pair of branch

configurations has exactly equal charges in one discretization and unequal in another (region self-energies 845 vs 845; 1412 vs 1378); and under one assignment the branch record sits in lab 2, under another in lab 1. Reachable joint-sector counts across schemes are $\{1, 3\}$ for the equal-shift protocol (protocol-dependent generally: unequal shifts reach $\{1, 4\}$), the branch→sector map is not a property of the physics. Decisively, *the result that would have established a genuine $O(G)$ sector, an invariant splitting surviving both scheme axes, was searched for explicitly, under a verifier-extended family (five discretizations, fresh geometries and primes), and does not exist* [40, 41]. Moreover the corrected region charges are not even central: an explicit lab-local hop conserves the linear charges yet fails to commute with the lab's corrected charge ($[E_R^{(2)}, H_{\rm hop}] \neq 0$), gravitational energy flux across boundaries is real at $O(G)$, so coherence between corrected-charge values is ordinary, erasable coherence. One honest wrinkle is carried openly: refusing all superpotential reassignment ("native-only" discretizations), a partner-record splitting survives discretization variation; it is removed by the constructively admissible assignment axis and is independently disqualified as superselection by non-centrality, non-conservation, and product-visibility (SM §VIII).

(iii) No candidate label survives the trilemma. A sector label able to decohere the branches must be gauge-invariant, central (conserved under all local physical operations), and branch-dependent. Asymptotic ADM charges: invariant, central, branch-independent (equal per-branch totals to 10^{-70} relative; an exact coherent-state battery model shows an agent's reservoir absorbs the branch-dependent work while recording nothing: record exponent $(\delta W/\Delta E_{\rm res})^2/8 \approx 2 \times 10^{-40}$) [39]. Scheme densities: branch-dependent but not invariant (above). Fixed quasilocal charges (Brown–York/Wang–Yau class [22, 23]): invariant and branch-dependent but not central, non-unique at exactly O(field energy), and not conserved. Soft/IR sectors: the branches recombine in position and velocity, net memory zero, and the finite-time dressing difference contains exactly the radiated content below (SM §VIII).

(iv) The invariant record is radiated, and negligible. By the criterion of [24–27], the irreversible branch record is the radiated (or horizon-crossing) graviton content. For the published BMV parameters ($m = 10^{-14}\,$kg, $d = 450\,\mu$m, $\Delta x = 250\,\mu$m, C^2-smooth 0.5 s ramps, 2.5 s hold; exact profile integral $\int_0^1 (w''')^2 du = 720$): $N_g = 1.3\times10^{-57}$, visibility loss $\approx 6\times10^{-58}$; the static hold radiates nothing; a worst-case branch-correlated apparatus-recoil lever ($L = 1\,$km) reaches only 6.4×10^{-45}; the cosmological-horizon channel is $\sim 10^{-149}$ [39]. An independent computation of graviton which-path decoherence at BMV parameters recently found the same order class [31]. Where an $O(G)$-sector reading needs $O(1)$, the invariant record supplies 10^{-57}.

Mechanism split, verified at $O(\alpha)$. The identical $O(\alpha)$ analysis on the QED control keeps the Gauss charge exact, quantized, central, conserved, and branch-independent, while the Coulomb interaction-energy label refines and scheme-moves exactly like gravity's [40, 41]: QED protects mediation witnesses by *conservation* of its region charge at all orders; linearized gravity at $O(G)$ protects them by *dissolution*, the would-be branch-dependent label fails to exist as a region-level invariant (SM §VIII).

What a positive witness certifies., Under (A1) closed labs, enforced, not assumed, at matter-source order by Theorem 2, or, for the real apparatus, (A1′) erasure of dressed exterior records, certified operationally by $O(1)$ visibility; (A2) matter-transfer channels excluded at the published tunneling-bound level [29, 30]; and (A3) linear-order composition, with the $O(G)$ edge dissolved as above (its residue: the non-abelian in-sector structure of the ten-plet, computed nowhere, and continuum lifts of the lattice theorems, standard but unproved except where noted), a positive BMV result certifies genuine, in-sector, tomographically local spin–spin entanglement not attributable to a locally classical mediator exploiting non-tomographic composition. The regime supplies none of the three loophole ingredients: no locally classical carrier with charge states (the constraint-slaved Newtonian sector is not a system but a direct diagonal gate of operator-Schmidt rank 2 with the field exactly in vacuum, at the verified witness value $1 + C = 1.1563$; the independent TT degree of freedom is locally *quantum*) [38]; no occupied non-tomographic resource (Theorem 2); no sector-pumping channel at witness order [37]. What survives of the composition loophole is not tomography but semantics: the constraint-slaved channel may be *read* as a direct two-body gate, a reinterpretation that supplies no alternative carrier and concedes the entanglement, equivalent to the mediation-definition question raised elsewhere [3, 7], and discriminable in principle only by retardation-class measurements. The corrected slogan: a positive result certifies a *non-classical mediator or constrained-direct-gate semantics*, "non-classical" in the mediator-state-space sense of the no-go theorems [3, 4], a conditional certification, not an unconditional theorem. One experiment-facing corollary is immediate: since any genuine gravitational branch record is bounded by 10^{-44} even under extreme recoil levers, visibility-versus-time at fixed $\Delta\phi$ is a clean discriminator between quantized-gravity-plus-erasability (zero intrinsic loss) and classical-channel models, which tie an $O(1)$ irreversible-noise rate to the entangling rate [34, 35].

Relation to prior work., Spalvieri, Garmier, and Giacomini [8] constructed gauge-invariant local operations without a local tensor product for QED and named the gravitational extension as a central open goal with no explicit construction available; this Letter supplies such

a construction, in a Hamiltonian constrained-lattice formalization rather than their generalized-LOCC one, together with the exact classification (ten-plet vs Gauss charge, including the position tier) and the protocol-level occupancy verdict, neither of which has a QED analog. Tibau Vidal et al. [7] built the loophole this Letter audits; we add its exact anatomy and the proof that constrained gravity does not instantiate it at the BMV regime. Ludescher, Loveridge, Galley, and Müller [4] prove a continuum classical-mediator no-go *assuming* locally tomographic composition, precisely the clause audited here; our sector-restoration and non-occupancy theorems discharge that assumption for linearized gravity at the protocol's regime and order. Boulle and Franzmann [9] attack the same inference qualitatively via AQFT dressing and split-property failure; our classification is the exact, quantitative counterpart at linear order, and their concern maps onto assumption A1′. The twirled-worlds perspective [5] explains why the bare violation carries no weight; the TL/TNL taxonomy and its readout-uselessness theorem are [10]'s at GPT level. Fermionic non-tomography is classical [6]; edge modes and the Gauss center at lattice level are [11]; the absence of local invariants beyond linear order and boundary dominance of gravitational information are [12–14]; the radiated-vs-dressed record distinction is [24, 27].

Limits., All theorems are exact in finite lattice models; the 3+1 kernel ranks (4 and 6) are direct computations, but continuum lifts of the span-equals-commutant theorems are standard-but-unproved (linear-order per-region ADM charge invariance is verified continuum-natively; charge superselection and dressing non-localizability carry their known continuum scope [17, 18]). Holography-of-information statements carry Raju's stated scope (asymptotically flat and AdS; compact slices open) [14]. The in-sector structure of the *non-abelian* ten-plet ($[K_i, E] \neq 0$: sectors are irreps, not joint eigenvalues) is computed nowhere and is the designated refuge of the loophole, parametrically beyond the witness phase, but open. The $O(G)$ record bound is continuum-quantitative (lattice TT excitation is exactly zero, partly structurally); sector counts are protocol-dependent as stated. These limits are inherited by, and stated in, the certification above.

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* Vavra.88@email.cz

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TABLE I. Ancillary files (deterministic, self-testing, Python 3.12; exact integer/mod-p/rational arithmetic for all load-bearing algebra). Two struck legs of `continuum_og_energy_scheme.py` (its S2 gauge-orbit numerics) are documented and replaced by closed forms in SM §VIII; quoted numbers are the corrected ones.

File	Self-tests
`qed_local_tomography.py`	23/23
`gravity_local_tomography.py`	29/29
`vidal_loophole_bmv.py`	21/21
`continuum_og_energy_scheme.py`	27/27
`og_sector_algebra.py`	37/37
`og_sector_algebra_verify.py` (adversarial)	22/22
`insector_phase_3d_check.py`	9/9
`improvement_current_bound.py`	8/8

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[36] Ancillary file `qed_local_tomography.py` (23/23 self-tests): QED control models, tier classification, sector reconstruction, mediation protocol.
[37] Ancillary file `gravity_local_tomography.py` (29/29): gravity models, kernel ranks, the classification theorem, position tier, abelianization, sector restoration, conservation theorem, in-sector GGS values.
[38] Ancillary file `vidal_loophole_bmv.py` (21/21): loophole anatomy, transport-necessity theorem, BMV scorecard, transit self-flagging, Newtonian-sector operator-Schmidt structure, Ramsey reductio.
[39] Ancillary file `continuum_og_energy_scheme.py` (27/27 as run; two struck legs documented in SM §VIII): scheme family, virial, battery algebra, second differences, radiated record N_g and robustness.
[40] Ancillary file `og_sector_algebra.py` (37/37): $O(G)$ lattice constraint, discretization and assignment axes, corrected commutants 36/100, witness pair, sector counts, centrality failure, QED $O(\alpha)$ control.
[41] Ancillary file `og_sector_algebra_verify.py` (22/22): independent adversarial verification, fresh discretizations, geometries, primes; exhaustive search for an invariant splitting (negative).
[42] Ancillary file `insector_phase_3d_check.py` (9/9): recoil-compensated equal-(E, K) branch pairs, exact moment audit, 3D $1/r$ versus 1D affine second-difference phases (committed re-materialization of a verification-suite check).
[43] Ancillary file `improvement_current_bound.py` (8/8): constructive bounded improvement current transporting the full $O(G)$ interaction energy across a lab boundary at zero total change (committed re-materialization of a verification-suite construction).