Supplemental Material for "Local tomography of linearized gravity under its constraints"

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CONTENTS

Models and conventions 3
QED control models 3
Gravity models 3
Physical algebra construction 4
Exactness methodology 4
Verification protocol 5

Hygiene lemmas 5

QED control (full) 6

The loophole engine, exact anatomy 7

Gravity classification (proofs) 8
Kernel of the adjoint constraint map 8
Main theorem 8
Tier table 9
The position tier 9
Non-occupancy and the operational disanalogy 10

Sector restoration and in-sector certification 11

Transit, self-flagging, and the real apparatus 12
Engineered transit 12
Self-flagging 12
The unique $O(1)$-transport channel 12
In-sector phase reality 13
The real apparatus (criterion A1′) 13

The $O(G)$ dissolution (full) 13
Continuum scheme analysis 13
Struck legs, documented (correction history) 15
The lattice $O(G)$ algebra route 15

The honest wrinkle (native-only splitting) 17
QED $O(\alpha)$ contrast (mechanism split) 17
The trilemma 18
The radiated record (DSW) computation 18
Reconciling $\delta E\, t/\hbar = \Delta\phi$ 19

Claim → evidence table 20

Honest-limits ledger 21

References 21

MODELS AND CONVENTIONS

QED control models

Three exactly solved compact-$U(1)$ lattice gauge models, Gauss law imposed exactly, no gauge fixing [`qed_local_tomography.py`, 23/23 self-tests]: (A) a chain with two charge labs and a slaved Coulomb field (physical dimension 11); (B1) a two-path geometry whose field region carries a genuine propagating photon mode, the local gauge-invariant field strength (Wilson loop) is an element of $\mathcal{A}_F$ and does not commute with the local electric operators (physical dimension 39); (B2) a ring, on which Gauss's law forces $Q_{\text{tot}} = 0$ and a global holonomy exists (physical dimension 18).

Gravity models

Four exactly solved constrained lattice models [`gravity_local_tomography.py`, 29/29]. The constraint is the discretization of the linearized Hamiltonian constraint in the form used by Raju [9], $(\partial_i\partial_j h_{ij} - \nabla^2 h)/16\pi G = \rho$, second order in derivatives, sourced by matter *energy*, together with the momentum constraint; on the 1D chain it reads $\Delta^2\pi_i = \rho_i$, solved exactly by the right-tail Green function $G(i,j) = \max(i-j, 0)$: (K) a kernel-window model isolating the adjoint-kernel computation; (GA) a single-mass two-lab chain (physical dimension 9; the field fully slaved, the constrained sector proper); (GB) a two-lab chain hosting the recoil-compensated "breathing" pair $A = (0,2,0)$, $B = (1,0,1)$ (equal region energy and

dipole) and the dipole-differing configuration $C = (2, 0, 0)$, with a transverse-traceless (TT) qubit as the independent field degree of freedom, added by hand and declared as such (dimension $18 = 3 \times 3 \times 2$); (GN) a BMV spin model, (2 positions $\times$ 2 spins)^2 (dimension 16). The $O(G)$ extension adds the models M1/M1b and the exact-rational kernel model of `og_sector_algebra.py` (Sec.).

Declared 1D artifacts, carried throughout: the affine Green function (no propagating constrained modes; the constrained channel is exactly diagonal), and the TT mode added by hand as the independent field degree of freedom. Their quantitative consequences are flagged where they enter (Secs. , ,).

Physical algebra construction

For a region R, the local physical algebra $\mathcal{A}_R$ is the algebra of constraint-invariant operators supported on R (for fermions, additionally invariant under global parity), never a bare-subsystem algebra. Each "part" is matter plus its local field; Dirac/ADM dressing is what populates the algebra.

Exactness methodology

All operators are exact matrices on $\mathcal{H}_{\text{phys}}$ (integer entries in the constrained bases). Every span-equals-commutant claim was upgraded from float numerics to exact integer linear algebra with modular-rank certification at two primes by the adversarial verification suites (E below), which re-ran every model. The certification logic: since $\text{rank}_p \leq \text{rank}_{\mathbb{Q}} \leq \dim(\text{commutant})$ (the last step by exact generator-wise containment), equality of the mod-p rank with the combinatorial commutant dimension $\sum_k n_k^2$ at either prime forces equality over $\mathbb{Q}$. Within the *committed* ancillary package this exact integer/mod-p certificate is carried by `og_sector_algebra.py` ($p_1 = 10^6 + 3$, $p_2 = 998244353$), which re-proves the flagship linear-order geometry (dimension 100, the main-theorem count) exactly and certifies every $O(G)$ algebra claim, re-certified by `og_sector_algebra_verify.py` at fresh primes (10^9+7, $2^{31}-1$); the $O(G)$ kernel model runs entirely in `fractions.Fraction`. The other committed scripts (`qed_local_tomography.py`, `gravity_local_tomography.py`, `vidal_loophole_bmv.py`) compute the same ranks in floating point (SVD at declared tolerances) on the exact op-

erator matrices; their exact-arithmetic certification was performed in the verification suites, which ran from session scratchpads and are not part of the committed package. Floating point otherwise appears in declared demonstrations (protocol negativities, continuum quadratures). All scripts are deterministic (byte-identical reruns, Python 3.12); fresh runs on 2026-08-03 reproduce every self-test count quoted in the Letter.

Verification protocol

Each program stage was audited by two independent adversarial verification suites (proof rigor; physics), which re-ran all scripts, re-derived closed forms, upgraded floating-point claims to exact arithmetic, added and killed steelmen (e.g. the soft-graviton sector, Sec. F), and searched explicitly for the results that would have overturned the conclusions. Two verification-suite results cited by the Letter are re-materialized as committed ancillary scripts: `insector_phase_3d_check.py` (9/9; Sec.) and `improvement_current_bound.py` (8/8; Sec. A). Struck and corrected items are documented in Sec. B and Sec. ; the Letter quotes only corrected numbers.

HYGIENE LEMMAS

Lemma S 1 (Localization is construction-independent)**.** *In every model, each part algebra equals the commutant of its complement's algebra (lattice Haag duality), verified exactly on both labs, including the suspect slaved-field side, where field labels could have smuggled in the other lab's matter. They do not: charge-class conditioning is carried by the local Coulomb/ADM tail itself, i.e. by the Gauss/ADM center.*

Lemma S 2 (Equivalence of separation criteria)**.** *"Products of local physical observables separate physical states" and "the local algebras generate the global physical algebra" coincide in all models here (the compressed part algebras mutually commute).*

Two senses of "superselected." Every claim in this SM is tagged with its sense: the *kinematic* sense (exact-constraint invariant algebra: which coherences no part-local physical operator can see) versus the *operational* sense (which labels behave as superselection rules at witness order once internal reference frames are available [14–16]). The two senses come

apart for gravity precisely where the Letter says they do (Sec. E); conflating them produces contradictions that are verbal, not physical.

QED CONTROL (FULL)

Theorem S 1 (QED classification)**.** *With the Gauss law imposed exactly, on every geometry,* $\mathrm{span}\{AFB\} = \{Q_1, Q_2\}'$ *exactly: chain* $27 < 121$*; two-path with photon mode* $383 < 1521$*; on the ring the span equals* $\{Q_1, Q_2, \Phi\}'$ *(holonomy flux* Φ *adjoined), dimension* $54 < 164 = \dim\{Q_1, Q_2\}'$*, the extra restriction being purely topological.*

Tier classification. Every deficit is a coherence between superselection-type labels. *Tier 0* (total charge): no operator away from the boundary detects total-charge coherence, the lattice shadow of the continuum charge superselection rule [12, 13]. *Tier 1* (inter-region charge coherence): a witness pair of charge-dressed states at trace distance 1 is identical on all products of locals to machine precision, separated only by the charge-transporting Wilson string with full contrast ($|\langle X\rangle_+ - \langle X\rangle_-| = 1.0$); local tomography returns the sector-dephased state (coherence deficit exactly 1/2). This is structurally the fermionic parity non-tomography of [3] with local parity replaced by local Gauss charge, the direction the loophole of [4] exploits. *Tier* $1'$ (topology): holonomy-flux coherences on the ring; absent in $\mathbb{R}^3$ and in BMV geometry (in the two-path model, plaquettes close the gap).

Edge modes / Gauss center. The lab's charge and its boundary flux are the *same operator* on $\mathcal{H}_{\mathrm{phys}}$ (checked exactly; the lattice-exact center structure of [6]). The center restricts *which sectors exist*, not in-sector composition.

Sector restoration. Within any joint (Q_1, Q_2) eigensector, parts compose as a standard tensor product (checked operator-by-operator: lab operators act as $\mathrm{op} \otimes \mathbb{1} \otimes \mathbb{1}$); arbitrary in-sector states, including matter–photon entangled states, are reconstructed exactly by linear inversion from product-of-local correlations (deviation 0 at machine precision). A two-lab mediation protocol coupled only through the field delivers lab–lab negativity 0.474 (at $t = 2.40$ in lattice units), remains exactly in one charge sector throughout (out-of-sector weight 0), and the entangled state is recovered exactly from product correlations; on the ring only the global holonomy phase is lost (topological, absent in $\mathbb{R}^3$).

Continuum caveats, carried. Charge superselection and non-localizability of charged states [12, 13]; the split property for QED; sector-wise reconstruction in the continuum is a

statement about the family of product correlations, any single truncated dressing attaining full contrast only asymptotically in its support.

THE LOOPHOLE ENGINE, EXACT ANATOMY

Faithful reproduction. The fermionic protocol of [4] is reproduced exactly [`vidal_loophole_bmv.py`, 21/21]: physical algebra dimensions $2^{2n-1} = 8/2/8$ for the three parts; the protocol delivers negativity 1.5 with the mediator diagonal throughout ($\max|\text{offdiag}| = 0$); the paper's Eq.-12 product witness value $\langle \hat{X}_1 \hat{X}_2 \rangle = -1/2$ is recovered.

Ingredients. (N1) a *locally classical mediator*: a commutative local physical algebra generated by its superselected local charge, with ≥ 2 locally distinguishable charge states; (N2) a *populated non-tomographic resource*: physical coherence between values of a joint sector label, here $\text{span}(\text{products}) = \{P_1, P_M, P_2\}' = 128 < 512$, and the mid-protocol transit pair is orthogonal (trace distance 1) yet identical on all products (deviation 0); (N3) *sector-pumping local dynamics*: gates that fail to commute with the hidden label.

Anatomy. Of the delivered negativity 1.5: a local particle-number twirl retains 0.25 (genuine in-sector entanglement); a parity twirl retains 0.50; the remaining 1.25 is charge-transfer coherence. The protocol's own product-observable witness $\langle \hat{X}_1 \hat{X}_2 \rangle = -1/2$ is reproduced *entirely* by the sector-dephased state: where the engine exists, the loophole fakes witnesses built from products of local observables, the escape "gravity may violate local tomography, but only harmlessly" is closed.

Theorem S 2 (Transport necessity)**.** *Physical circuits through a locally classical mediator whose gates conserve the mediator's charge mediate exactly nothing.*

Proof sketch. The mediator algebra is generated by its charge; charge-conserving physical gates therefore act on the mediator only through that charge and cannot build lab–lab coherence through it: a charge-conserving locally classical mediator is a GGS-classical system [2], and the classical-mediator no-go applies. Numerically: 40 random mediator-charge-conserving physical circuits give maximum lab–lab negativity 2×10^{-15} (float zero). The protocol of [4] evades this only by *transporting* the superselected charge, N3 is load-bearing. □

Convention audit. The script's mediator trace is a Jordan–Wigner qubit trace, not the ex-

act fermionic partial trace; the two differ by a local parity conjugation confined to unphysical odd $\otimes$ odd blocks. All physical (parity-even) product expectations agree exactly; both conventions give negativity exactly 1.5. The transfer-coherence *sign* is convention-dependent, itself an illustration of why those directions are non-tomographic.

GRAVITY CLASSIFICATION (PROOFS)

Kernel of the adjoint constraint map

Lemma S 3 (Charge content)**.** *Let $C^\dagger$ be the adjoint of the constraint map on smearing functions. For the first-order Gauss law,* ker $C^\dagger$ *on a region consists of the constants: one charge $Q(R)$. For the linearized Hamiltonian constraint,* ker *of $f \mapsto (\partial_i\partial_j - \delta_{ij}\nabla^2)f$ consists of the affine functions $\{1, x^i\}$, rank exactly 4 in 3+1 dimensions, giving region energy $E(R)$ and the energy-dipole/boost triple $K_i(R)$. The momentum constraint contributes* ker *of the symmetrized gradient = the flat Killing fields, rank exactly 6, giving $P_i(R)$ and $J_i(R)$.*

Proof. Tracing $(\partial_i\partial_j - \delta_{ij}\nabla^2)f = 0$ gives $(1-d)\nabla^2 f = 0$, hence $\nabla^2 f = 0$; substituting back, $\partial_i\partial_j f = 0$ for all i, j, so f is affine: the span of $\{1, x^i\}$, dimension 4 in $d = 3$. A vector field with $\partial_{(i}\xi_{j)} = 0$ has $\partial_i\partial_j\xi_k = 0$ by the standard Killing identity, so ξ is affine with antisymmetric gradient: translations plus rotations, dimension 6. Both ranks are also direct machine computations (window-rank census in 1D; explicit 3+1 rank computations in the verification suite). □

Each charge is a linearized-ADM surface integral on the region's boundary [10, 11]. Dimension-luck note: a scalar (Poisson-type) toy in higher dimensions would over-protect, its kernel contains all harmonic moments; it is the *tensor* structure that protects exactly the ten-plet, so the 1D affine kernel is the correct analog.

Main theorem

Theorem S 3 (Failure classification; Letter Eq. (3))**.** *In each model, exactly,* span$\{AFB\} = \{E(R_a), K_i(R_a), P_i(R_a), J_i(R_a)\}'_{a=1,2}$*, with every part-local physical operator conserving all region charges, part algebras equal to the commutant of their complement, and all charges exterior-collar field operators on $\mathcal{H}_{\text{phys}}$.*

Proof structure. (*Containment.*) Two lines: each region charge is a boundary term plus a multiple of the constraint (Regge–Teitelboim structure [11]); an interior-supported constraint-invariant operator commutes with the boundary term (disjoint support) and with the constraint (invariance), hence with the charge. Checked exactly, generator-wise, on integers. (*Saturation.*) The commutant dimension is the combinatorial $\sum_k n_k^2$ over joint charge blocks; the span rank equals it by mod-p rank at two primes (Sec. D), forcing equality over $\mathbb{Q}$. In the largest linear-order model: $\dim = 100 < 324 = \dim \mathcal{B}(\mathcal{H}_{\text{phys}})$. (*Duality and collar support.*) Verified exactly on both labs; each charge acts as a field operator supported on the exterior collar. □

Tier table

TABLE SI. Failure tiers, QED versus linearized gravity.

Tier	QED	Linearized gravity
0 (boundary)	total charge SSR	total $\{E, K\}$ superselected; "H is a boundary term" [9, 11]
1 (charge)	inter-region charge coherences	region-energy coherences (clock superpositions $\leftrightarrow$ charge superpositions)
1 (position)	*absent*	*gravity-only: position itself is charged* (energy dipole)

The position tier

A single mass in a superposition of two interior positions is invisible to every product of strictly local physical observables (deviation 0 at trace distance 1); local tomography returns the charge-dephased state (deficit exactly 1/2); the pair is detected only by a dressed displacement whose dressing reaches the boundary (contrast 2). In the QED control the same pair is locally distinguished (position is not charged under the Gauss law).

Lemma S 4 (Dressing obstruction)**.** *A matter rearrangement admits a compactly supported (interior) dressing iff its monopole and dipole vanish (gravity); monopole only (QED).*

Verified exhaustively over all 125 *source rearrangements on the GB lattice.*

This is the lattice-exact content of the gravitational no-screening property of Donnelly and Giddings [7] (our phrasing paraphrases their content; their $O(\kappa^2)$ spacelike dressing commutators, verified against the published paper, are consistent with what the models show). The recoil-compensated breathing flip $A \leftrightarrow B$ has a one-site interior dressing (residual 2×10^{-16}: an element of the lab algebra); the uncompensated shift $A \leftrightarrow C$ requires a step dressing to the boundary (residual 1.0 against the lab algebra).

Corollary S 1 (Classicalization of small systems)**.** *A region too small to host internal multiplicity at fixed charges has an abelian physical algebra: constrained linearized gravity classicalizes isolated small systems. In-sector quantumness requires charge-degenerate internal multiplicity (recoil pairs, spins).*

Non-occupancy and the operational disanalogy

Theorem S 4 (BMV non-occupancy; Letter Theorem 2)**.** *Every part-local physical operator conserves every region charge exactly (the two-line containment proof above). Hence all branch-dependent operations generated in the labs' interiors keep the four BMV branches* $\{LL, LR, RL, RR\}$ *in a single joint charge sector; at matter-source order the protocol never occupies the non-tomographic directions. In the GB model this is verified concretely: all four branches lie in one joint sector with recoil accounting, and the two lab-1 branches have identical field profiles outside the lab.*

(The "identical exterior profiles" clause is itself a 1D-affine artifact; the correct general statement is "no which-branch record in *protected* boundary data." In 3D, equal-(E, K) branches still differ in unprotected exterior multipole tails, which is exactly how the in-sector phase arrives; Sec. D.)

Operational disanalogy (why energy is not charge). Three exact facts. (i) The one mapped operational SSR (matter number/parity) is branch-independent at BMV: the local-number twirl acts as the identity on the witness state. (ii) A referenceless energy-superselection argument proves too much: an energy twirl kills ordinary gravity-free Ramsey fringes ($0.454 \to 0$); interior clock coherences live at fixed region charge via internal reference frames [14–16]. (iii) Unlike Gauss charge, regional energy is pumped by the joint coupling

itself: $||[V_{\text{Newton}}, H_{\text{lab}}]|| = 0.57 \neq 0$, energy flux needs no matter crossing, charge flux does. This is a structural disanalogy cutting toward *less* usable sector structure than QED's. The kinematic conservation theorem and these operational facts are statements about different objects (Sec.); both are exact, and the Letter's verdicts hold in both senses.

SECTOR RESTORATION AND IN-SECTOR CERTIFICATION

Within a joint eigensector of all region charges, composition is a standard tensor product and tomography is exact: on the BMV composite of `vidal_loophole_bmv.py` the product span is complete (576/576), and arbitrary in-sector states are reconstructed by exact linear inversion from product correlations (maximum deviation 1.4×10^{-16}), including a field-excited, matter–field-entangled state at TT coupling $g_{\text{rad}} = 0.4$ ($\langle n \rangle = 0.149$, matter–field negativity 0.335). In the gravity models proper [`gravity_local_tomography.py`]: arbitrary states in the fixed-charge sector of the GB model ($2 \times 2 \times 2$ including TT entanglement) are reconstructed exactly (deviation 0), and the GN spin model's in-sector freedom (the spins, span 64 of 256) is exactly the witness readout.

At linear order the field carries a genuine noncommutative local invariant algebra, the TT sector, the analog of $F_{\mu\nu}$ ($||[x, p]|| = 4$ on the TT qubit), which is what makes the mediator a self-contained tomographic part. Beyond linear order no local invariants exist at all [7, 8]; the certificate is linear-order-only.

GGS in-sector. The no-go theorem of [2] applies in-sector with full force: a classical field mediator (three-state diagonal carrier, 20 random 6-round mediated circuits) delivers maximum negativity 4×10^{-16}; a quantum mode mediator through the same round structure delivers 0.500 [`vidal_loophole_bmv.py`; the TT-mediated closed-lab protocol of `gravity_local_tomography.py` independently gives final lab–lab negativity 0.500 in one sector, reconstructed exactly from products]. License note: in the constrained description the in-sector tensor structure is licensed by the closed-lab conservation theorem (Sec. E), not by postulating an unconstrained tensor product.

TRANSIT, SELF-FLAGGING, AND THE REAL APPARATUS

Engineered transit

Branches engineered to differ in a region charge (dipole-differing A/C-type branches) acquire a charge-coupled second-difference phase ($\Delta_2 = 16$ exactly in the GB toy). Mid-protocol, the entangling coherences are invisible to *every* product of local physical observables (product deviation 0; coherence weight at trace distance 0.75) while the product-visible state carries zero entanglement, mediated entanglement *can* transit the non-tomographic directions.

Self-flagging

Such transit (a) broadcasts a classical which-branch record into *protected* exterior data (the branch label is readable at the boundary); (b) requires split/recombine unitaries that are boundary-anchored strings, not lab-local, the spin-conditioned splitting unitary has residual 0.707 against the lab algebra, the uncompensated shift dressing residual 1.0; and (c) its mid-flight coherences are tomographically nonlocal (TNL), hence unusable for Bell/steering-grade readout, while the final BMV entanglement is tomographically local (TL). The TL/TNL import is scoped: what is used here is the sector-dephasing projector mechanism instantiated in these models (the mid-protocol state agrees with its sector-dephasing on all products, and the dephased state is unentangled); the general GPT theorem is [5]'s and is cited, not re-derived, instrument-level closure for constrained field theory is deferred to a companion paper and not claimed here.

The unique $O(1)$-transport channel

The only $O(1)$-transport channel on the BMV composite is coherent matter transfer through the field region. It survives with gravity switched off (certifying nothing about the mediator) and is exponentially suppressed by tunneling bounds through the inter-lab barrier, as established independently in [24, 25]; assumption A2 of the Letter excludes it at that published level.

In-sector phase reality

The 1D models' equal-charge branches acquire exactly zero in-sector phase, a declared artifact of the affine kernel, which couples only to the protected moments (E, D). That the real in-sector phase survives protection is verified directly [`insector_phase_3d_check.py`, 9/9; committed re-materialization of the original verification-suite check]: collinear labs at separation $d = 10$ (lattice units), proof mass $m = 1$ split to ± 0.3, compensator mass $M = 4$ recoiling about a pivot offset $p = 1.05$ from the lab center, so per-lab (E, D) are branch-independent *exactly* (Fraction arithmetic) while the branch difference carries a nonzero quadrupole. Results: the compensated pair acquires a 3D $1/r$ second-difference phase -1.7×10^{-4}, same order as the uncompensated (dipole-distinct) reference -7.2×10^{-4}; the 1D affine kernel gives exactly 0 (Fractions) for both; far-field scaling exponents confirm the mechanism (compensated $\sim d^{-5}$, quadrupole–quadrupole; bare $\sim d^{-3}$, dipole–dipole). The 3D kernel couples to unprotected multipoles (quadrupole and up), the in-sector phase with closed labs is real, and this link rests on computation, not extrapolation.

The real apparatus (criterion A1′)

Laboratory Stern–Gerlach splittings are Earth-anchored: recoil compensation lodges outside any lab-sized region, so real branches are weakly dipole-distinct and real splitting unitaries weakly boundary-anchored, formally in the weak transit class. The operative protection is *erasability*: the exterior record is dressed and slaved (recombinable, false decoherence in Unruh's sense [23]), not radiated or amplified, the distinction made sharp by Danielson, Satishchandran, and Wald [19, 21]. Operational certificate: observed $O(1)$ interference visibility certifies that no resolvable branch record formed. A1′ is a criterion with an operational certificate and a quantitative floor (Sec. G), not a theorem.

THE $O(G)$ DISSOLUTION (FULL)

Continuum scheme analysis

[`continuum_og_energy_scheme.py`, 27/27 as run; two struck legs documented in B below; `improvement_current_bound.py`, 8/8.]

Linear order (continuum-native confirmation of the lattice theorem). For any closed surface ∂R, the linearized ADM charge $E(R) = (c^4/16\pi G) \oint (\partial_j h_{ij} - \partial_i h)\, dS_i$ is invariant under all linearized diffeomorphisms: the gauge shift of the integrand is identically $-[\nabla \times (\nabla \times \xi)]_i$, and the closed-surface flux of a curl vanishes (numeric anchor: lab-sphere charge $= mc^2$ to 10^{-5}).

The scheme family (exact shares). Admissible energy localizations, densities differing by superpotential divergences [27, 28], agree exactly on the total $-Gm_1m_2/d$ and disagree at $O(1)$ on every region share:

- *Field scheme* ($-|\nabla\phi|^2/8\pi G$ cross term): the share inside a mass-centered ball of radius a is exactly $\max(0, 1 - d/a)$, zero for every lab-sized ball; the share beyond radius r falls exactly as d/r, so half of δE lies beyond $r = 2d \approx 1\,\mathrm{mm}$.

- *Source scheme* $\frac{1}{2}(\rho_1\phi_2 + \rho_2\phi_1)$: half in each lab. *Asymmetric scheme* $\rho_1\phi_2$: all in one lab.

- *Stress-inclusive (Tolman) schemes*: routed through the holding hardware, exact static virial $\int T_{kk} = +Gm_1m_2/d$ (symbolic, `sympy`).

- *Divergence transport, constructive*: with $u' = u - \nabla\cdot j$, $j = f(r)\hat{r}$ about the lab center, $f(r) = -\delta E/4\pi r^2$ on $[r_0, R_{\mathrm{out}}]$ (ramped to zero inside r_0 and beyond R_{out}), the ball share of every sphere $r_0 \le r \le R_{\mathrm{out}}$ shifts by exactly δE at exactly zero total change, with $\max|f| = \delta E/4\pi r_0^2 = 1.2 \times 10^{-28}\ \mathrm{J/m^2}$ at the Bose geometry ($m = 10^{-14}\,\mathrm{kg}$, $d = 450\,\mu\mathrm{m}$, $r_0 = 100\,\mu\mathrm{m}$). The entire interaction energy crosses any lab boundary under a bounded current [`improvement_current_bound.py`; this replaces an in-script placeholder check, see B].

A quantity with no scheme-independent region-level value is not a region observable of the constrained theory; a non-observable cannot superselect. The invariant residue: the asymptotic charges, branch-independent for the closed protocol, and the second difference $\Delta_2 E_{\mathrm{int}}$, which *is* the witness phase (0.63 rad at $200\,\mu\mathrm{m}/2\,\mathrm{s}$; 0.31 rad for the published geometry [1], branch phases $\Delta\phi_{RL} = 0.439$, $\Delta\phi_{LR} = -0.126$).

Struck legs, documented (correction history)

- The route's S2 gauge-orbit numerics were finite-difference noise passing their own tests for the wrong reasons; the originally reported $\xi^* = 5.2\times10^{-36}\,\mathrm{m}$ and $|\partial\xi|^* = 2.6\times10^{-31}$ are *retracted*. Corrected statement (all the argument needs): a collar diffeomorphism of amplitude $\xi \sim 3 \times 10^{-43}\,\mathrm{m}$ already shifts a truncated finite-region reading by the full δE_{int} scale, while the asymptotic charge shifts by exactly 0, the finite-region truncated charge is gauge-soft, the boundary charge rigid; invariantly completed quasilocal charges respond to surface moves only at geometry scale. Effective verified count: 25/27 sound + 2 struck; the Letter quotes only corrected numbers.

- The in-script divergence-transport check was a placeholder; replaced by the constructive bounded improvement current above (now the committed `improvement_current_bound.py`).

- Joint-sector reachability for the equal-shift BMV protocol was corrected from $\{1, 2, 4\}$ to $\{1, 3\}$ (LL/RR always energy-degenerate there); see C for the protocol-dependence of counts.

- A slogan the route leaned on ("energy differences are phases unless a third system records them") is false as a general principle in constrained dynamics, QED charge coherence is the counterexample (the constraint itself is the recorder when the label is central). The verdict never uses the slogan; it rests on the trilemma (F) and the chain (H).

The lattice $O(G)$ algebra route

[`og_sector_algebra.py`, 37/37; adversarial verification `og_sector_algebra_verify.py`, 22/22.]

Construction. The corrected constraint $\Delta^2\pi = \rho + g\,\varepsilon(\pi)$ is solved exactly at first order in the formal parameter g (integer arithmetic). The discretization of ε is the operator-ordering axis: `SITE` ($\varepsilon_i = \pi_i^2$), `SPLITL` ($\pi_i\pi_{i+1}$), `SPLITR` ($\pi_{i-1}\pi_i$); the verifier adds symmetric point-split and second-neighbor. The assignment (superpotential) axis at fixed discretization: `NATIVE` (region gets $\sum_{i\in R}\varepsilon_i$), `SRCSYM` (source-symmetric), `SRCASYM` (leftmost source), `BOUNDARY` (all to the exterior collar), the last made *constructively* admissible by an explicit

bounded lattice improvement current (a pure discrete divergence telescoping across every region boundary at zero total change, $\max|\delta| = 1040 \leq 1840$; the lattice analog of the continuum current above). The corrected charge is $E_R^{(2)} = E_R + g\, W_R^{\text{scheme}}$; sectors at generic formal g are labeled by the joint tuple.

Results (all exact; dimensions mod-p at two primes, re-certified by the verifier at $10^9 + 7$ *and* $2^{31} - 1$*):*

- The linear-order theorem re-proves: span $= \{E, D\}' = 100$.
- The *corrected* commutant is 36 under every interior scheme (all five discretizations) and 100 under `BOUNDARY`: the commutant itself moves with the scheme. Per fixed scheme the machinery survives formally: rebuilding the local algebras with the corrected charge gives span $= \{\text{corrected}\}' = 36$ exactly.
- *Strictness*: the breathing hop $X_1 = |B\rangle\langle A| + \text{h.c.}$ is literally an element of the linear-order lab algebra (exact matrix match), conserves (E, D) exactly, and fails to commute with the lab's corrected charge: products of local physical operators *connect* different corrected "sectors."
- *Centrality/conservation failure*: $[E_R^{(2)}, H_{\text{hop}}] \neq 0$ exactly (maximum commutator entry $2 = |W(A) - W(B)|$): $O(G)$ energy flux across the lab boundary under interior dynamics is real; corrected-charge coherences are ordinary, erasable coherences, and product-visible with contrast exactly 1 (no operational superselection).
- *Witness pair* (pre-derived, M1b): lab patterns $X = (5, 2, 7, 0)$, $Y = (0, 13, 0, 1)$ with equal $(E, D) = (14, 44)$ per lab: equal `SITE` self-energy 845 per lab site-sum ($2W = 1690$), unequal `SPLITL` ($2W = 2824$ vs 2756, i.e. 1412 vs 1378): whether the branch states even have equal internal energy is discretization-dependent. Under `SITE/NATIVE` lab-2's charge carries the record (83450 vs 84706) and lab-1's is branch-constant; under `SITE/SRCASYM` the record relocates to lab 1 (143074 vs 144330): the same branch distinction is carried by different regions under admissible reassignment.
- *Sector counts*: BMV-faithful kernel setup (rigid masses, mandatory trap-work ledger, internal/external agent placement, exact rationals): reachable joint-sector counts across schemes are exactly $\{1, 3\}$ for the equal-shift protocol; with unequal branch

shifts $\{1, 4\}$; the lattice conformation-branch setup reaches $\{1, 2, 4\}$. Counts are protocol-dependent; the dissolution (count-1 schemes reachable; no invariant splitting) is geometry-independent.

- $\Delta_2 E_{\text{int}} = -\frac{1}{10} + \frac{1}{11} + \frac{1}{9} - \frac{1}{10} = +\frac{1}{495}$ exactly (scheme-invariant), annihilating every per-lab-additive ledger $f(b_1) + g(b_2)$: the pump is the only invariant branch-difference energy object; every scheme's total ledger is exactly zero on every branch (per-branch total energies equal).

- *Negative search (decisive)*: the one result that would have established a genuine $O(G)$ sector, an invariant splitting surviving both scheme axes, was searched for explicitly, under the verifier-extended family (five discretizations, fresh geometries, fresh primes, extended kernel schemes), and does not exist.

The honest wrinkle (native-only splitting)

Restricted to native-only discretizations, refusing all superpotential reassignment, a partner-record splitting survives all five lattice discretizations. It is removed by the constructively admissible assignment axis, and is independently disqualified as superselection by the kills that never use that axis: the label is non-central, not conserved, and its coherences are product-visible with full contrast. Bookkeeping, not superselection, exhibited, not buried. Related scope notes: "every admissible ledger is a function of pair distance alone" holds for the imported comoving/relational scheme subclass (a fixed-partition-surface field scheme would generically break the LL/RR degeneracy, direction of error dissolution-safe); the kernel model's field-scheme shares are imported from the continuum closed forms of A, not re-derived; the lattice TT-zero is partly structural (exactly diagonal 1D constrained channel), so the quantitative record bound rests on the continuum computation (G).

QED $O(\alpha)$ contrast (mechanism split)

The Abelian Gauss law has no quadratic field source (the photon is uncharged): $\varepsilon_{\text{QED}} \equiv 0$ structurally, and the region charge stays the exact, quantized matter charge at $O(\alpha)$, central, conserved by every local physical operation (including the interior dressed hopper), branch-

independent in mediation. The same hopper fails to conserve the Coulomb *energy* label, which refines and scheme-moves exactly like gravity's W: energy bookkeeping misbehaves identically in both theories; the difference is solely which object the constraint promotes to a charge, Q (exact, central) in QED, energy (scheme-soft) in gravity. QED protects mediation witnesses by *conservation*; linearized gravity at $O(G)$ protects them by *dissolution*.

The trilemma

A sector label able to decohere the branches must be (i) gauge-invariant, (ii) central/conserved under all local physical operations, and (iii) branch-dependent. Every candidate satisfies at most two (Table SII).

TABLE SII. No candidate label occupies all three corners.

Candidate	invariant	central	branch-dependent / disposition
Asymptotic ADM charges	yes	yes	*no*: per-branch totals equal to 10^{-70} relative; battery record exponent $(\delta W/\Delta E_{\rm res})^2/8 \approx 2\times10^{-40}$
Scheme densities	*no*	,	yes: $O(1)$ share disagreement (A); bounded divergence transport reaches any assignment
Fixed quasilocal (Brown–York/Wang–Yau [17, 18])	yes	*no*	yes: non-unique at O(field energy); not conserved, $[E_R^{(2)}, H_{\rm hop}] \neq 0$ exhibited operator-level
Soft/IR sectors	yes	yes	*no*: branches recombine in position and velocity, net memory zero; horizon-regularized version $\sim 2\times10^{-149}$
Non-abelian ten-plet Casimirs	?	?	no mechanism found; the declared open refuge (Sec.)

The radiated record (DSW) computation

By the criterion of [19–22], the irreversible branch record is the radiated (or horizon-crossing) graviton content. For the published BMV parameters [1] ($m = 10^{-14}\,$kg, $d =$

450 μm, $\Delta x = 250\,\mu$m, C^2-smooth quintic ramps of 0.5 s, hold 2.5 s), with branch-difference quadrupole $\delta Q = 2m\,d\,\Delta x\,w(t)$ and the exact profile integral $\int_0^1 (w''')^2 du = 720$ (symbolic; re-derived by hand by both verifiers):

- $N_g = 1.3 \times 10^{-57}$ (mass motion), visibility loss $1 - V \approx N_g/2 \approx 6 \times 10^{-58}$; frequency-resolved cross-check (verification suite) $\int (dE/d\omega)/\hbar\omega\, d\omega = 5.0 \times 10^{-58}$.
- Robustness (verification-suite scans): C^3 septic profile $\times 1.56$; one-sided-split worst pair $\times 0.20$; $\bar{\omega}$ convention $\times 2$.
- The static hold radiates exactly nothing (no horizon $\Rightarrow$ no secular growth); branch-difference tidal kinematics during the hold give a recombination-mismatch exponent $\sim 10^{-17}$.
- Worst-case branch-correlated apparatus-recoil lever: N_g scales by $(L/d)^2$; $L = 1\,\mathrm{m} \to 6.4 \times 10^{-51}$; $L = 1\,\mathrm{km} \to 6.4 \times 10^{-45}$, still ≥ 44 orders below $O(1)$.
- Cosmological-horizon channel (de Sitter, order estimate at cited coefficient class): $\sim 2 \times 10^{-149}$.
- Sudden-switch artifact class: $\sin^2$-type ramps have jerk deltas at the joins and their $\int (w''')^2$ is grid-divergent; C^2 protocols are mandatory in any lattice/numeric route, and no fixed inflation factor is quotable.

Independent anchor: a recent computation of graviton which-path decoherence at BMV parameters finds $\Gamma_{\mathrm{vac}} = 8.7 \times 10^{-61}$ on a different stage, the same order class [26]; we cite it and claim no novelty for the magnitude. Where an $O(G)$-sector reading needs $O(1)$, the invariant record supplies 10^{-57}.

Reconciling $\delta E\, t/\hbar = \Delta\phi$

Three steps, none using the struck slogan of B:

1. *The candidate recorder does not exist as an invariant*: the per-region $O(G)$ energy label is not a Dirac observable (A, C); the constraint-as-recorder mechanism, real in QED, real for gravity's linear-order ten-plet, has nothing to hold at this order. The

identity $\delta E\, t/\hbar = \Delta\phi$ is a statement about the scheme-invariant $\Delta_2 E_{\rm int}$, not about any region charge.

2. *The labels that are invariant and central are branch-independent*: exact coherent-state battery algebra, branch work δW booked into a reservoir orthogonalizes iff the agent energy-resolves it; at BMV numbers $\Delta E_{\rm res} = 8\times10^{-16}\,\mathrm{J} \gg \delta W = 3.3\times10^{-35}\,\mathrm{J}$, record exponent $\approx 2\times10^{-40}$ (a 300 K thermal reservoir records 10^{-50}-class). Per-branch total energies are equal (conservation, 10^{-70} relative) while the configuration sector keeps the full relative phase: energy differences between branches generate phase by the Schrödinger equation, every Ramsey fringe is a coherent superposition of different-energy branches.

3. *What remains branch-dependent and invariant is not central*, and for it the radiated-record criterion is the right test: the branch label lives entirely in unprotected, erasable multipole data (Secs. E, D); constraint-slaved dressing recombines (false decoherence [23]); the irreversible residue is the radiated content of G: 10^{-57}.

The constraint-slaved field is the battery reservoir writ large: the identity prices the *phase*; the record is priced separately, by radiation, and costs 57 orders less. This is also the content of the Letter's experiment-facing corollary: quantized gravity plus erasability predicts zero intrinsic visibility loss at fixed $\Delta\phi$ within any conceivable systematics, while classical-channel models [29, 30] tie an $O(1)$ irreversible-noise rate to the entangling rate.

CLAIM → EVIDENCE TABLE

Table SIII lists every number quoted in the Letter with its producing script and arithmetic class (I = exact integer, P = mod-p certified, F = exact rational/`Fraction` or symbolic, D = float demonstration). All scripts re-run fresh on 2026-08-03 with exactly the quoted self-test counts, exit 0. Where a class letter I or P attaches to a linear-order span/commutant rank, the committed Q/G/V scripts compute that rank in floating point on exact matrices and the exact certification is the verification suites' (§I D); the dimension-100 geometry additionally has its committed exact certificate in `og_sector_algebra.py`. Closed forms credited to C that are not `sympy` outputs of the committed script (the field-scheme share and tail) are verification-suite derivations numerically confirmed by C at multiple radii.

HONEST-LIMITS LEDGER

- *Continuum lifts* of the lattice theorems are standard-but-unproved, except linear-order per-region ADM invariance (continuum-native, §VIII A). Charge superselection and dressing non-localizability carry their known continuum scope [12, 13]; the split property for QED; a single truncated dressing attains full contrast only asymptotically.
- *Non-abelian ten-plet in-sector structure* ($[K_i, E] \neq 0$: sectors are irreps, not joint eigenvalues): computed nowhere; the designated refuge of the loophole; parametrically beyond the witness phase, but open. Only the energy member of the ten-plet is corrected at $O(G)$ in §VIII.
- *Declared 1D artifacts*: affine kernel (exact in-sector zero, §VII D); exactly diagonal constrained channel; TT mode added by hand. Lattice TT excitation exactly zero is partly structural, so the quantitative record bound is the continuum's (§VIII G).
- *Raju scope*: holography-of-information statements proved for asymptotically flat and AdS; compact slices open [9]. Donnelly–Giddings "no-screening" is a content paraphrase [7].
- *TL/TNL import* at GPT level [5]; instrument-level closure for constrained field theory deferred (§VII B).
- *A1′* is a criterion with an operational certificate, not a theorem (§VII E).
- *Parameter conventions*: 200 μm/2 s (design point) versus 450 μm/2.5 s (published [1]), both stated, each regression-consistent with its source.
- *Sector counts* are protocol-dependent ($\{1, 3\}$ equal-shift; $\{1, 4\}$ unequal; $\{1, 2, 4\}$ conformation branches); the dissolution is geometry-independent.
- $O(G)$ *claims* are at first order in formal g; no finite-g resummation claim.
- *The native-only wrinkle* (§VIII D) is exhibited, not buried.

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TABLE SIII. Claim-to-evidence ledger. Scripts: Q = `qed_local_tomography.py` (23/23), G = `gravity_local_tomography.py` (29/29), V = `vidal_loophole_bmv.py` (21/21), C = `continuum_og_energy_scheme.py` (27/27 as run; two struck legs documented in §VIII B), O = `og_sector_algebra.py` (37/37), Ov = `og_sector_algebra_verify.py` (22/22), Ph = `insector_phase_3d_check.py` (9/9), Im = `improvement_current_bound.py` (8/8).

Claim (Letter location)	Value	Script	Class
Vidal anatomy split	$1.5 = 0.25 + 1.25$; parity twirl 0.50	V	D (exact values)
Faked product witness	$\langle X_1 X_2 \rangle = -1/2$ from dephased state	V	D (exact value)
Transport necessity	$\leq 2 \times 10^{-15}$ over 40 circuits	V	D
QED span theorem	span $= \{Q_1, Q_2\}'$, all geometries	Q	D (I+P: suites, §I D)
QED mediation	negativity 0.474, one sector	Q	D
3+1 kernel ranks	4 (affine), 6 (Killing)	G + §V A proof	proof / D
Main theorem, Letter Eq. (3)	span $= \{\text{ten-plet}\}'$, dim 100	G, O	I+P (via O)
Position tier	dev 0 at TD 1; 125 rearrangements; monopole+dipole	G	I / D
In-sector completeness	576/576; reconstruction $\sim 10^{-16}$; $\langle n \rangle = 0.149$, $N = 0.335$	V (G: sector recon.)	D (§I D)
GGS in-sector	4×10^{-16} vs 0.500	V, G	D
L/R contrast	2.00	V	D
Ramsey reductio	$0.454 \to 0$	V	D
Energy pumping	$\lvert[V_{\mathrm{N}}, H_{\mathrm{lab}}]\rvert = 0.57$	V	D
Transit toy	$\Delta_2 = 16$; residuals 0.707/1.0	V, G	I / D
3D in-sector phase	-1.7×10^{-4} vs -7.2×10^{-4}; 1D exactly 0	Ph	D / F (1D zero)
Newtonian sector	OS rank 2; $N = C/2 = 0.0782$; $1 + C = 1.1563$	V	D
δE, phases	3.3×10^{-35} J; 0.63/0.31 rad; 0.439/ −	C	D